MODERN ELECTRIC PRACTICE

Prepared under the Editorship of

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NEW EDITION

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PREFACE

The wide subject of Modern Electric Practice is now so extensive that no single writer could hope to treat it comprehensively in a book which would at once be suitable for the use of practical engineers and serviceable to students attending Technical Colleges and Polytechnics. Accordingly I have obtained for this treatise the cooperation of contributors, each of whom treats of work in his own department of study and practice.

The Book is divided into two sections, as follows:

SECTION I.—THE MAKING OF GENERATORS, TRANSFORMERS, [illegible] AND [illegible]

1. [illegible] of Direct-current Machines
2. Alternating-current Machines
3. Direct-current Power Generators
4. Alternating-current Generators
5. Direct-current Motors
6. Alternating-current Motors
7. [illegible] Transformers and Rotary Converters
   [illegible] Testing

SECTION II.—ELECTRIC LIGHTING AND ELECTRIC POWER DISTRIBUTION

1. [illegible] and Gas
2. Electric Mains
3. Distribution and Measurement of Electric Energy
4. Electric Fittings and Wiring
5. Electric Lamps
6. Arc Lamps
7. [illegible] Lamps

SECTION III.—ELECTRIC [illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
   [illegible]
8. [illegible]

SECTION IV.—BOILERS AND PRIME MOVERS—

1. Steam-Boilers.
2. Steam-Engines and other Prime Movers.
3. Feed-Pumps, Condensers, Economizers, &c.

SECTION V.—MISCELLANEOUS APPLICATIONS OF ELECTRICITY—

1. Electrochemistry and Electrometallurgy.
2. Telegraphy.
3. Telephony.
4. Electricity in Mining.
5. Electric Cranes.
6. Electro-Medical Appliances.
7. Modern Electrical Appliances employed in the operation of Railways.

The question of notation and units has been considered, but I thought it, meantime, advisable not to restrict the contributors to any fixed set. Hence rises of temperature are given sometimes in degrees Fahrenheit and sometimes in degrees Centigrade; dimensions of machines occasionally in feet and inches, but more often in centimetres; magnetic flux density in lines per square inch in one article, and in lines per square centimetre in another. The relations between the C.G.S. system of units and the F.P.S. system of units will be found on page 5 and following pages.

In the case of one or two of the topics discussed I have retained the slight overlapping of treatment observable, deeming it rather an advantage in these particular instances, as each contributor deals with the subject from a different point of view, and hence to the benefit of the reader.

My cordial thanks are due to the authors for their hearty co-operation, and for their readiness to accept suggestions made with a view to the symmetrical arrangement and completeness of the whole treatise.

I have also to acknowledge gratefully the willingness with which manufacturers supplied prints, tracings, and blocks of the various pieces of apparatus described in the text.

MAGNUS MACLEAN.

PREFACE TO SECOND EDITION

In the comparatively short period since *Modern Electric Practice* was first published new facts have come to light, and considerable progress has been recorded in various departments of Electrical Engineering. Among other changes may be noted the introduction of the high-speed turbine-driven generator, the recently improved types of switch gear and of electric illuminants, and the central battery system of telephony.

So many advances and of so important a character have rendered desirable a second edition of this work. In view of these facts and of the gratifying success that has attended the first issue, the publishers have given full scope for the revision of the whole treatise. It has therefore been my aim to embody the new information available, in order to bring the different volumes up to date and render them as serviceable as possible. In consequence, considerable additions have been made to most sections of the work, while not a few of the articles have been entirely rewritten.

Of the additions, mention may here be made of two chapters in the section dealing with current generators—one on commutating-pole machines, the other on turbo-generators; a chapter on motor generators, and a concise treatment of single-phase commutator motors. Telegraphy and telephony have been further expanded—the former in a chapter on radio-telegraphy, the latter in a full description, with numerous illustrations, of the central battery system, while a new chapter dealing with mining cables and accessories appears in the article devoted to electric mains.

So many improvements have been made in switch-gear apparatus, that the article on this subject has been rewritten and new illustrations introduced. That [illegible] mission of electric energy has been [illegible] information regarding modern [illegible] schemes adopted. Details of [illegible] in the article on incandescent lamp[illegible]

While the chapter on electric [illegible] those on steam turbines have been [illegible] tion dealing with Motive Power [illegible]

It is hoped that this work [illegible] will prove more valuable than ever [illegible] Practice.

TECHNICAL COLLEGE,
GLASGOW, *January*, [illegible]

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Section I

he Measuring, Generating, Transforming and Storing of Electric Currents

1. Electric and Magnetic Measurements

CHAPTER I

MAGNETIC UNITS

Introduction.—Perhaps not the least striking feature of our present knowledge of electromagnetic phenomena, and of the numerous important applications of that knowledge to electric lighting, electric traction, electrical power transmission, telegraphy, telephony, and many other branches of engineering, is the comparative suddenness with which such knowledge has been acquired and applied to useful purposes. Very little reflection is needed to convince the most unimaginative of persons that the conditions of modern everyday life would, to one living about a century ago, have appeared well-nigh impossible, and resembling the product of some wild romancer's dream rather than the facts of sober reality. Yet the wonderful transformation witnessed by the nineteenth century has been accomplished within a period whose duration, regarded from the historical point of view, is almost negligible. It is practically to a single century that we owe the existence of modern electrical science.

The knowledge of the ancients regarding electrical and magnetic, and, indeed, all physical phenomena, was extremely limited. The use of the mariner's compass in the far East is said to date very far back, although in Europe it seems to have been unknown until about the twelfth century. The magnetic properties of the lodestone and artificial magnets, and the electrical properties of amber when subjected to friction, were known to the ancient Greeks and Romans. During the Middle Ages, wild speculations regarding electrical and magnetic phenomena were rife, and very little advance was made until the appearance, in the sixteenth century, of Dr. Gilbert of Colchester's great treatise, *De Magnete*, which laid the foundation of the modern sciences of electricity and magnetism. In 1791 Galvani published a book in which he gave an account of his remarkable researches on the physiological effects produced under certain conditions by weak electrical stimuli. The year 1800 is memorable as the date of Volta's epoch-making discovery, and his invention of the "crown of cups". The phenomenon of the electric arc became known soon afterwards. In 1820 Oersted discovered the magnetic effect of the electric current, and this was followed by the discovery, in 1822, of thermo-electricity by Seebeck. In 1827 Ohm published the law known by his name. The year 1831 ranks as one of the most important in the history of electrical discovery: two brilliant experimenters—Michael Faraday in England and Joseph Henry in America—independently and almost simultaneously discovered the fact that an electric current could be produced by other than chemical means, namely, by the relative motion of a magnet and a conductor. It is a knowledge of this latter fact that has rendered the production of large electric currents commercially possible.

Useful practical inventions soon followed in the wake of the brilliant scientific discoveries to which we have briefly referred. In 1825 Sturgeon constructed the first soft-iron electromagnet. In 1837 Cooke and Wheatstone introduced their needle-telegraph. The art of dynamo construction received a powerful impulse in 1870, when Gramme introduced his machine with a smooth-core ring-wound armature. The invention of the first telephone by Philip Reis in 1861, of the magnetotelephone by Graham Bell in 1876, and of the microphone by Hughes in 1878, laid the foundations of modern telephony. In 1879 Swan succeeded in constructing the first carbon glow-lamp, thus solving the hitherto unsolved problem of the "subdivision" of the electric light. Within the last quarter of a century developments have taken place in the various practical applications of the electric current which have not only completely revolutionized many existing branches of engineering, but have also created new industries, whose rapid progress is unparalleled in the history of applied science.

This rapid progress is undoubtedly due to the use, almost from the birth of the electrical industry, of scientific methods and a scientific system of units, both of which have largely contributed to the advance of electrotechnology by rendering accurate measurements possible. The introduction (in 1832) of the present system of "absolute" units we owe to Gauss. Gauss's proposal was followed up by W. Weber, who devised the two systems of units known as the "electrostatic" and "electromagnetic" ones. Weber's work was strongly supported and extended by William Thomson (afterwards Lord Kelvin), and it was largely owing to his exertions that a committee of the British Association was appointed in 1861 for the purpose of dealing with the subject of electrical units.

The importance to every student of electrical science of an exact knowledge of electrical units and their mutual relationship can hardly be over-estimated. In what follows we shall attempt to furnish the reader with a connected account of the present practical system of electrical units, to explain how the more important electrical and magnetic measurements may be carried out, and to describe the instruments employed in such measurements.

Dynamical Units in the C.G.S. System.—In order to facilitate calculations it is extremely convenient to choose our system of physical units so that there is a definite simple connection between them, and that the use of awkward multipliers and divisors is eliminated. Not only is the purely arithmetical part of many problems thereby simplified, but the memory is relieved of the burden of numerous unnecessary "constants". Our meaning will be rendered clear by a simple example. In the British system, if we take the unit of volume to be a cube whose edge is 1 foot, then the mass of a unit volume of water will be 62.4 lbs. Hence an engineer, in dealing with hydraulic problems, always has to bear in mind the constant 62.4, which reduces cubic feet to lbs. of water. In the metric system, however, the unit of mass is taken to be that of unit volume of water, and the use of an awkward factor is thereby done away with. The same principle is followed throughout in the system of magnetic and electrical units known as the "C.G.S. electromagnetic" system.

In building up a system of physical units on this principle, it is found necessary to choose arbitrarily *three* units as a basis of the system; all the other units may then be very simply derived from the three whose magnitudes have been arbitrarily fixed, and which are called the three *fundamental* units. If we attempted to choose more than three, say four, units *arbitrarily*, then we should find that such a random choice of the magnitudes of four units would no longer enable us to maintain the simplicity aimed at, and inconvenient constants would be introduced in referring the other units to the four chosen at random. *Any* three units might be selected as the fundamental units; but for the sake of simplicity and convenience, the three units which have been adopted as *fundamental units* are those of *length*, *mass*, and *time*. Not only are these units known to everybody, but measurements in terms of them are, in common everyday life as well as in scientific investigations, more frequent than any other measurements. All other physical units may now be derived from the three fundamental units, and are for this reason termed *derived* units.

The unit of length is the *centimetre*; 1 centimetre = .3937 inch, and 1 inch = 2.54 cms.

From the unit of length we at once derive the units of area and volume: the unit of area being the area of a square whose side is 1 cm., and the unit of volume the volume of a cube whose edge is 1 cm.

The unit of *mass* is the *gramme*; this represents the mass of a cubic cm. of water at its point of maximum density (about 4° C.). The advantage of so selecting the unit of mass is that the volume of any given space, in cubic cms., also corresponds to the mass of an equal volume of water (at 4° C.).

One pound avoirdupois = 453.59 (or, in round numbers, 454) grammes.

The unit of *time* is the *second*, which is defined as a certain fraction of the mean solar day.

From the units of length and time we derive the kinematical units of *velocity*—1 centimetre per second—and of *acceleration*—1 cm. per sec. per sec.—respectively.

We have next to consider the following dynamical units:—

Force Unit.—The unit of force, called the *dyne*, is a force producing an acceleration of 1 cm. per sec. per sec. in a mass of 1 gramme. Otherwise, it represents the force which, if allowed to act for one second on a mass of 1 gramme, originally at rest, will impart to it a velocity of 1 cm. per sec.

If a mass of 1 gramme be allowed to fall freely under the action of its own weight at the earth's surface, it is found to move with an acceleration of 981 cms. per sec. per sec. Hence the gravitational pull exerted by the earth on a mass of 1 gramme, or the *weight* of a gramme-mass, is equal to 981 dynes. The gramme-weight is the *gravitational* unit of force, and is, as we have just seen, much larger than the absolute unit.

Work Unit.—The absolute unit of work is the *erg*, and is the work done by a force of 1 dyne acting through a distance (in its own direction) of 1 centimetre.

The British engineer's unit of work is the foot-pound. Since 1 pound-weight = 453·59 grammes-weight, and 1 gramme-weight = 981 dynes, 1 pound-weight = 453·59 × 981 dynes. Since, further, 1 foot = 12 × 2.54 cms., we see that 1 foot-pound = 453·59 × 981 × 12 × 2.54, or about 13.56 million ergs. The erg is thus an extremely small fraction of a foot-pound.

Activity or Power Unit—The unit of activity or power is the rate of doing 1 erg of work per sec. This unit has not received any special name.

The British engineer's unit of power is the horse-power, which corresponds to 550 foot-pounds per sec. Hence 1 horse-power = $550 \times 13.56 \times 10^6$, or 7.46×10^9 ergs per sec.

Magnetic Units.—We are now in a position to apply the dynamical units derived above to the definition of some important magnetic units in the absolute C.G.S. system.

A magnetized piece of iron or steel possesses the property of attracting pieces of iron, steel, and certain other magnetic materials (cobalt, nickel, and a few compounds). It also possesses the property of taking up a definite position when suspended or pivoted horizontally so as to be capable of rotation about a vertical axis. The end of the magnet which points approximately north is termed the north-seeking or north pole of it; the other end being termed the south-seeking or south pole. The property of attraction is mainly confined to the poles or ends of the magnet. Experiment shows that the north ends or south ends of two magnets repel each other, while a north end attracts a south end.

In order to enable us to study *quantitatively* the laws of magnetic attraction and repulsion, it becomes necessary to define a *unit magnetic pole* (also called unit quantity of magnetism). If we attempt to isolate a single magnetic pole by breaking a magnet, we find that this merely results in the production of two complete magnets, each with a north and south pole. In order to enable us to study the effects due to a *single* pole, we have to use an extremely long magnet. If then we confine our attention to points in the immediate neighbourhood of one of the poles, the effect due to the other may, on account of its distance, be neglected.

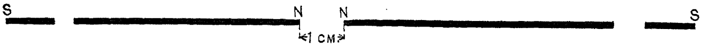

Fig. 1

Imagine, then, two very long magnets, similar in every respect, placed in line, and with their like poles facing each other, as in fig. 1, and let suitable arrangements be provided for measuring accurately the force exerted between them when the poles are 1 cm. apart.[1] Suppose, further, that by suitable means we are able to vary simultaneously and equally the degree of magnetization of the two magnets, and let this be so adjusted that the repulsion between them amounts exactly to 1 dyne. We then agree to say that each pole is a *unit magnetic pole*. Hence the following definition:—

A unit magnetic pole is one which, when placed at a distance of 1 cm. from an equal pole, acts on it with a force of 1 dyne, the intervening medium being supposed to be air.

If we suppose that a unit pole is placed at a distance of 1 cm. from another pole, and that the force exerted between them amounts to m dynes, we should say that the strength of the second pole is m. From this definition it follows that the force exerted between two poles varies as the product of their pole-strengths.

As a result of experiment, we find that the force exerted between two poles of given pole-strength varies *inversely as the square of the distance between them.*[2]

Consider a straight bar magnet of pole-strength m, and let the distance between the poles be l. Then the product $m\,l$ is termed the *magnetic moment* of the magnet.

If v be the volume of the magnet, then $\frac{m\,l}{v}$ is the *intensity of magnetization* (or magnetization simply) of the magnet (= magnetic moment per unit volume).

Let a = area of cross-section of the magnet. Then since $v = l\,a$, we have $\frac{m\,l}{v} = \frac{m\,l}{l\,a} = \frac{m}{a}$, *i.e.* the intensity of magnetization may also be defined as the *pole-strength per unit of area* of cross-section of the magnet.

If we imagine a unit north pole (*i.e.* one end of a very long magnet of unit pole-strength) placed at any point of space, then, on account of the earth's magnetic action, or that due to magnets in the neighbourhood, or to conductors conveying electric currents, the unit pole will experience a force of definite magnitude and direction, which is defined to be the *magnetic intensity*[3] (or *magnetic force*) at that point.

Any region throughout which magnetic force is exerted is termed a *magnetic field*, and the magnetic force or intensity is sometimes called *field intensity*.

Any line drawn so that the direction of the tangent to it at every point gives the direction of the magnetic force at that point is called a *line of magnetic force*. It is obvious from this definition that a line of magnetic force is the curve which would be traced out by a magnetic pole if free to move in the field..

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[1] The medium surrounding the magnets is supposed to be air, and their lengths are assumed to be very great in comparison with 1 cm., so that the action due to the distant poles S S may be left out of consideration.

[2] The law of the inverse square of the distance is sometimes called the *law of nature*, on account of its very general applicability. It holds not only for magnetic, but also for gravitational and electric attractions and repulsions, and in other cases. The researches which led up to the experimental verification of this law are fairly numerous, the most important being those of Coulomb and Gauss, especially the latter. A detailed account of Gauss's proof will be found in J. J. Thomson's *Elements of the Mathematical Theory of Electricity and Magnetism* (Cambridge University Press).

[3] *Magnetic intensity* (more generally called *magnetic force*) must be carefully distinguished from *intensity of magnetization.*

An infinite number of magnetic lines might be drawn in any given region. For purposes of measurement, however, it is convenient to adopt a convention according to which the number of lines is regarded as limited. Let a surface be drawn through any given point so as to be everywhere at right angles to the direction of the lines. Then—simply as a convenient way of expressing certain facts—we agree to say that the number of lines passing through a square centimetre of the surface in the immediate neighbourhood of the point is numerically equal to the magnetic force (or intensity) at that point.[1] Thus, magnetic force, field intensity, and number of lines per square centimetre of a surface drawn at right angles to the lines, are all equivalent expressions.[2]

Consider now the lines of force due to a unit north magnetic pole Since another unit pole if placed in its neighbourhood would experience a force which is along the line joining the two, it follows that the lines of force due to the unit pole are everywhere radial, as shown in fig. 2. In order to ascertain their *number*, according to the convention explained above, imagine a spherical surface described of 1 cm. radius and having the unit pole for its centre. It is evident that every line which proceeds from the unit pole cuts the spherical surface, so that the total number of lines to which the unit pole gives rise is equal to that crossing the spherical surface. Now (by the definition of unit pole and unit field intensity) the field intensity at every point of the spherical surface is unity; and since this surface is everywhere at right angles to the lines, it follows that *one* line passes through every square cm. of the surface. But the total surface has an area of 4π square cms.[3] Hence *a unit pole gives rise to 4π lines.*

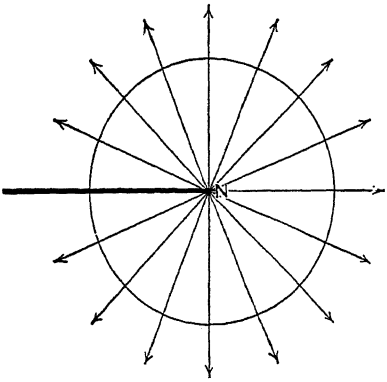

Fig. 2

Our next magnetic unit, that of *difference of magnetic potential*, is one of extreme importance. Consider any two points in space, A and B, fig. 3, and let a unit north magnetic pole be transferred along any path, such as ACB, from A to B, against the action of any magnetic forces which may happen to exist in that region. In moving the unit pole from A to B against the action of the magnetic forces, a definite amount of work will be done. This is *independent of the path* joining the two points,[1] and (expressed in ergs per unit pole) is defined to be *the difference of magnetic potential* between A and B. If the work done is positive, *i.e.* requiring expenditure of energy, B is said to be at a higher magnetic potential than A; but if the work is negative—*i.e.* if the magnetic forces themselves tend to produce the transference from A to B, then A is said to be at a higher potential than B. Otherwise, a unit north pole tends to move from places of high to places of low potential.

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[1] Instead of speaking of magnetic *lines*, some writers prefer the expression *tubes* of magnetic force. A tube of force may be defined as a tubular surface so drawn that the lines of force lie *along* it at every point.

[2] The medium being supposed to be an air space.

[3] The area of a sphere is $4\pi r^2$, r being the radius ($=1$ in the case considered).

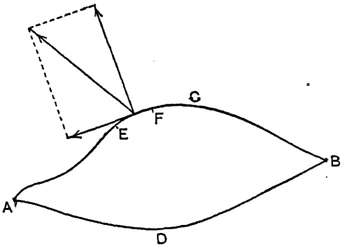

Fig. 3

In order to determine the total amount of work done, we may imagine the path ACB divided up into a large number of small parts or elements, such as EF. By resolving the magnetic force into two components, one along, and the other at right angles to EF, and taking the product of the length of EF and the component of magnetic force along it, we get the work done in moving over the distance EF. The sum of all such products will give the magnetic potential difference (or P.D.) between A and B. It is obvious from this that *the mean value of the magnetic force along any curve is equal to the difference of magnetic potential between its ends, divided by the length of the curve.*

Let us next suppose that instead of transferring our unit pole from A to B, we take it round a closed curve such as ACBDA, thus returning to the starting-point. Since—so long as we exclude the case of magnetic fields due to electric currents—the work done in passing from A to B along ACB is equal and opposite in sign to that done in passing from B to A along BDA, it follows that the total work done in describing the closed path vanishes. *The work done in carrying a unit pole round a closed curve is defined to be the magnetomotive force round that curve*, and, so long as we exclude the case of fields due to conductors conveying electric currents, the magnetomotive force vanishes round every closed curve.

The unit of magnetomotive force (or M.M.F.) is obviously identical with that of magnetic P.D., and is represented by 1 erg per unit pole.

CHAPTER II

UNITS OF ELECTRIC CURRENT AND ELECTRIC QUANTITY

Effects of an Electric Current.—When we say that a wire conveys an electric current, we mean thereby that the wire exhibits certain peculiar properties not possessed by any ordinary wire. Among the most important of the effects produced by an electric current are the following:—

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[1] Just as, in the case of gravitational attraction, the work done in transferring a unit mass from one point to another against the action of gravity is independent of the path followed.

1. *The Magnetic Effect.*—The space surrounding the wire is found to be the seat of lines of magnetic force. In the case of a long straight wire, these lines take the shape of concentric circles having their centres on the axis of the wire. The magnetic effect of an electric current was first discovered in 1820 by Oersted, who noticed that when a current was sent through a wire a magnetic needle placed near it tended to set itself at right angles to the wire. The magnetic effect of a current forms one of the most convenient and most delicate methods of detecting the presence of an electric current in a conductor. If we suppose a wire bent into a circle, then the magnetic force in the plane of the circle is everywhere perpendicular to that plane. Hence, if a magnetic needle be pivoted or suspended at the centre of the circle, and the position of the circular conductor adjusted so that the plane of the conductor contains the axis of the needle, then, on sending a current through the circular conductor, a deflection of the needle is produced. This deflection may be rendered more easily visible by either attaching a long pointer to the needle, or, better still, by using a long weightless pointer—a ray of light which is allowed to fall on a small mirror attached to the needle and is then received, after reflection, on a graduated scale placed at a suitable distance from the mirror. The sensitiveness of such an arrangement may be increased by using a coil of wire having a large number of turns instead of a single circular conductor. Such an arrangement of a *fixed coil* of wire and a *movable magnet* constitutes what is known as a *needle galvanometer*, and by means of various devices, the details of which we cannot enter into here, the sensitiveness of a needle galvanometer may be increased to a wonderful extent.

The practical applications of the magnetic effect of an electric current are extremely numerous. A piece of soft iron or other magnetic material is always found to move from weak to strong parts of the field. If we take a long coil of wire, or *solenoid*, and fit it with a soft-iron plunger capable of sliding inside the coil, then, since the most intense part of the magnetic field due to the coil is in its interior, the plunger will tend to take up a position of symmetry with respect to the coil when a current is sent through it. The coil will exert a *sucking action* on the plunger if the latter is displaced from its position of symmetry. Such an arrangement is known as a *coil-and-plunger* mechanism, and has received numerous applications in various electromagnetic devices.

Another important arrangement is that in which a coil of insulated wire is fitted with a *fixed* soft-iron core. Under the influence of the magnetic field, due to the current in the coil, the core becomes strongly magnetized. Such an arrangement constitutes an *electromagnet.* The first electromagnet was constructed by Sturgeon in 1825. By means of electromagnets much more intense fields may be produced than by any other method, and electromagnets are invariably employed nowadays for producing the magnetic fields of dynamos.

2. *The Heating Effect.*—A wire carrying an electric current is always found to be at a higher temperature than its surroundings. By sufficiently

increasing the current extremely high temperatures may be reached. The heating effect of an electric current is utilized in electric lighting, in the construction of electric furnaces, in electric welding, and other applications.

3. *The Chemical Effect.*—If a portion of the path followed by the current consists of a liquid chemical compound, the compound is split up into its constituents by the action of the current, and such a process of splitting up by electrical means is termed *electrolysis*. The compound itself forms the *electrolyte*, and the two constituents into which it is decomposed are termed *ions*. The chemical effect of an electric current is applied in copper refining, the electrolytic production of alkalies, electro-plating, &c.

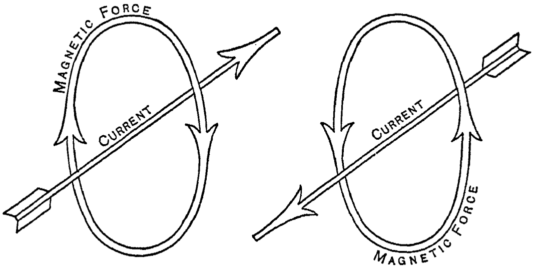

Fig. 4

Definition of C.G.S. Unit of Current.—If by means of a magnetic needle we determine the direction of the magnetic force produced by a conductor conveying a current, and then reverse the connections of the conductor to the source producing the current (battery or dynamo), we find that the magnetic force has also been reversed. An electric current therefore possesses a definite directive property, which is expressed by saying that the current flows in a definite direction along the conductor. It must be clearly understood that this expression does not necessarily imply the flow or transference of any material substance along the wire. By a generally adopted convention, the direction of the current is taken to be such that the relation connecting this direction with the direction of the magnetic force is identical with the relation which exists between the thrust and the twist of a right-handed screw. This relation is shown in fig. 4 for the case of a straight wire. The same "right-handed-screw rule" enables us to find the connection between the direction of the current in a coil or solenoid and the direction of the magnetic force due to it. This case is illustrated in fig. 5.

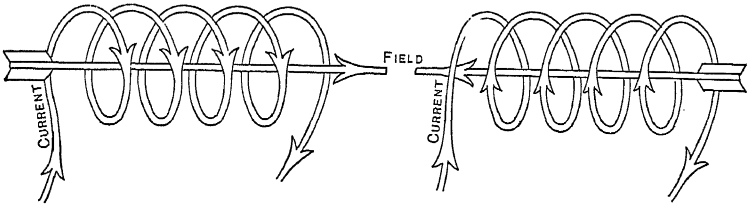

Fig. 5

The magnitude of a current is taken to be proportional to the magnetic force which it produces. Thus, if a certain current flowing in a given coil produces a definite magnetic force at a given point, and if we increase the current until the magnetic force at the same point is doubled, we agree to say that the current in the second case is twice as great as in the first.

The C.G.S. or "absolute" unit of current is arrived at as follows:—Imagine a circular conductor, of 1 cm. radius, as shown in fig. 6, to be conveying a current whose magnitude may be varied at will, and let a unit north pole be placed at the centre of the circular conductor. The force acting on this pole will, by a simple application of the right-handed-screw rule, be from right to left, as shown in the sketch. Let us now suppose that the magnitude of the current is adjusted until the force acting on the unit pole becomes equal to 2π dynes.[1] The current is then said to be a unit C.G.S. current. We thus have the following definition:—

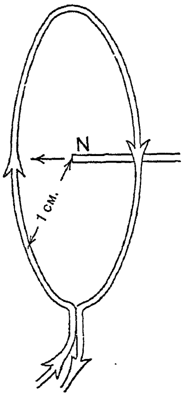

Fig. 6

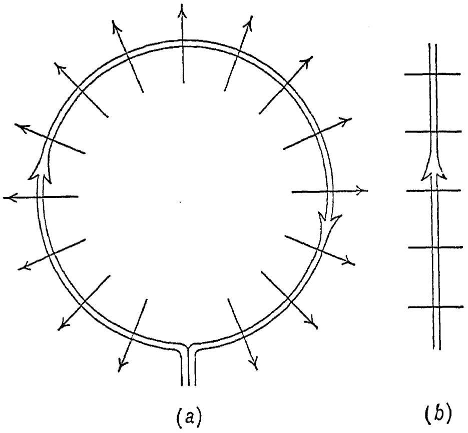

Fig. 7

The absolute unit of current is a current which, flowing round a circular conductor of 1 cm. radius, produces a magnetic force of 2π units at the centre of the circle.

At first sight it might appear as if a more convenient constant than 2π—say unity—were preferable. The reason for choosing 2π is, that the total length of the circular conductor is 2π cms., so that, corresponding to unit length of the conductor, we have unit magnetic force at the centre.

Dynamical Action on Conductor Conveying Current when Placed in Magnetic Field.—The action between the circular conductor and the unit pole at its centre is, by Newton's Third Law of Motion, a mutual one. The conductor tends to move the unit pole from right to left, and the unit pole tends to move the conductor from left to right. But the only effect produced by the presence of the unit pole at the centre of the conductor is a magnetic field at every point of the conductor which, as shown in fig. 7 (*a*), is everywhere radial, and therefore everywhere perpendicular to the length of the conductor. Further, since every point of the conductor is at unit distance from the unit pole, it follows that the field intensity (due to the pole) at every point of the conductor is unity. We may say that the conductor tends to move *because it is placed in a magnetic field*, and we see that the direction of motion is perpendicular to both the current and the field. The total force acting on the conductor is 2π, and since this is distributed over a length of conductor equal to 2π, the force per unit length of conductor is unity. Since this force is solely due to the presence of a unit magnetic field at every point of the conductor, it follows that if the conductor be opened out straight, as in fig. 7 (*b*), and if, by suitable means, a unit field intensity, which is everywhere perpendicular to the conductor, be maintained as before, the force acting on the conductor will remain unaltered. We thus have the following important result:—

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[1] The force is supposed to be that due solely to the current in the circular conductor; the earth's field, or any other magnetic fields, being supposed to be either non-existent or suitably compensated.

A conductor of unit length and conveying a unit current when placed in a field of unit intensity which is everywhere at right angles to the conductor, experiences a force of 1 dyne in a direction perpendicular to both the conductor and the field.

If the current amount to i units, the length of the conductor to l cms., and the field intensity to H, the total force acting on the conductor is ilH dynes.

A very convenient rule for determining the relation connecting the direction in which the conductor tends to move with the direction of the field and current has been given by Dr. Fleming, and is illustrated in fig. 8. If the thuMb, the Forefinger, and the mIddle[1] finger of the *left* hand be held so as to form a system of three mutually perpendicular lines, the forefinger pointing along the direction of the magnetic field, and the middle finger along that of the current, then the thumb will indicate the direction of the force acting on the conductor.

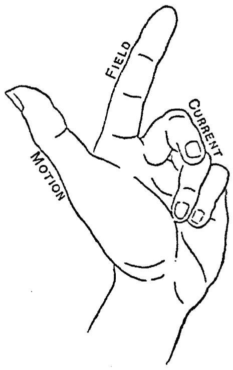

Fig. 8

Work Done in Increasing Magnetic Flux Through Circuit.—Let us suppose that a conductor of length l cms., conveying a current of i units, and placed in a field of intensity H (with the length of the conductor perpendicular to the field), is moved parallel to itself through a distance d in a direction opposed to that of the force acting on it. Since this force amounts to ilH dynes, and is overcome through a distance d, the work done in displacing the conductor is $ilHd$ ergs. Now ld represents the area of the rectangle swept out by the conductor, and since H also denotes (by the definition of a unit line) the number of lines passing through every sq. cm. of that area, ldH represents the total number of lines cut by the conductor during the motion. We thus see that the work done equals the product of the current into the number of lines cut. This result holds generally, so that in every case in which a circuit conveying a current has the number of lines linked with it increased, whether by mechanical displacement, increase of area of the circuit, or by increase of current, the corresponding expenditure of energy in bringing about this increase is given by the product of the current into the increase in the number of lines.

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[1] The letter i is very generally used as the symbol for *current*.

Moving-Coil Galvanometers.—The fact that a conductor conveying a current is subject to a definite dynamical force when placed in a magnetic field is utilized in the construction of a type of galvanometer known as the *moving-coil galvanometer*. An instrument of this type was devised originally by Lord Kelvin in connection with submarine telegraphy; it was modified for use in the laboratory by Deprez and D'Arsonval,[1] and is nowadays very largely employed in preference to a needle instrument. The construction of one form of such instrument is shown in fig. 9. Essentially, it consists of a coil of insulated wire suspended between the poles of a powerful steel magnet, a mass of soft iron (marked "iron core" in the figure) being introduced between the pole-pieces with the object of concentrating the lines and so obtaining a more intense field in the annular space which is occupied by the coil. The suspensions of the coil consist of two very fine strips of phosphor-bronze, which also serve the purpose of conveying the current into and out of the coil. The deflections are read off by means of the usual lamp and scale arrangement. One of the main advantages of this type of instrument lies in its being practically unaffected by external magnetic disturbances, so that it may be used in the neighbourhood of dynamos and under other conditions which would render the use of an ordinary needle galvanometer extremely troublesome, if not impossible.

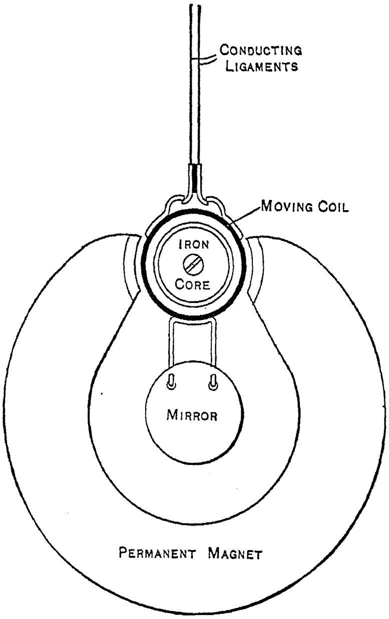

Fig. 9.—Moving-Coil Galvanometer

Practical Realization of Absolute Unit of Current.—The arrangement described in connection with the definition of the unit of current is not one which could be conveniently employed for the experimental determination of currents in absolute measure.[2] For this purpose several methods may be employed, and these are diagrammatically represented in fig. 10. At (*a*) we have a *standard galvanometer*, consisting of two fixed coils, C_1 and C_2, placed parallel to one another and at a distance apart equal to their mean radius; between them is a suspended or pivoted magnetic needle. If the dimensions of the coils and number of turns in them are known, the value of the current in absolute measure may be calculated from the observed deflection of the needle, provided the value of the horizontal component of the earth's field is known; this latter absolute measurement must thus precede the current measurement. At (*b*) we have a *standard electrodynamometer*; this differs from (*a*) in having a small suspended coil, C_m, instead of a needle. At (*c*) is shown the most satisfactory method for the absolute measurement of current. The instrument, known as a *current balance*, consists of two fixed horizontal coils, C_1 and C_2, and a smaller movable coil, C_m, which is suspended from the beam of a balance. The three coils are connected in series, and the directions of the currents are indicated by arrows. A laborious mathematical calculation enables the pull exerted by the fixed on the movable coil to be determined in terms of the current and the constants of the coils (their dimensions and number of turns); and if this pull be experimentally found by ordinary weighing, we have a means of finding the current.

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[1] Hence such an instrument is frequently called a Deprez-D'Arsonval galvanometer.

[2] An *absolute* measurement is one involving the measurement of a length, a mass, and a time only.

Instruments such as those briefly described above are expensive to construct, and require a considerable amount of care and skill in their use. They form the ultimate or primary standards which have enabled us to realize experimentally the theoretically defined absolute unit of current. Some simpler method of carrying out a current measurement in absolute measure becomes imperative, and such a simple method is furnished by making use of the *chemical effect* of a current.

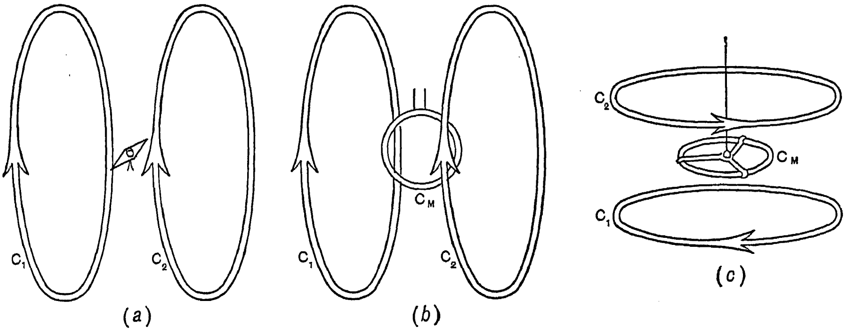

Fig. 10

Definition of Practical Unit of Current.—For practical purposes the C.G.S. unit of current is inconveniently large, and in practice one-tenth of this is taken as a unit, and is called an *ampere*. Thus:

$$\text{One ampere} = 10^{-1} \text{ C.G.S. unit.}$$

Laws of Electrolysis and Electrochemical Equivalents.—We have already stated that the two constituents into which an electrolyte is split up by a current are termed *ions*. The current is led into and out of the liquid by means of suitable conducting plates termed *electrodes*. The electrode by which the current enters the electrolyte is termed the *anode*, the other being the *cathode*. The products of decomposition or ions never appear in the body of the liquid, but are always given off *at the electrodes*. The ion which comes off at the anode is termed the *anion*, that at the cathode the *cation*. In the case of solutions of metallic salts or hydroxides, one of the ions consists of the metal, and this *always appears at the cathode*.

It is an experimental result that the amount of an ion liberated from a compound by a constant current flowing through the electrolyte is proportional to the time during which the current is allowed to flow. It is further found that if different currents be passed through a given electrolyte during equal intervals of time, the amounts of either ion liberated are *proportional to the currents.* The total amount of chemical decomposition is therefore proportional to the product of the current into the time. It is convenient to give to this product a special name: it is called the *quantity of electricity.* Thus, we say that when a current of c absolute units has been allowed to flow for t seconds, ct absolute units of electric quantity have been passed round the circuit. Making use of this definition of electric quantity, we may express the two facts stated above as follows:

The amount of an ion liberated is proportional to the quantity of electricity which has passed through the electrolyte. This result was first established by Faraday, and is known as Faraday's First Law of Electrolysis.

The practical unit of quantity is the *ampere-second* or *coulomb.* One coulomb = 10^{-1} C.G.S. unit of quantity.

The amount of an ion liberated by the passage of 1 coulomb is called the *electrochemical equivalent* of that ion.

Faraday's Second Law of Electrolysis states that *the electrochemical equivalent of an ion is proportional to its chemical equivalent.* Thus, if we once for all carefully determine the electrochemical equivalent of *one* ion, the electrochemical equivalents of all the other ions may be easily calculated from the known atomic weights of the elements and the valencies of the ions.

If the electrochemical equivalent of an ion is known, then we have a means of measuring a current by determining the amount of that ion decomposed during a given time, and dividing this by the product: time × electrochemical equivalent.

The most suitable electrolytes for this purpose are silver nitrate ($AgNO_3$) and copper sulphate ($CuSO_4$). The former may be used for the measurement of small, and the latter of large currents. An arrangement which enables a current to be measured by means of its chemical effect is termed a *voltameter.*

The determination of the electrochemical equivalent of an ion is one of the most important measurements in physics, furnishing us as it does with a cheap and comparatively simple and easy mode of measuring currents in absolute measure. Hence this determination has formed the subject of numerous researches, one of the most important of which is that carried out by Lord Rayleigh and Mrs. Sidgwick, who, using a current balance of the form briefly described above, determined the electrochemical equivalent of silver. The value arrived at by them was about .001118 gramme per coulomb (or .01118 gramme per absolute unit of quantity). The corresponding value for copper is about .000328 gramme per coulomb.

In using a silver or copper voltameter certain precautions must be observed, which we shall consider when dealing with the calibration of ammeters.

CHAPTER III

FUNDAMENTAL THEOREMS REGARDING M.M.F. AND E.M.F.

Fundamental Theorem regarding M.M.F.—In many of the practical applications of electromagnetism it becomes necessary to provide magnetic fields of definite intensity, and this is generally accomplished by means of coils carrying currents. It is a matter of extreme importance to the designer to be able to predetermine, either with perfect accuracy or with a sufficiently high degree of approximation, the magnetic intensity in any given region which will be produced by a coil of given size and shape when supplied with a known current. Such calculations are rendered possible by the use of an extremely important theorem, which we now proceed to consider.

Imagine a simple circuit, ABC (fig. 11), conveying a current of i absolute units, and consider any closed curve, DEF, which is so drawn as to be *linked* with the electric circuit. Let a unit magnetic pole be carried round the closed curve. By the definition of M.M.F., the work done in carrying the unit pole round DEF gives the M.M.F. round that curve. Now, as the pole is carried round the curve, every magnetic line proceeding from the pole will cut the circuit, and the total number of lines cut will be equal to that proceeding from the unit pole, *i.e.* to 4π. But in order to make the circuit cut 4π lines, an expenditure of energy is required of amount $4\pi i$. Hence the M.M.F. round the closed curve is equal to $4\pi i$.[1]

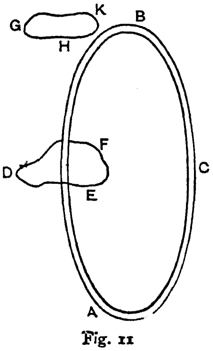

Fig. 11

The result obtained, it will be observed, is quite independent of the *shape* of the curve, since the reasoning employed does not involve a consideration of the shape. The essential point is the *linkage* of the closed curve with the circuit; for if we consider a closed curve, such as GHK, which is *not* linked with the circuit, then although, in carrying a unit pole round it, the lines of force cut the circuit in a certain direction during part of the journey, they cut it (or sweep across it) *in the opposite direction* during the remainder of the journey. Thus, if the first part of the journey involves an *expenditure* of energy, the second corresponds to a *gain* of energy; the total gain or loss of energy becoming zero, *i.e.* the M.M.F. round the closed curve vanishing.

If instead of a simple circuit we consider a coil of S turns conveying a current i, and determine the M.M.F. round any closed curve which is linked with *every* turn of the coil, such as the curve KLM in fig. 12, then since the work done in carrying a unit pole round a single turn of the coil is $4\pi i$, and since the coil consists of S turns, the total work done in going round KLM is $4\pi i S$.

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[1] The algebraical sign of the M.M.F. is determined by the direction followed in going round the curve: a positive sign being taken if the M.M.F. corresponds to a *gain* of energy, and a negative sign if it corresponds to an *expenditure* of energy.

The expression iS still denotes the total flow of current through the closed curve (or total current linked with it). The relation established is a perfectly general one, and may be briefly put thus:

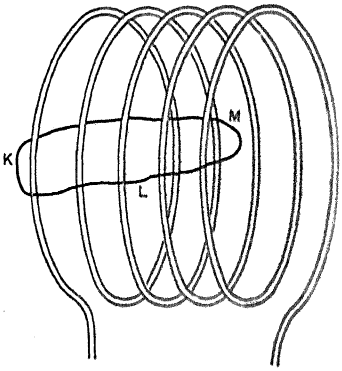

Fig. 12

The M.M.F. round any closed curve is equal to 4π times the total current linked with it.

In practical calculations it is usual to estimate M.M.F. in C.G.S. units, while the current is measured in amperes. If we are considering a coil of wire, then the product of the current in amperes and the number of turns is spoken of as the *ampere-turns* of the coil. Since an ampere $= 10^{-1}$ C.G.S. unit, we see that the M.M.F., in C.G.S. units, round any closed curve linked with a given coil is equal to $\frac{4\pi}{10}$, or *1.257 times the ampere-turns* of the coil. Conversely, the ampere-turns required to produce a given M.M.F. equal $\frac{1}{1.257}$, or, roughly, .8 time the M.M.F.

Applications of Theorem regarding M.M.F.—In order to illustrate the importance of the theorem just considered, we shall apply it to the solution of a few important problems.

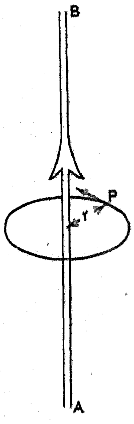

Fig. 13

The first of these consists in finding the magnetic intensity (or magnetic force) at any point outside a long straight wire conveying a current of i C.G.S. units. Let, in fig. 13, AB be the wire, and P the point at which the intensity is required. Through P draw a circle having its centre on the axis of the wire, and its plane perpendicular to it. We know as a result of experiment that the lines of magnetic force will form circles surrounding the wire, and having their centres on its axis. It is clear from symmetry that the intensity at every point of the circle passing through P is the same; let H stand for this intensity. Then the M.M.F. round the circle is $H \times 2\pi r$, where r is the distance of P from the axis of the wire. But by the theorem regarding M.M.F., the M.M.F. is also equal to $4\pi i$. Since the two expressions for the M.M.F. must give the same result, we have

$$H \times 2\pi r = 4\pi i, \text{ or } H = \frac{2i}{r}.$$

The magnetic force is thus seen to vary inversely as the distance from the wire.

The second problem is that of finding the magnetic intensity at any point inside a coil wound in the form of an anchor-ring, as shown in fig. 14.

Let P be the given point. Through P draw a circle whose plane is perpendicular to, and whose centre is on, the axis of the ring. Let r be the radius of this circle (= distance of P from axis of ring). As before, considerations of symmetry indicate that the intensity will have the same value at every point of the circle, the intensity being tangential to the circle. If H = intensity, then M.M.F. = $H \times 2\pi r$. But M.M.F. also = $4\pi S i$, S being number of turns in coil and i current in C.G.S. units. Hence

$$H \times 2\pi r = 4\pi S i, \text{ or } H = \frac{2 S i}{r}.$$

Thus the intensity is not quite constant over the cross-section of the anchor-ring, but diminishes as the distance from the axis increases. If, however, the radius of the ring is large in comparison with its cross-section, the intensity may be taken as approximately constant over the cross-section.

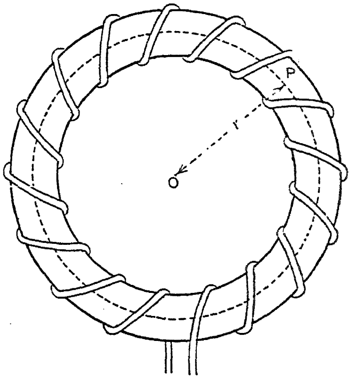

Fig. 14

The third problem which we shall consider is that of determining the intensity inside a straight coil of length great in comparison with its cross-section. Such a coil, or *solenoid*, may be regarded as forming part of an anchor-ring of infinite radius. Applying the result obtained above for an anchor-ring, we get $H = \frac{4\pi S i}{2\pi r}$. This may be written in the form $H = 4\pi \frac{S i}{2\pi r}$. Now $\frac{S}{2\pi r}$ denotes the number of turns S_1 per unit of length of the ring, and $H = 4\pi S_1 i$. So long, therefore, as $S_1 i$ is constant, H is independent of the value of r. We may now suppose r to increase indefinitely; then any finite portion of the ring, of length great in comparison with the cross-section, may be regarded as approximately straight, forming a *solenoid*. If—which is permissible in the case of a very long solenoid—we neglect the effect of the distant portions of the anchor-ring, we may suppose these to be removed without materially affecting the intensity inside the solenoid at points not near the ends. We thus get $H = 4\pi S_1 i$, approximately, for points well inside the solenoid.

If, in the last problem, the current be measured in amperes, we see that H = 1.257 time the ampere-turns per cm. length of the solenoid.

Difference of Electric Potential and E.M.F.—If a conductor be connected by means of a wire to one terminal of a battery or other source of current, a *momentary* current is found to flow, the duration of which is in general extremely short. The conductor which has been so connected is found to possess peculiar properties, and is said to be *electrified*, or to have received an *electric charge*. The amount of this charge, or the *quantity of electricity* which has passed into the conductor, is measured by the product of the average value of the current into the time during which the current has lasted.

Electric charges are of two kinds: positive and negative. A conductor is said to receive a positive charge if this is communicated by the flow of a current *into* the conductor; if the current flows *out of* the conductor, the corresponding charge acquired by the conductor is said to be *negative.*

Conductors which have both acquired either positive or negative charges are found to repel each other, while a positively charged conductor is found to attract a negatively charged one. In this respect, charged conductors behave precisely like magnetic poles. But while electrical repulsions and attractions may be exhibited by all known substances, magnetic polarity is confined to the relatively small class of "magnetic" substances.

The law of electrical attraction and repulsion is also identical with the corresponding magnetic law—it is that of the inverse square of the distance. The analogy between the two cases holds still further. For just as it is impossible to produce a north-seeking pole without at the same time producing a south-seeking one of equal strength, so it is impossible to impart a positive charge to any conductor without at the same time imparting an equal negative charge to some other conductor: positive and negative charges being always produced in equal amount.

The charge of a conductor is found to reside entirely on its surface. If, by means of suitable experimental arrangements, the outer layers or skin of a conductor be removed, the conductor ceases to exhibit any electrification, thus showing that its charge resided entirely in its surface layers.

The phenomena of electrical attraction and repulsion are exhibited by conductors which have been momentarily, or are permanently, connected to the terminals of a battery. In general, however, these attractions or repulsions are in this case so feeble as to require specially-constructed instruments for their detection. There is, however, another way of producing electrification—by friction. In this latter case the attractions and repulsions are so powerful that they may be readily studied without the use of any very delicate instruments. The method of electrification by friction has been known for ages, long before the existence of electric currents was even suspected.

We have already had occasion to consider the mathematical abstraction to which we gave the name of unit magnetic pole. It will now be convenient to introduce a corresponding conception into the electrical case, to which we shall give the name of a *unit electric charge* or *unit quantity of electricity.* Imagine a conductor of indefinitely small dimensions (say a very small sphere), and suppose that by connecting it to a battery a momentary current is allowed to flow into the conductor. Suppose, further, that either the strength of the battery or the duration of the contact is so adjusted that the product of the average value of the current into the time during which it has been flowing is equal to unity. Then the conductor has received a unit positive charge, and since the dimensions of the conductor are assumed to be indefinitely small, the charge is concentrated into a point, forming a *point charge*.

If such a unit point-charge be brought into the neighbourhood of any other charged conductors, it will experience a definite force acting in a definite direction, and this force (measured in dynes) gives the value of the *electric intensity* at the point where the unit charge is placed.[1] Any region throughout which electric intensity is exerted is spoken of as an *electric field*.

Consider any two points, A and B, in an electric field, and let a unit charge be taken from A to B along any path. In moving the charge from A to B against the action of the electric forces, a definite amount of work will be done, and this amount (measured in ergs) gives us the *difference of electric potential*, or electric P.D., between the two points. Hence the definition:—

The difference of electric potential between two points is the work done in transferring a unit electric quantity from the one point to the other.

If the transference of unit charge from A to B involves expenditure of energy, B is said to be at a higher potential than A; and *vice versa*.

Let us next consider the case of a closed curve, and let unit electric quantity be carried completely round the closed curve. The work done is then defined to be the *electromotive force*, or E.M.F., round the curve.

Practical Unit of P.D. and E.M.F.—The unit of P.D. and E.M.F. employed in practice is termed the *volt*, and is a very large multiple of the C.G.S. unit.

$$1 \text{ volt} = 10^8 \text{ C.G.S. units.}$$

Capacity and Units of Capacity.—Any arrangement of two conductors placed a definite distance apart forms what is known as a *condenser*. If the two conductors forming the condenser be connected to the terminals of a battery, a momentary current will flow, each conductor acquiring a charge—positive in the one case, negative in the other. If we divide the charge on either conductor by the P.D. between them, we get the *capacity* of the condenser. The capacity of a condenser depends not only on the size and shape of the conductors and their relative positions, but also on the nature of the intervening medium.

If we measure the quantity in coulombs (or ampere-seconds), and the P.D. in volts, then the ratio $\frac{\text{coulombs}}{\text{volts}}$ gives us the capacity in terms of a unit known as the *farad*, which is taken as the practical unit of capacity. Since 1 coulomb = 10^{-1} C.G.S. unit, and 1 volt = 10^8 C.G.S. units, it follows that 1 farad = $\frac{10^{-1}}{10^8}$ C.G.S. unit, or

$$1 \text{ farad} = 10^{-9} \text{ C.G.S. unit.}$$

The capacity represented by a farad is very large, and hence in practice it is usual to express capacities in terms of the *microfarad*.

$$1 \text{ microfarad} = 10^{-6} \text{ farad} = 10^{-15} \text{ C.G.S. unit.}$$

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[1] Electric intensity is measured in *dynes per unit charge*.

Activity or Power in an Electric Circuit, and Units of Power.—Let a current of i C.G.S. units flow in a circuit, and let A and B be any two points in the circuit, between which there is a P.D. of V units (C.G.S.). In accordance with the definition of unit electric quantity, we may say that i units of quantity are transferred in every second from A to B. But by the definition of P.D., the transference of *each* unit from A to B involves the expenditure of V ergs of work. Thus the total work per second, or the power, in the part of the circuit lying between the points A and B, is Vi ergs per second. We therefore see that

$$\text{Activity or Power} = \text{P.D.} \times \text{current.}$$

If, instead of considering a portion of the circuit, we consider the entire closed circuit, then the work done on every unit in carrying it completely round the circuit gives us the E.M.F., which we shall denote by E. Hence the power in the entire circuit is Ei.

If the P.D. be expressed in volts, and the current in amperes, then the product P.D. × current will give us the power in *volt-amperes* or *watts*. The watt is taken as the practical unit of power, and since one volt = 10^8 C.G.S. units, and 1 ampere = 10^{-1} C.G.S. unit, we find that 1 volt-ampere = $10^8 \times 10^{-1}$, or 10^7 C.G.S. units.

$$1 \text{ watt} = 10^7 \text{ ergs per sec.}$$

One horse-power corresponds to 550 ft.-lbs. per sec.; and since we have found that 1 ft.-lb. = 13.56×10^6 ergs, it follows that 1 horse-power = $550 \times 13.56 \times 10^6 = 746 \times 10^7$ ergs per sec. approximately. Hence

$$1 \text{ horse-power} = 746 \text{ watts.}$$

The watt forming only a very small fraction of a horse-power, it is usual to employ 1000 watts, or a *kilowatt*, as a unit of power.

$$1 \text{ kilowatt} = 1000 \text{ watts} = \frac{1000}{746}, \text{ or } 1.34 \text{ H.P.}$$

Units of Energy.—Electrical energy is generally reckoned in terms of the Board of Trade unit (frequently called "unit" simply), which corresponds to a *kilowatt-hour*, or 1000 watt-hours (= 1.34 H.P. hours).[1]

Fundamental Theorem regarding E.M.F.—In 1831 Faraday and Henry made the important discovery that every change in the number of magnetic lines passing through a circuit results in the production or *induction* of an E.M.F. in that circuit. This E.M.F. lasts only so long as the number of lines is varying, and disappears as soon as that number becomes constant. Imagine that into a circuit which originally contains no E.M.F. there are introduced N lines during t seconds, and that the average current i is due to an average induced E.M.F. of amount E.[2] The

[1] One kilowatt-hour = $1.34 \times 33{,}000 \times 60$ ft.-lbs. Another unit of energy which has been occasionally used is the *joule*; this is equal to a *watt-second*.

[2] C.G.S. units being used in each case.

mechanical energy expended in introducing N lines into a circuit in which the current is i is given by $N i$ (see Chapter II). But since the introduction of the N lines has taken t secs., the average power is $\frac{N i}{t}$. This power is transformed into electrical power, and must be equivalent to this latter. Now the electrical power is given by $E i$. Hence $E i = \frac{N i}{t}$, or

$$E = \frac{N}{t}.$$

In other words, *the E.M.F. induced in a circuit by variations in the number of magnetic lines passing through the circuit is equal to the rate at which the number of lines is changing.*

In cases where the change in the number of lines is produced by the displacement of a circuit in a magnetic field in such a manner that one of the conductors forming the circuit is made to cut the magnetic lines, while the motion—if any—of the remainder of the circuit does not result in any cutting of lines, the rate at which the number of lines is changing is equal to that at which the *active* conductor is cutting lines. We may then regard the active conductor as being the seat of an E.M.F., and say that the E.M.F. induced in the conductor is equal to the rate at which it is cutting lines.

The direction of the E.M.F. induced is always such as to tend to oppose the change which gives rise to it. If, for instance, the change consists in an increase in the number of lines, the E.M.F. tends to produce a current which will decrease the number of lines. A convenient "hand rule" has been given by Dr. Fleming for the case of a conductor moving parallel to itself perpendicularly across a magnetic field. This rule corresponds to that already explained in Chapter II in connection with the direction in which a conductor conveying a current tends to move across a field. In the present case, however, the *right* hand is used, the forefinger pointing along the field, and the thuMb along the direction of Motion; the mIddle finger (which is at right angles to the other two) then gives the direction of the Induced E.M.F.

If a conducting mass be rotated in a magnetic field, the induced E.M.F.s give rise to currents which tend to stop the motion, and thus exert a powerful *damping effect.* This damping effect is very frequently made use of in practice.

Practical Realization of Absolute Unit of E.M.F.—One method of practically realizing the absolute unit of E.M.F. is the following[1]:—Let a permanent magnet of known magnetic moment[2] be mounted inside a solenoid so as to be capable of rotation about an axis passing through its middle point and perpendicular to the axis of the solenoid and that of the magnet. If the magnet be rotated, its lines will cut the various turns of the solenoid, inducing in the latter an alternating E.M.F. *i.e.*

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[1] This method was employed by C. Limb (*Journal de Phys.* v. pp. 61–70, 1896) to determine in absolute measure the E.M.F.s of a Clark and a standard Daniell cell.

[2] The determination of the magnetic moment of a magnet is easily carried out by a method due to Gauss; the horizontal intensity of the earth's field being determined at the same time.

one whose direction is periodically reversed). For a certain position of the rotating magnet, the rate at which its lines cut the solenoid becomes a maximum. This maximum rate of cutting, or maximum E.M.F., may be calculated from the known moment of the magnet, its speed of rotation, and the size and shape of, and number of turns in, the solenoid.

Such a standard of E.M.F. would not, however, be very convenient, and in practice the standards of E.M.F. take the form of primary cells whose E.M.F. at a given temperature may be relied on to have a *definite* value which does not change in the course of time. The E.M.F. of such a constant or standard cell may be compared, by suitable experimental arrangements, with the maximum value of the alternating E.M.F. induced in the solenoid by the rotating magnet, and thus the value of the E.M.F. of the cell may be obtained in absolute units.

The cell most frequently used as a standard is that known as the Clark cell;[1] its E.M.F., at 15° C., is 1.434×10^8 C.G.S. units, or 1.434 volt.

CHAPTER IV

RESISTANCE AND ITS MEASUREMENT

Definition of Resistance.—Let a current of i C.G.S. units flow in a conductor between whose ends there exists a P.D. of V units (C.G.S.). The ratio $\frac{V}{i}$ is then defined to be the *resistance* of the conductor to the current, and is usually denoted by r. Thus $r = \frac{V}{i}$.

Since the P.D. is V, and the current i, the power is $Vi = ri^2$ This power is employed entirely in heating the conductor, since no other transformation of energy is supposed to be taking place.[2] Hence ri^2 gives us the rate at which heat is being produced in the conductor. We thus arrive at the following *alternative* definition of resistance:—

By the resistance of a conductor is meant the coefficient which, when multiplied by the square of the current, gives the rate of heat production in the conductor.[3]

Ohm's Law and Units of Resistance.—There is no *a priori* reason why the quantity which we have just defined as the resistance of a conductor should not have different values for different currents; and whether such is the case or not is a matter to be decided entirely by experiment. Very careful experimental researches have shown that, so long as the *physical state* of the conductor (as regards temperature and stress) remains unaltered, *its resistance is independent of the current.* This extremely important result is known as *Ohm's Law*, and the reader must be careful to discriminate between the mere definition of resistance and Ohm's Law.

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[1] For a description of various types of standard cells, see Chapter VIII.

[2] It is assumed that the conductor is not the seat of an E.M.F.

[3] The heat being measured in dynamical units (ergs).

If Ohm's Law did not hold, then in dealing with the resistance of a conductor we should have to specify the current for which the resistance has been determined. But since the resistance is found to be independent of the current, the latter need not be specified.

The resistance of a conductor is thus a definite physical quality of it, which depends solely on the size and shape of the conductor and on its physical state.

The absolute unit of resistance is the resistance of a conductor in which unit current is produced by unit P.D. between its terminals.

If the P.D. be measured in volts, and the current in amperes, then the ratio $\frac{\text{volts}}{\text{amperes}}$ is taken as the resistance of the conductor in practical units or *ohms.*

$$1 \text{ ohm} = \frac{10^8}{10^{-1}} \text{ or } 10^9 \text{ C.G.S. units.}$$

Practical Realization of Unit of Resistance, the B.A. Unit and Legal Ohm.—On account of its great importance, the determination of a given resistance in absolute measure has engaged the attention of a large number of physicists. We shall briefly indicate two important classes of methods which may be employed for this purpose.

In one of these, due to Lorentz, a copper disc is placed coaxially with a coil of wire, and is fitted with rubbing contacts near its centre and at the circumference. The resistance r whose value is to be determined is joined in circuit with the coil, and a current i is sent through them. This produces a fall of potential over the unknown resistance of amount ri. The disc is now rotated until its E.M.F. is found to balance the fall of potential over the unknown resistance. Now E, the E.M.F. of the disc, is proportional to the current i producing the field of the coil in which the disc is rotating, and to the speed of rotation ω. Hence we may write $E = Mi\omega$, where M is a constant whose value may be calculated from the dimensions and relative positions of the coil and disc. We thus get $Mi\omega = ri$, or $r = M\omega$ when the speed has been so adjusted as to produce balance between E and ri.

In another class of methods, due to Joule, a current of known value i (which may be measured in absolute units by a current balance) is sent through the unknown resistance r, and the amount of heat produced in a given time t is determined. This heat is given by ri^2t, and so enables us to find r.

The method originally employed by the Committee of the British Association (that of a coil revolving in the earth's field and acting as its own galvanometer) did not prove very satisfactory, and led to a large error in the determination of the practical unit of resistance. The unit so determined is known as the *B.A. unit,* and

$$1 \text{ B.A. unit} = .9866 \text{ ohm.}$$

At a later stage, an attempt was made to correct the error made by the B.A. Committee, and a new unit, known as the *legal* ohm (which, however, never was legalized in England), was adopted:

$$1 \text{ legal ohm} = .9977 \text{ ohm}.$$

The importance of the B.A. unit and the legal ohm at the present time lies solely in the fact that many of the older resistance boxes, which are still in use, are constructed in terms of these two units.

Series and Parallel Arrangement of Resistances Shunts. Let two resistances, r_1 and r_2, be connected end to end, or *in series* with each other, as in fig. 15 (*a*), and let a current i be sent through them. Let V, V_1, and V_2 stand respectively for the P.D.s across the two resistances, and across the first and second taken separately. Then from the definition of P.D. it follows that $V = V_1 + V_2$; but by the definition of resistance, $V_1 = r_1 i$, and $V_2 = r_2 i$. Hence $V = (r_1 + r_2) i$, or $\frac{V}{i} = r_1 + r_2$. Again, by the definition of resistance, $\frac{V}{i}$ is the total resistance of the two resistances connected in series. If this be denoted by r, then we have $r = r_1 + r_2$, *i.e.* the total resistance is the *sum* of the two resistances. Similar reasoning would apply to any number of resistances joined in series, so that in general

The total resistance of any number of resistances connected in series with each other is equal to the sum of the separate resistances.

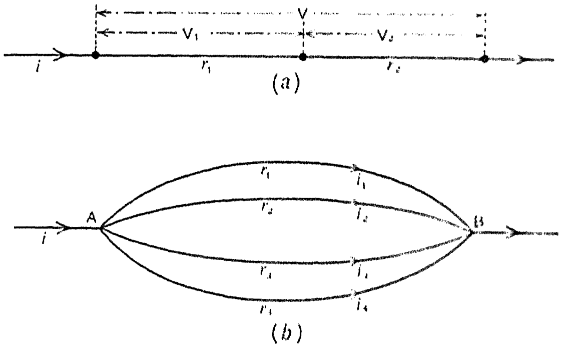

Fig. 15

From this it follows that the resistance of a conductor of uniform cross-section is *proportional to the length* of the conductor, and that the fall of potential over it when conveying a given current is also proportional to its length.

Consider next a number of conductors, whose resistances are r_1, r_2, r_3, r_4, placed side by side, as in fig. 15 (*b*), and connected between two common points A and B. Such an arrangement is called a *parallel* connection of conductors. If x = joint resistance of the system of conductors, V = P.D. between A and B, and i = total current, then $x = \frac{V}{i}$. But if i_1, i_2, i_3, i_4 denote the currents in the various conductors, then $i = i_1 + i_2 + i_3 + i_4$, and $i_1 = \frac{V}{r_1}$; $i_2 = \frac{V}{r_2}$; $i_3 = \frac{V}{r_3}$; $i_4 = \frac{V}{r_4}$, so that $i = V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}\right)$, whence

$$x = \frac{V}{i} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}}.$$

The reasoning may be extended to any number of conductors; hence the following general rule:—

The joint resistance of a number of conductors connected in parallel is equal to the reciprocal of the sum of the reciprocals of the separate resistances.

From this it follows that the resistance of a conductor of given length varies *inversely as the cross-section* of the conductor.

In practice, we very frequently have to deal with a parallel connection of two conductors. In such a case, one of them is said to form a shunt to, or to be shunted by, the other. Applying the rule given above, we easily find that in this special case the joint resistance is equal to the product of the two resistances, divided by their sum.

The reciprocal of the resistance of a conductor or system of conductors is termed the *conductance*, and in some problems it is more convenient to consider conductances than resistances.

Standards of Resistance.—Standards of resistance generally take the form of coils of wire of definite length and cross-section. There is, however, a certain objection to the use of such standards. For the molecular structure of the material of which the wire is composed is liable to undergo slow changes, and these bring about corresponding changes of resistance. The standard is therefore liable to change. Although in ordinary commercial work such changes need hardly be considered, yet in accurate determinations they must be guarded against. Hence it is that in defining the practical unit of resistance, the Board of Trade has chosen to refer it to a material which possesses a perfectly definite molecular structure — pure mercury. The Board of Trade definition of the ohm reads as follows:—

The ohm has the value 10^9 *in terms of the centimetre and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice,* 14.4521 *grammes in mass, of a constant cross-sectional area, and of a length of* 106.3 *centimetres.*

Resistivity and Temperature Coefficient.—We have already seen that, so far as its dimensions are concerned, the resistance of a conductor is directly proportional to its length, and inversely proportional to its cross-section. The other two factors determining the resistance are the nature of the material forming the conductor, and the temperature.[1]

If we consider a conductor of length l and cross-section a, then its resistance r may be written in the form

$$r = \rho \frac{l}{a},$$

where ρ, which depends on the material and temperature of the conductor, represents the resistance of a conductor of the same material, but of unit length and cross-section, and is termed the *resistivity* of the material at the given temperature. The *resistivity* of a material may be defined as the resistance between opposite faces of a cube of it whose edge is unity.

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[1] We omit the consideration of the effects of mechanical *stress*.

The reciprocal of resistivity is termed *conductivity*.

The resistivity of a material in general depends on the temperature, and for moderately small changes of temperature the changes of resistivity may be taken as proportional to changes of temperature. It is usual to take 0° C. as the standard temperature, and if ρ_0 stand for the resistivity of a material at this temperature, its resistivity ρ_t at any other temperature t may be written as

$$\rho_t = \rho_0 (1 + \alpha t),$$

where t = temperature, in degrees C., and α is a constant (roughly) known as the *temperature coefficient* of resistance.

As a general rule, α is positive for good conductors (metals), and negative for bad conductors (electrolytes and insulators). In the case of certain metallic alloys, α is so small that within a certain range of temperature the resistivity may be regarded as independent of temperature. Such alloys are particularly suitable for the construction of standard resistance coils, since it then becomes unnecessary to apply any "temperature corrections".

The following table gives the approximate values of the resistivities and temperature coefficients of some important substances:—

Name of Substance.	Resistivity in ohms per cm. cube.	Temperature Coefficient.
Copper	1.6×10^{-6} at 0° C.	+ .004
Iron	9.6×10^{-6} at 0° C.	+ .0048
Mercury	94×10^{-6} at 0° C.	+ .00072
Platinum	8.2×10^{-6} at 0° C.	+ .0032
Platinum silver ... ... (33% of platinum, 64% of silver)	26.8×10^{-6} at 0° C.	+ .00018
German silver ... ... (4 parts Cu, 2 Ni, 1 Zn)	21.2×10^{-6} at 0° C.	+ .00044
Platinoid ... ... (German silver, with 1 or 2 % of tungsten)	43.6×10^{-6} at 0° C.	+ .00025
Manganin ... ... (84% Cu, 12% Mn, 4% Ni)	42×10^{-6} at 0° C.	+ .00002
Phosphor bronze	8.5×10^{-6} at 0° C.	+ .0006
Aluminium	3×10^{-6} at 0° C.	+ .004
Carbon (gas retort)	67×10^{-2} at 0° C.	− .0005
Sulphuric acid (density 1.17)	1 at 15° C.	—
Sulphate of copper (saturated)	25 at 15° C.	—
Sulphate of zinc (saturated)	21 at 15° C.	—
Mica	$.084 \times 10^{30}$ at 20° C.	—
Gutta-percha	$.45 \times 10^{30}$ at 24° C.	—
India-rubber	10.9×10^{30} at 24° C.	—
Ebonite	28×10^{30} at 46° C.	—
Paraffin-wax	34×10^{30} at 46° C.	—
Shellac	9×10^{30} at 28° C.	—

Measurement of Resistance — Very Low Resistances.—The methods of measuring resistance are so numerous and varied, that it will be impossible here to attempt more than an outline of some of the more important of them.

If the resistance is a very low one (such as that of the armature or series coils of a dynamo), one of the most satisfactory methods is to send a known current i through the resistance, and to measure the corresponding fall of potential V. The resistance is then given by V/i. Instruments for the measurement of currents and P.D.s are described in Chapter V.

Moderately High Resistances—Wheatstone's Bridge.—For the measurement of moderately high resistances, the most satisfactory method is that known as the Wheatstone's Bridge. The usual diagrammatic way of representing this arrangement is shown in fig. 16, where r_1, r_2, and r are three resistances having known values, and x is the unknown resistance. A battery and key is connected between A and D, and a galvanometer and key between B and C. In carrying out the measurement, r is varied until the galvanometer shows no deflection when both battery and galvanometer keys are closed. Balance is then said to have been obtained, and there is no P.D. between B and C. In order that this condition may be satisfied, it is obvious that the fall of potential from A to B must equal that from A to C, and the fall from B to D that from C to D. These conditions are expressed by the equations

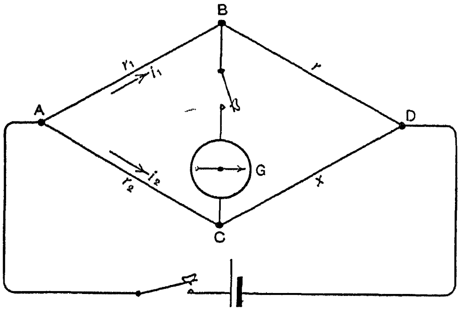

Fig. 16

$$r_1 i_1 = r_2 i_2$$
$$\text{and } r i_1 = x i_2,$$

where i_1, i_2 stand for the currents along A B D and A C D respectively. Dividing the first equation by the second, we get for the condition of balance

$$x = \frac{r_2}{r_1} r,$$

which gives us x in terms of the known resistances. In order to avoid spurious and misleading deflections of the galvanometer, *the battery key is always pressed before the galvanometer key, and released after.*

In practice, the known resistances usually take the form of coils of platinum silver, German silver, platinoid, or manganin silk-covered wire wound on bobbins of boxwood and thickly coated with paraffin-wax. These bobbins are attached to the lower surface of a horizontal slab of ebonite, which is polished on its upper surface, and the ends of the coils are connected to a set of heavy brass contact blocks arranged on the upper surface of the ebonite. Neighbouring contact blocks may be connected by

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means of a plug, whereby the coil between them is short-circuited. It is a matter of the highest importance to have the plugs thoroughly clean and fitting well into the conical spaces intended to receive them.

Insulation Resistances—Direct-Deflection Method. One of the most commonly employed methods for the measurement of very high resistances is that known as the direct-deflection method. A high E.M.F. is generally necessary in such cases, and the resistances dealt with are so high that the resistance of the battery and galvanometer may be neglected in comparison with them. The galvanometer and testing battery are joined in series with the unknown high resistance x, and the galvanometer reading d_x is noted. The unknown resistance is next replaced by a known high resistance r, and the new reading d_r is noted. If the galvanometer and battery resistances be negligible, then, assuming the same E.M.F. to be used in both cases, and the galvanometer readings to be proportional to the currents, we have $\frac{x}{r} = \frac{d_r}{d_x}$, or $x = \frac{d_r}{d_x} r$. In practice, if the same E.M.F. is used in obtaining the two readings, it is generally necessary to shunt the galvanometer, in which case its reading must be multiplied by the "multiplying power" of the shunt;[1] if, in addition, the E.M.F.s are different in the two cases, say E_x and E_r, then the factor $\frac{E_x}{E_r}$ must be introduced into the formula. Thus, in the most general case we have

$$x = \frac{E_x}{E_r} \cdot \frac{d_r}{d_x} \cdot s \cdot r,$$

where s stands for the multiplying power of the shunt.

In carrying out any measurements of very high resistances, it is extremely important to have all the instruments and connecting wires thoroughly well insulated.

Silvertown Portable Testing Set.—This testing set, which is very convenient for *approximate* measurements of resistance, is capable of measuring from .01 ohm upwards. The general appearance of the instrument is illustrated in fig. 17, and fig. 18 gives a diagram of the connections. For resistances up to 9900 ohms the Wheatstone's Bridge method is used. The "ratio arms", corresponding to r_1 and r_2 of fig. 16, are represented by the coils marked 10, 100, and 1000 in the lower part of the diagram of connections, fig. 18, and the adjustable arm (r in fig. 16) is represented by the two sets of coils marked "tens" and "units". The coils in the ratio arms are thrown into circuit by *taking out* the corresponding plugs; but in the case of the adjustable arm, one plug is provided for each set of circular contact blocks, and the amount of resistance introduced is given by the number opposite the plug. If either of these two plugs be withdrawn, a complete break results. The unknown resistance is connected to the two terminals marked "bridge terminals" in the diagram. The battery is connected to the two sockets marked "bridge" by means of flexible wires ending in

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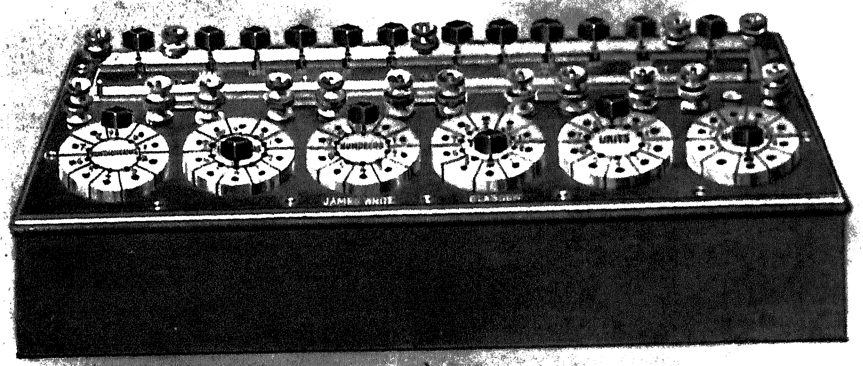
[1] The "multiplying power" of a shunt is the number by which the current passing through the galvanometer must be multiplied in order to obtain the current in the main circuit. Corresponding to shunts of $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ the multiplying powers are 10, 100, and 1000.



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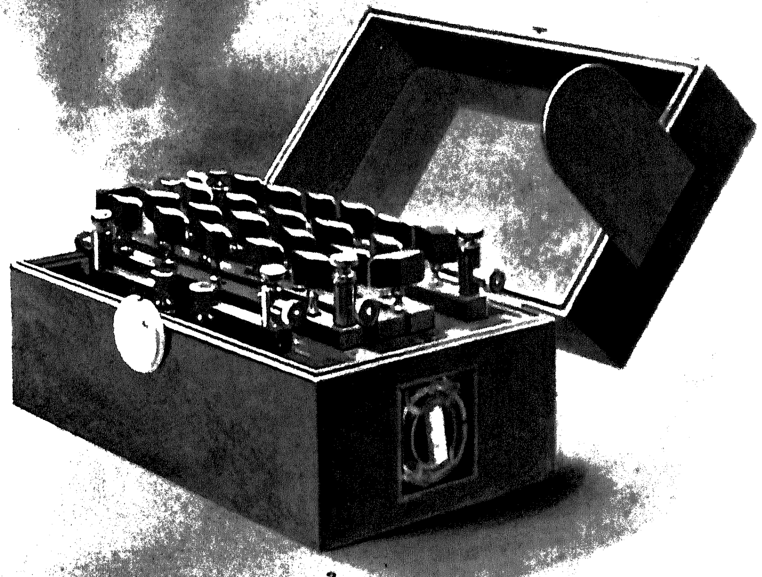

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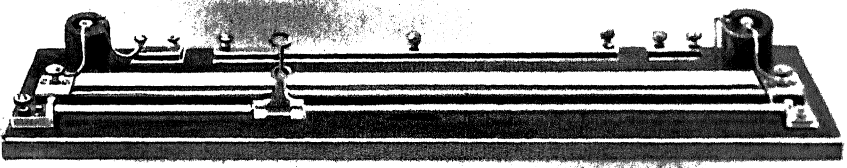

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1, Standard Six-Dial Pattern, Wheatstone Bridge; 2, Post-Office Pattern, Wheatstone Bridge; 3, Metre Bridge

SOME MEASURING INSTRUMENTS. I

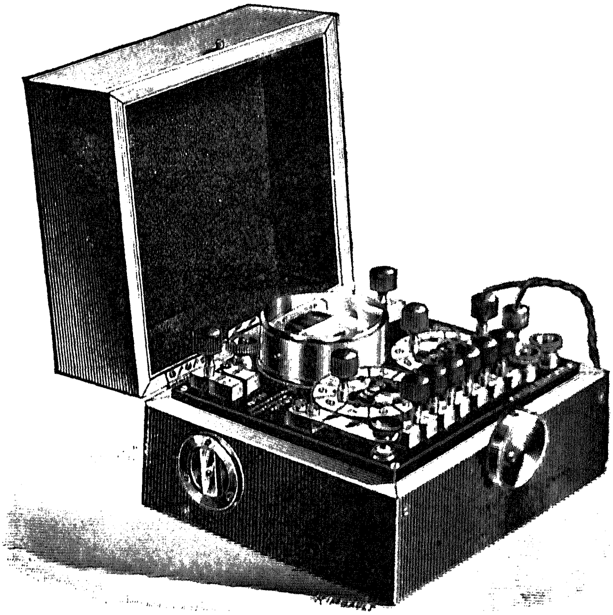
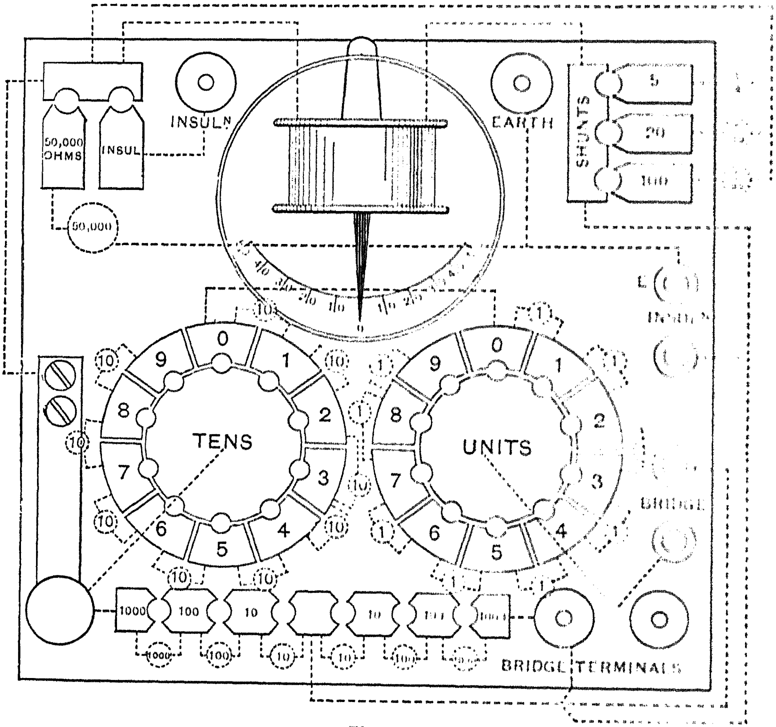


Fig. 17.—Silvertown Testing Set

plugs. No battery key is provided, the insertion or withdrawal of either battery plug when the other is in, making or breaking the battery circuit. The key seen at the left-hand side is the galvanometer key.

If the resistance to be measured exceeds 9900 ohms, the direct-deflection method is used in place of the Wheatstone's Bridge. The battery plugs are transferred to the sockets marked "insul^n", and the ends of the unknown resistance are connected to the two upper terminals marked "insul^n" and "earth". Two readings



are now taken: one with a plug inserted into the hole marked "insuln", which introduces the unknown resistance into circuit; the other with the plug in the hole marked "50,000 ohms", this disconnects the unknown resistance and for it substitutes 50,000 ohms. The galvanometer scale is marked so that the readings are proportional to the currents. It is in general necessary to shunt the galvanometer when taking the second reading, and it may further be necessary to reduce the E.M.F. by using a smaller number of cells in the testing battery.

When using the instrument it must be levelled and set so that the pointer attached to the needle stands at zero; this position is found by trial, the instrument being turned about a vertical axis. By means of the little permanent magnet let into the case of the instrument (clearly seen in fig. 17) and capable of rotation about a horizontal axis, the sensitiveness of the galvanometer may be varied.

A portable battery of "dry cells" for use with this instrument is supplied by the makers.

Evershed's Ohmmeter.—The idea of an instrument capable of giving the values of resistances directly in ohms was originally suggested by Professors Ayrton and Perry. The general principle of such an instrument is illustrated in fig. 19:—Imagine a magnetic needle placed in a field which is the resultant of a constant field and a variable field of constant direction. Then as the variable field (of constant direction) assumes all possible values between a zero and a certain maximum value, the position of the needle, which is always along the resultant field, will vary continuously between certain limits depending on the relative directions of the two fields. Since the direction of the resultant remains unaltered if both fields be altered in the *same ratio*, the position of the needle will depend simply on the *ratio* of the two fields.

Suppose, next, that one of the fields is produced by a coil of constant resistance connected to a source of E.M.F., while the other is produced by a coil joined in series with a variable resistance and connected to the *same* source of E.M.F. As this variable resistance is indefinitely increased from a zero to a very large value, the position of the needle steadily changes, and from what has been said above it follows that the value of the E.M.F. supplying both coils does not affect the position of the needle, which is determined solely by the value of the variable resistance. By using a set of known resistances the instrument may be "calibrated" or made direct-reading, and on then substituting any unknown resistance the value of this latter may at once be read off.

The great defect of such an instrument is the comparatively small degree of accuracy attainable, since the scale extends from zero to infinity. Hence, although there is no reason why such an instrument should not be used for the approximate measurement of any range of resistances, practically it is only employed in cases where a very high degree of accuracy is not required, viz., for the measurement of very high or insulation resistances, such as that of the wiring in a building. An instrument devised for this purpose by Mr. Evershed, and known as Evershed's "Megger", is very largely employed.

The general appearance of the instrument is shown in fig. 20. The unknown high resistance is connected across the two terminals seen to the left, and marked "line" and "earth" respectively. The handle in front is for driving the small high-voltage generator which supplies the testing voltage. The generator and measuring instrument are combined in a single case, measuring 6¼ × 6¼ × 12 inches. The entire "Megger" weighs about 18 lb. The scale is fitted immediately below the top of the case, and is viewed through a plate-glass window provided with a suitable cover. The driving shaft is fitted with an automatic clutch, which begins to slip when a certain speed is reached.

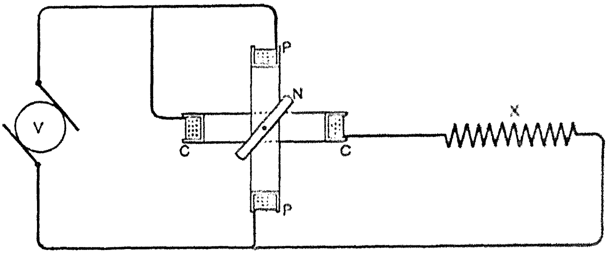


Fig. 19.—Diagram showing Principle of Ohmmeter

v, Source of Pressure; cc, Current Coil in series with External Resistance. pp, Pressure Coil directly connected to Pressure; n, Magnetised Needle. x, Resistance under test.

The instrument is used as follows:—It is stood on a steady base, and the driving handle is turned in a clockwise direction, the speed being gradually increased until the clutch is felt to slip. The value of the unknown high resistance is then read off on the scale.

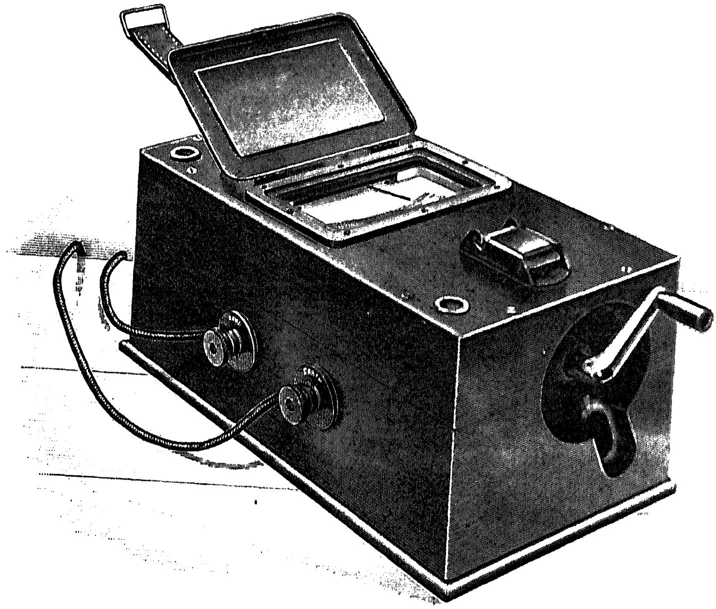


Fig. 20.—Evershed's Patent Megger

In the original "ohmmeter" devised by Ayrton and Perry, and shown in fig. 19, two *fixed coils* and a *moving magnet* or needle were provided. The instrument was thus of the "needle" type. In Evershed's Megger this arrangement is inverted, there being a powerful *fixed magnet* and two

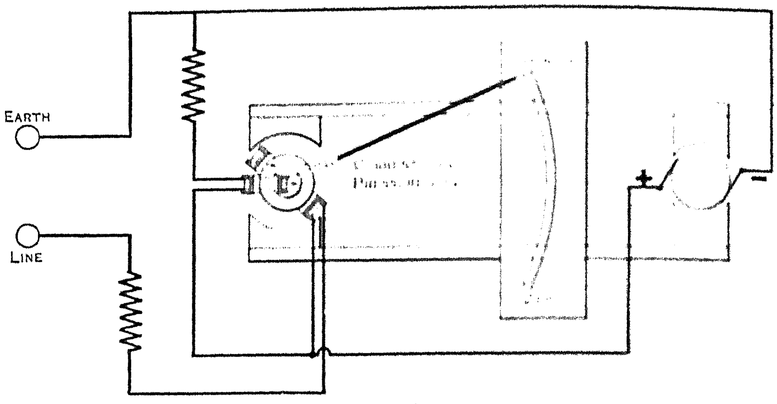


Fig. 21.—Diagram showing Principle of Megger.

moving coils. The "Megger" thus belongs to the moving coil type of instrument so largely represented nowadays by voltmeters and ammeters

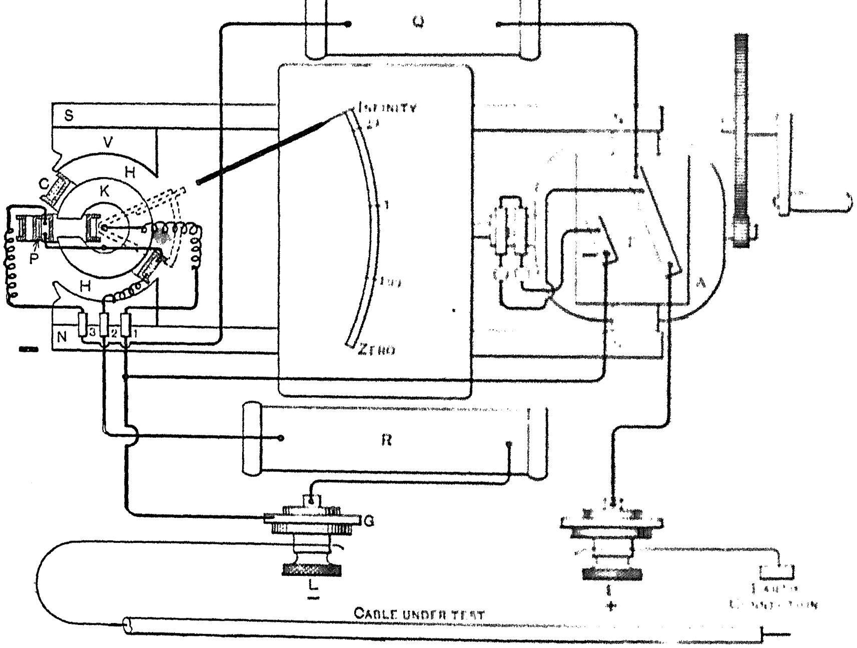


Fig. 22.—Evershed's Patent Megger. Explanatory Diagram.

A, Generator Armature, B, Generator Commutator; C, Ohmmeter Current Coil; E, Earth Terminal; G, Guard Plate; HH, Annular Air Gaps; K, Iron Core; L, Line Terminal; NS, Field Magnets; P, Pressure Coils; Q, Resistance in series with Pressure Coils; R, Resistance in series with Current Coils; T, Terminals of Generator; V, Pole-pieces of Ohmmeter; 1, Common Terminal of Current and Pressure Coils; 2, Terminal of Current Coil; 3, Terminal of Pressure Coil.

for continuous currents (see p. 39). The general arrangement of the instrument is shown in fig. 21. It will be noticed that the two permanent bar magnets are fitted with pole-pieces at each end. The pole-pieces at the left-hand end provide the instrument field, while those at the right hand end provide the generator field. By making the same set of magnets serve this double purpose, great economy of space is effected, and a very compact form of construction is secured. The two moving coils are rigidly attached to a common axle, which carries the pointer of the instrument. The coils make a certain angle with each other. One of them, known as the *current coil*, is connected in series with the unknown resistance and the generator, a safety resistance being also included in the circuit. The other coil, known as the *pressure coil*, is connected in series with a very high resistance and across the generator terminals. The hollow, soft-iron cylindrical core placed between the pole-pieces is slotted on one side as shown, so as to admit one half of the pressure coil. It is evident that, since this coil will tend to move so as to make its axis coincide with that of the field in which it is placed, it will take up the position shown in the figure when there is no current flowing through the current coil; this will happen when the external resistance is infinite, and the pointer will then stand at the "infinity" division of the scale. If, however, the unknown resistance is not infinite, a current will pass through the current coil and this coil will be deflected (as in a moving-coil galvanometer, p. 39) in a clockwise direction, the deflection being opposed by the couple acting on the pressure coil, which moves into a field of increasing intensity as the deflection increases. The coil system is quite free to move under the action of the resultant couple due to the two coils, there being no control, as the currents are led into the coils by means of extremely fine strips, whose control is negligible.

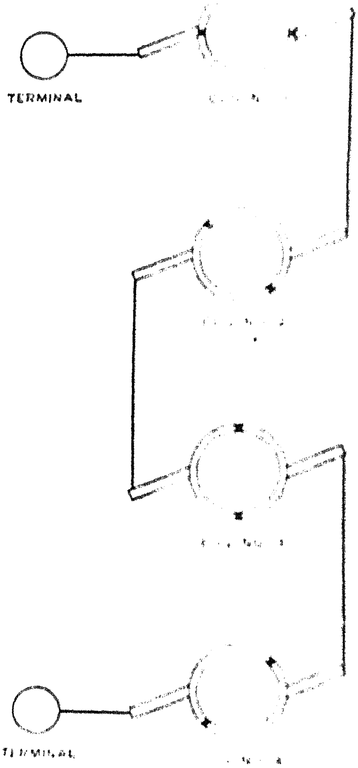


Fig. 2[illegible]. Diagram of [illegible] shed's Portable Megger [illegible]

Some further details of the instrument are shown in fig. [illegible]. In order to prevent the instrument from being affected by external fields, the pressure coil is made astatic[1] with respect to such fields by having a small external compensating coil attached to it on one side, as shown in fig. [illegible]. The number of turns in this compensating coil is so adjusted that the couple exerted on it by any uniform field is equal and opposite to the couple exerted by the same field on the main pressure coil.

The connections of the generator are shown in fig. 23. Depending on the useful range of the instrument, the generator is constructed to give voltages of 100, 250, 500, and 1000 at the speed when slipping begins to take place.

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[1] *I.e.*, constructed so as to be unaffected by any *uniform* field.

A high-testing voltage is essential, as a defect in the insulation under test may not be detected with a low-testing voltage. The armature of the generator consists of several independent coils spaced at equal angular distances from each other, and each coil is provided with a two-part commutator. The several commutators are then connected in series, as shown in fig. 23. The E.M.F. developed by such a generator is not absolutely steady, but fluctuates between certain limits. Double reduction toothed gearing is used for driving the generator, as shown in fig. 22. The normal speed of the driving handle is 100 revolutions per minute.

Instruments of this type are made having a useful range up to 2000 megohms.

CHAPTER V

AMMETERS AND VOLTMETERS

Definition of Practical Unit of Current. The following is the Board of Trade definition of the ampere:

The ampere, which has the value $\frac{1}{10}$ in terms of the centimetre, the gramme, and the second of time, is represented by the unvarying electric current which, when passed through a solution of nitrate of silver in water, . . . deposits silver at the rate of ·001118 of a gramme per second.

Instruments intended for the measurement of currents are termed *ammeters.*

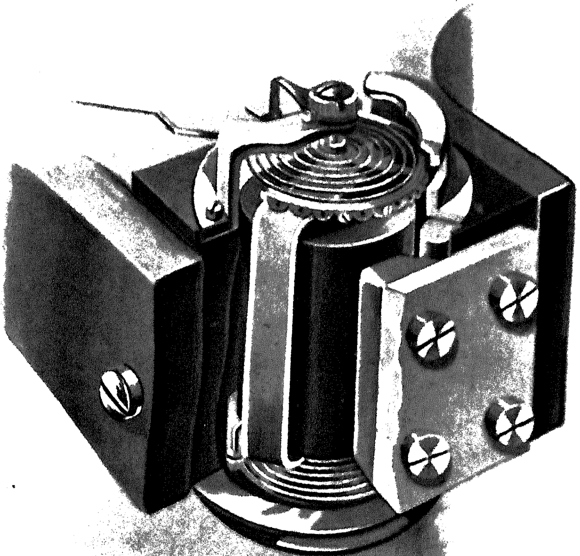
Ammeters.—In order that the introduction of an ammeter may not disturb the existing conditions of a circuit, the resistance of the instrument must be negligible in comparison with that of the remainder of the circuit. This is one reason why it is desirable to make the resistance of an ammeter *as low as possible.*

There is also, however, another reason. The power absorbed by an ammeter is spent in producing heat in the instrument, and the amount of power so used is given by ri^2, where r is the resistance of the ammeter, and i the current. The power wasted is therefore reduced to its smallest practicable amount by making r as small as possible.

In the construction of ammeters, we may make use of either the heating or the magnetic effect of an electric current.

Hot-Wire Ammeters.—A hot-wire ammeter consists essentially of a short piece of platinum-silver wire which is heated by the current, and a suitable arrangement for magnifying the expansion of the wire when hot. Such instruments are of special importance in connection with alternating currents, and a detailed description of their construction will be found in the section of this treatise which deals with alternating-current measurements.

Electrodynamic Ammeters—Siemens Electrodynamometer. The dynamical action (explained in Chapter II.) which exists between two conductors



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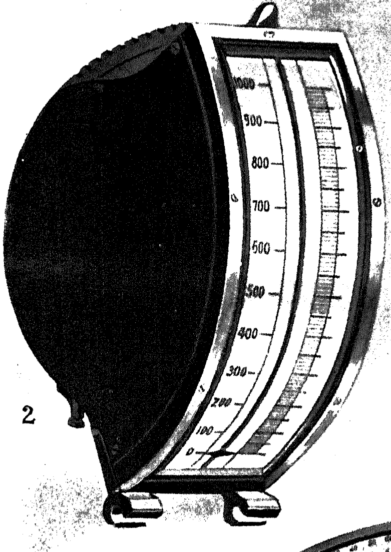

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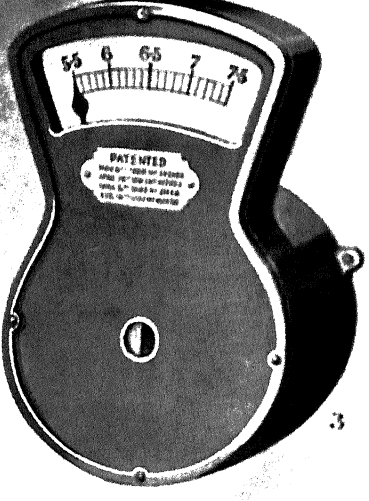

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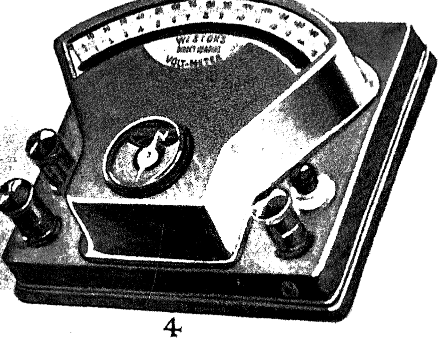

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1, Weston Permanent-Magnet Voltmeter; 2, Weston Permanent-Magnet Ammeter (Edgewise Pattern); 3, Weston Arc-Lamp Ammeter (Sector Pattern); 4, Weston Laboratory Type Moving-Coil Voltmeter

SOME MEASURING INSTRUMENTS. II

conveying currents is utilized in the construction of an important class of instruments, which we may call *electrodynamic* instruments. One of the oldest and best-known of these is the *Siemens electrodynamometer*, illustrated in fig. 24. It consists of a fixed and a movable coil arranged with their planes at right angles to each other, and connected in series. The movable coil is suspended by a silk thread, and the current enters and leaves it by two mercury cups into which the ends of the coil dip. This coil carries a pointer bent so that its extremity is capable of moving, through a small range limited by two stops, over a graduated horizontal circular scale. Over the same scale may be moved by hand a pointer attached to the torsion head of a fine gilt steel spiral spring, whose lower end is rigidly fixed to the top of the movable coil. When a current passes through the instrument, the pointer attached to the movable coil is thrown against the right-hand stop; but by turning the torsion-head and so twisting the spiral spring, the movable coil may be restored to its normal or zero position.

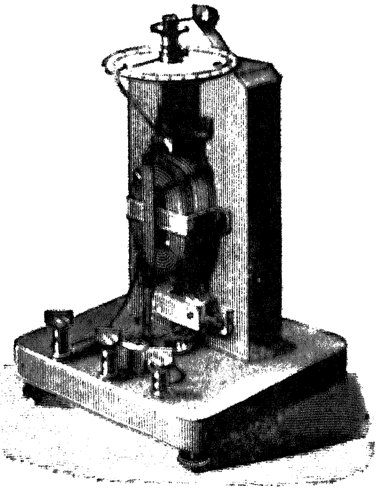

Fig. 24.—Siemens Electrodynamometer.

The Siemens electrodynamometer obeys a very simple law. When the movable coil is in its zero position, the couple due to the torsion of the spring, which is proportional to the angle of torsion, balances the electrodynamic couple due to the current. This latter, however, varies as the *square* of the current; for it is proportional to the product of the current through the movable coil into the field in which this coil is placed; the field, however, is that produced by the fixed coil, and is proportional to the current flowing through it, which is the same as the current through the movable coil. Thus the current $= \text{constant} \times \sqrt{\text{angle of torsion}}$.

In order to prevent the earth's field from exerting a couple on the movable coil, the instrument must be so placed that the plane of this coil is perpendicular to the earth's field,

In the illustration, three terminals are shown. The middle terminal, which is always used, is connected to the lower mercury cup, and so to one end of the movable coil. The other end of this coil dips into the upper cup, which is connected to the junction of two fixed coils, only one of which is in use at a time. The other ends of these coils are connected to the two extreme terminals. One of the fixed coils consists of a comparatively large number of turns of thin wire, and would be used for the measurement of small currents, while the other is wound with thick wire, has only a few turns, and is intended for the measurement of larger currents.

Lord Kelvin's Current Balances.—Electrodynamic instruments known as Current Balances have been constructed by Lord Kelvin. They possess a very high degree of accuracy and are frequently used as *standard* instruments. The general appearance of such an instrument will be seen fro fig. 25. As in the Siemens electrodynamometer, we here have to dea with a system of fixed and movable coils. By using two movable coil which carry currents in opposite directions, an *astatic* movable system i obtained. The two movable coils are mounted at the ends of a balanc

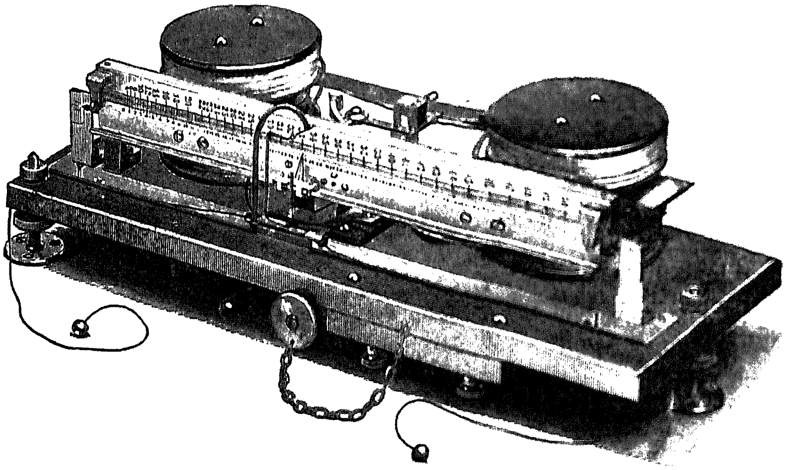

Fig. 25.—Kelvin's Deciampere Balance

beam, and above and below each movable coil is a fixed coil. The direction of the currents is shown in fig. 26, and it will be seen that each pair of fixed coils produces a radial horizontal field in which is placed the movable coil. The connections are such that the left hand coil tends to move down while the right-hand one tends to move up. By reasoning similar to that employed in the description of the Siemens electrodynamometer, it is easy to see that the deflecting couple acting on the movable system is proportional to the *square* of the current. This deflecting couple is balanced by sliding a weight along the finely-divided or movable scale attached to the balance beam. Since the balancing couple varies as the displacement of the weight, it follows that the current is proportional to the *square root* of the displacement. The fixed or inspectional scale seen above the finely-divided scale in fig. 23 has divisions proportional to the square roots of the corresponding divisions on the movable scale, and the value of the current is obtained by multiplying the fixed scale readings by a certain constant.

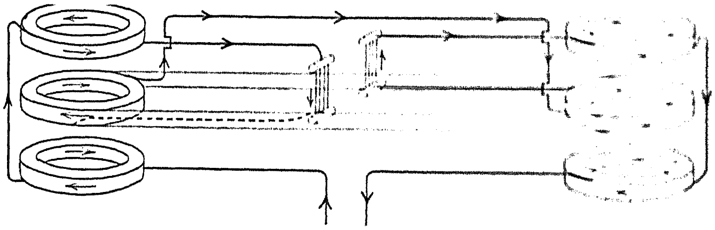

Fig. 26

One of the chief features of these instruments is the ingenious manner in which large currents may be conveyed into and out of the movable system of coils without interfering with the flexibility of the suspensions. The method adopted is shown diagrammatically in fig. 26. The weight of the balance beam is borne by a double set of suspending "ligaments", which consist of a number of very fine copper wires laid side by side and soldered to suitably insulated contact-pieces. The current enters the movable coils by one set of ligaments, and leaves by the other. Hundreds of fine copper wires are required in the case of instruments intended for the measurement of very large currents.

Lord Kelvin's current balances are among the most accurate and reliable electrical measuring instruments constructed, and are very frequently employed for checking other instruments. They are not intended to be used as ordinary commercial measuring instruments.

The scale of a Kelvin balance is rendered very open over the greater portion of the range of the instrument by the simple device of providing three or four different sets of sliding weights which are simple multiples of each other. A different constant will then correspond to each sliding weight. In order to maintain equilibrium of the balance beam in the zero position of each weight, a set of counterpoise weights is provided. These latter are made cylindrical, and are supported in the triangular trough seen to the right of fig. 25, a cross-pin which passes through the counterpoise fitting into a hole at the bottom of the trough and thus giving the weight a perfectly definite position with respect to the beam. The sliding weights fit over a carriage (clearly shown in fig. 25), which may be moved along the beam by pulling one or other of the two strings attached to a self-releasing pendant and passing outside the glass case which is placed over the instrument when in use.

The Siemens electrodynamometer and the Kelvin balance are both examples of *zero instruments*, *i.e.* instruments requiring an adjustment before a reading can be taken. In an instrument intended for ordinary commercial work the necessity for such an adjustment would be considered a very serious disadvantage. Hence all ordinary commercial instruments are direct-reading *deflectional* instruments; *i.e.* they indicate the value of the current by the position of a pointer moving over a scale marked in amperes, no preliminary adjustment or reference to tables or multiplication by a constant being required.

An important feature of the electrodynamic class of instruments (which they share with instruments of the hot-wire type) is that they are capable of giving correct readings on alternate-current circuits without requiring recalibration.

Permanent-Magnet Ammeters—Weston Ammeter.—One of the most commonly used types of ammeter is that in which a coil of wire is pivoted between the poles of a permanent magnet, and by its deflection when conveying a current indicates the amount of current passing. This type of ammeter has recently come much into favour, and is being manufactured in a variety of different forms. Such an instrument is really a moving-coil galvanometer, similar to the one illustrated in fig. 9. The deflecting couple due to the current is balanced by the controlling couple due to the coili of two fine hair-springs, by one of which the current enters the coil, a leaves it by the other. It is evident that the cross-section of the sprin must always be small, otherwise a very powerful deflecting couple—d to the great rigidity or stiffness of the springs—would be required produce the necessary deflection; and a difficulty thus arises with lar currents, which would either fuse or seriously damage the fine sprin if allowed to pass through them. This difficulty is overcome in a ve

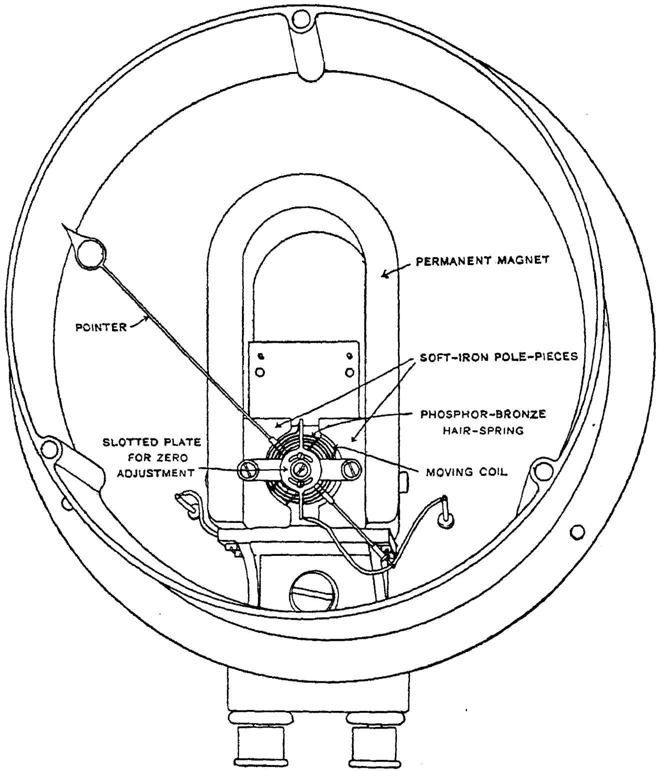
PERMANENT MAGNET
POINTER
SOFT-IRON POLE-PIECES
PHOSPHOR-BRONZE HAIR-SPRING
SLOTTED PLATE FOR ZERO ADJUSTMENT
MOVING COIL

Fig. 27.—Weston Ammeter

simple and effective manner by allowing only a small fraction of the current to pass through the moving coil, the remainder flowing through a special resistance or shunt placed across the terminals of the instrument. Where very large currents, amounting to hundreds of amperes, have to be measured, it is usual to make the shunt quite separate from the instrument proper, which is then connected across the shunt terminals by means of flexible leads supplied for the purpose. For moderately large currents, however (say below 100 amperes), it is usual to enclose the shunt in the ammeter case.

One of the best-known forms of permanent-magnet ammeter is the Weston ammeter. The construction of one of these instruments is shown in fig. 27, which represents a switchboard type of ammeter with the front cover removed. To the poles of a horse-shoe magnet are attached soft-iron pole-pieces, hollowed out so as to form a cylindrical cavity, into which is introduced a soft-iron cylinder whose diameter is less than that of the cavity, an annular space being thereby formed. The moving coil embraces the cylindrical core, its sides being capable of moving in the annular space. The hair-springs attached to the ends of the coil are made of phosphor-bronze. The outer end of the top spring is soldered to an arm projecting from a small slotted circular plate, clearly seen in the illustration. By slackening the two small screws which fix this plate the plate may be rotated through a small angle, and thus the position of the pointer adjusted to the zero of the scale when no current is passing through the instrument. The pointer is of aluminium, and the coil is wound on an aluminium frame which, by the damping effect produced during its motion across the field, renders the instrument extremely dead-beat. The springs are connected to two straight wires seen projecting through the back plate of the instrument, and on the other side of this plate the wires are soldered to the shunt terminals, as shown in fig. 28.

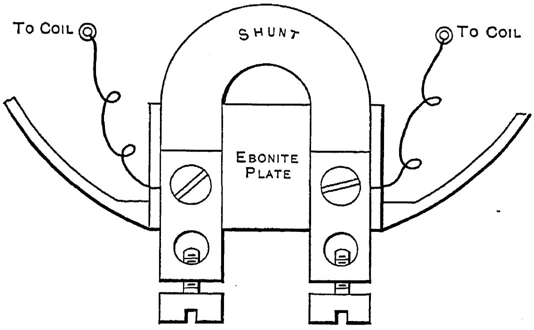

Fig. 28

Permanent-magnet instruments belong to the class of *polarized* instruments, *i.e.* a reversal of current produces a reversal of the deflection. In a well-constructed instrument there should be no appreciable weakening of the magnet with age; a sudden weakening may take place if an excessive current is momentarily passed through the instrument, as such a current exerts a demagnetizing effect on the magnet.

If the magnetic field in which the coil moves be uniform, the deflecting couple will vary in proportion to the current, and the scale will be one of equal divisions.

Soft-Iron Ammeters.—Ammeters in which soft iron is used form a very numerous class. They are generally of relatively simple construction, and some forms are obtainable at very low prices. All soft-iron instruments suffer more or less from a defect which is known as the *hysteresis error*, and on account of which the instrument gives higher readings when descending than when ascending the scale.[1]

The principle on which all such instruments act may be very briefly stated by saying that a mass of soft iron always tends to move from weak to strong regions of a magnetic field. The movable part of the instrument consists of a piece of soft iron, and the fixed part is some device—either a simple coil or a coil provided with a core—for producing a field of varying intensity by means of the current to be measured.

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[1] The reader will readily understand how the hysteresis error is caused by referring to Chapter VII, on magnetic measurements.

As an example of this type of instrument we shall consider

Lord Kelvin's Ampere Gauge.—This is illustrated in fig. 29, and consists of a fixed coil of wire and a movable, very thin, soft-iron wire suspended from a sector attached to a horizontal axis carrying a pointer. In the normal position of the movable part the lower end of the wire enters the coil; and this latter exerts a sucking action on the wire when a current is sent through it. The controlling force is furnished by gravity.

Calibration of Ammeters.—An ammeter may be *calibrated*, *i.e.* the true values of its readings ascertained, by one of the following methods:

1. By the use of a voltameter.
2. By comparison with a standard instrument.
3. By the potentiometer method.

Use of Copper Voltameter.—Although in the legal definition of the ampere special reference is made to the silver voltameter, and a schedule is appended which contains detailed instructions regarding the use of such a voltameter, we shall here consider only the copper voltameter, as this is more suitable for the measurement of large currents. The silver voltameter can be conveniently used for currents up to 1 ampere; its cost would be prohibitive for larger currents.

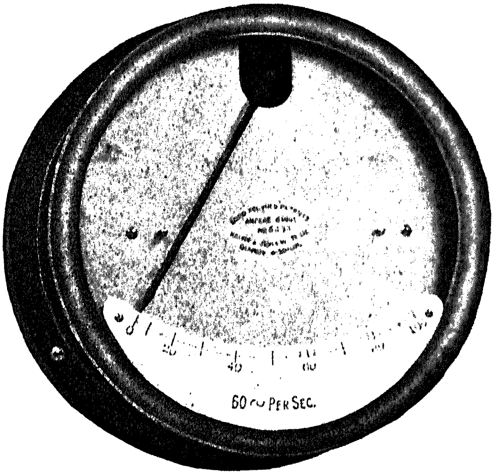

Fig. 29.—Kelvin's Ampere Gauge

Although the electrochemical equivalent of an ion is an absolute constant of nature, it is found that the values obtained by experiment in the case of copper deposited from a solution of copper sulphate vary appreciably under different conditions of working. Such variations are not real, but only *apparent*, and are due to the solvent action of the electrolyte on the freshly-deposited copper. This solvent action depends on the density of the solution and on the temperature, and in order to obtain reliable results with a copper voltameter the following precautions should be observed:—

The solution is prepared by dissolving pure recrystallized sulphate of copper in clean ordinary tap-water until a density of 1.18 is reached, and then adding 1% by volume of strong sulphuric acid. It should be replaced by a freshly-prepared solution when the aggregate time during which it has been in use amounts to ten hours.

The area of the cathode plate or plates should be such that there are not less than 50 sq. cms. per ampere of current, and the area of the anode plates should always be in excess of that of the cathodes.

The deposition is allowed to go on for about one hour, the current being maintained constant by means of a suitable rheostat.[1] The cathode plates are taken out of the solution immediately after the current has been switched off, and are plunged into a vessel filled with water which has been acidulated with a few drops of sulphuric acid. They are then thoroughly washed and dried, first by the application of warm white blotting-paper and then by being held in front of a clear fire or over a spirit-lamp, care being taken not to allow them to get appreciably hot.

The values of the *apparent* electrochemical equivalent of copper are given in the following table, prepared by Prof. T. Gray:—

Area of Cathode in sq. cms. per ampere.	Values of Apparent Electrochemical Equivalent of Copper.				
	2° C.	12° C.	23° C.	28° C.	35° C.
50	.0003288	.0003287	.0003286	.0003286	.0003282
100	.0003288	.0003284	.0003283	.0003281	.0003274
150	.0003287	.0003281	.0003280	.0003278	.0003267
200	.0003285	.0003279	.0003277	.0003274	.0003259
250	.0003283	.0003278	.0003275	.0003268	.0003252
300	.0003282	.0003278	.0003272	.0003262	.0003245

The copper voltameter method of calibrating an ammeter is especially suitable in the case of instruments obeying a known law, such as the Siemens electrodynamometer and Kelvin balance. In the case of other instruments, a single determination with the voltameter only enables us to verify the accuracy of the instrument for one particular scale reading. The voltameter method, which is the one actually employed in the standardization of Lord Kelvin's balances, is thus unsuitable for calibrating commercial instruments over their entire scale. For this purpose the next method is more convenient.

Calibration of Ammeters by Comparison with a Standard Instrument.—The instruments used as standards for this purpose are generally Lord Kelvin's current balances. The instrument to be calibrated is connected in series with the Kelvin balance, a suitable rheostat and a number of secondary cells, and a series of simultaneous readings is obtained with both instruments. A calibration curve may then be plotted, instrument readings being measured horizontally and their correct values vertically; or an "error curve" may be plotted instead, whose abscissæ (horizontal distances) represent instrument readings, and ordinates the amount to be added to, or subtracted from, these readings in order to obtain the correct values of the current as given by the standard instrument.

Potentiometer Method of Calibrating Ammeters.—In this method the ammeter is joined in series with a known standard resistance (preferably an exact submultiple of 10), and the value of the P.D. across this resistance is measured by means of an instrument known as a "potentiometer" (described below). The P.D. divided by the value of the standard resistance gives the true value of current corresponding to the reading of the ammeter.

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[1] Rheostats consisting of troughs of mercury of variable length are very convenient for this purpose.

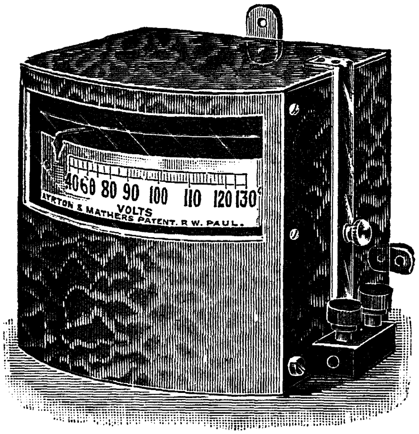


Fig. 30.—Ayrton and Mather Electrostatic Voltmeter

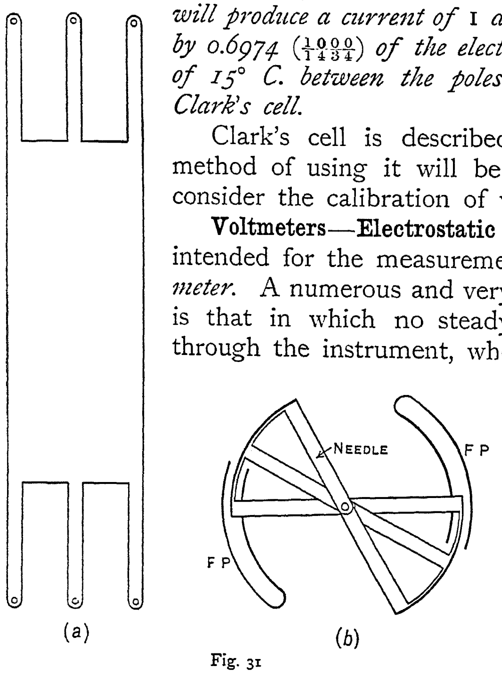
In the calibration of ammeters it is important to have all the connecting wires or cables arranged so that there is no direct magnetic effect produced on the instruments by the current flowing along the connecting wires. The latter should be preferably twisted so as to form a twin conductor, all large loops or coils being carefully avoided. Neglect of this precaution may give rise to serious error.

Definition of Practical Unit of P.D. and E.M.F.—The following is the legal definition of a volt:—

The volt, which has the value 10^8 *in terms of the centimetre, the gramme, and the second of time, is the electrical pressure that, if steadily applied to a conductor whose resistance is 1 ohm, will produce a current of* 1 *ampere, and which is represented by 0.6974* $\left(\frac{1000}{1434}\right)$ *of the electrical pressure at a temperature of 15° C. between the poles of the voltaic cell known as Clark's cell.*

Clark's cell is described in Chapter VIII, and the method of using it will be explained when we come to consider the calibration of voltmeters.

Voltmeters—Electrostatic Instruments.—An instrument intended for the measurement of P.D.s is termed a *voltmeter.* A numerous and very important class of voltmeters is that in which no steady current is allowed to flow through the instrument, whose deflection is produced by the electrical attractions and repulsions exerted between electrified bodies. Such instruments are called *electrostatic* voltmeters. Most of them may be regarded as modifications of the quadrant electrometer. They possess several advantages over other types of voltmeter: taking no current, they absorb no power; they are capable of reading correctly on alternate-current circuits; and they are free from certain errors which affect other types.



(a) (b)

Fig. 31

A description of Lord Kelvin's electrostatic voltmeters will be found in the section of this work which deals with alternate-current measurements. We shall here describe two other forms, viz. the Ayrton and Mather, and the Addenbrooke instruments.

Ayrton and Mather Electrostatic Voltmeters. — The general appearance of one form of this instrument is shown in fig. 30, and some details of construction are given in fig. 29. The movable part or *needle* of the instrument consists of two aluminium sheets stamped out to the shape shown in fig. 31 (*a*). The central portion of each is then bent so as to form part of the surface of a cylinder, and the projecting pieces are bent at right angles to the axis of the cylinder. The axle of the instrument is then threaded through these radial supports, and the needle appears as shown in plan in fig. 31 (*b*). The axle is pivoted in jewelled bearings, and the controlling couple is furnished by phosphor-bronze hairsprings similar to those used in the Weston ammeters. To the top of the axle is attached a light horizontal pointer of aluminium whose extremity is bent so as to move over the cylindrical scale of the instrument; this is clearly shown in fig. 30. The fixed plates, marked F P in fig. 31 (*b*), consist of two double cylindrical surfaces formed of sheet brass. These two plates are connected to one terminal of the instrument, and the needle to the other. When a difference of potential is established between the needle and the fixed plates, the former is sucked into the space enclosed by the latter, and a deflection is obtained.

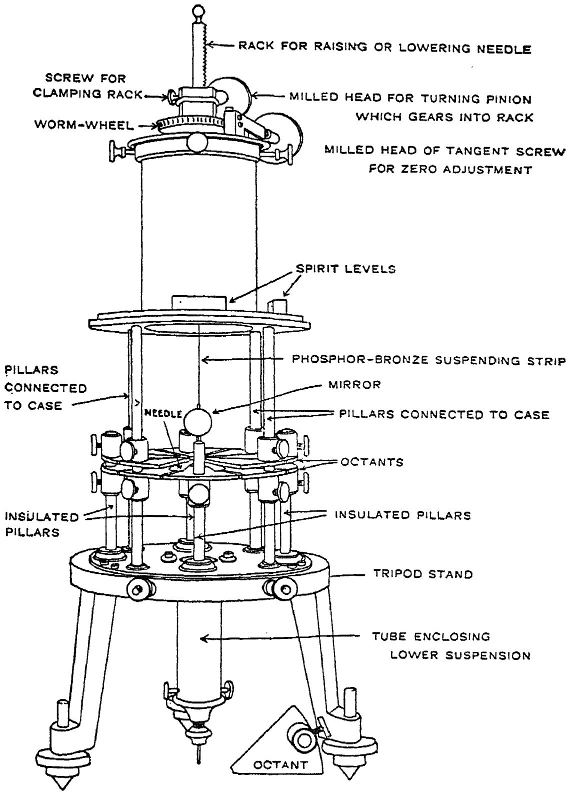

RACK FOR RAISING OR LOWERING NEEDLE
SCREW FOR CLAMPING RACK
MILLED HEAD FOR TURNING PINION WHICH GEARS INTO RACK
WORM-WHEEL
MILLED HEAD OF TANGENT SCREW FOR ZERO ADJUSTMENT
SPIRIT LEVELS
PHOSPHOR-BRONZE SUSPENDING STRIP
PILLARS CONNECTED TO CASE
MIRROR
NEEDLE
PILLARS CONNECTED TO CASE
OCTANTS
INSULATED PILLARS
INSULATED PILLARS
TRIPOD STAND
TUBE ENCLOSING LOWER SUSPENSION
OCTANT

Fig. 32.—Addenbrooke Reflecting Electrostatic Voltmeter

Instruments of the form described are made for various ranges from 40 up to 600 volts. For still higher P.D.s, a modified form, in which gravity furnishes the controlling force, is employed. The axle is in this case horizontal, and the construction of the needle and fixed plates is somewhat different.

Addenbrooke Reflecting Electrostatic Voltmeter.—The construction of a satisfactory electrostatic voltmeter capable of reading accurately small P.D.s (of the order of 1 or 2 volts), whether continuous or alternating, is a matter presenting considerable difficulty.[1] These difficulties hav been successfully surmounted by Mr. Addenbrooke, who has designe the instrument shown (with the lower part of the case removed) i fig. 32. The needle is a double one, each half consisting of a cross o very light sheet brass, as shown in fig. 33, and these two crosses ar displaced relatively to each other through an angle of 45°, and are insulate from each other by a disc of mica. Top and bottom suspensions of ver thin phosphor-bronze strip are employed. The top suspending stri supports the weight of the needle and serves to connect the upper ha of it to the case of the instrument. The height of the needle is adjustabl by means of a rack an pinion movement, the rac is seen projecting outsid the upper part of th case in fig. 32, and it position in a horizont plane may be roughl adjusted by rotating th circular plate which su ports the top suspensio and finely adjusted b means of a tangent screw The strip connected t the bottom half of th needle is left slack, an its lower end is soldered t the copper wire seen projecting in fig. 32 through the tube which enclose the strip. This tube i *insulated* from the case.

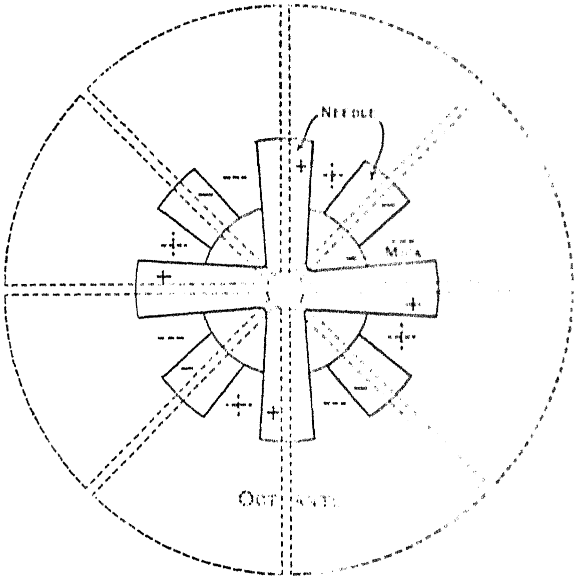


Fig. 33

The fixed plates consist of two circular brass plates divided into octants. One of these octants, removed from its usual position, is seen lying close t the right-hand levelling-screw in fig. 32; at the middle of its curved oute edge is a hollow cylindrical projection with a clamping screw, which enable it to be rigidly clamped to the supporting pillar. Each pillar carries tw octants, one being below and the other above the needle. The positions o the octants relatively to the needle are indicated by the dotted lines i fig. 33. By sliding the octants up or down the pillars, the distance of th octants from the needle, and with it the sensitiveness of the instrumen may be varied. One set of four alternate pillars is insulated and connecte to the bottom half of the needle and the insulated terminal of the instru ment, while the remaining set of four alternate pillars is connected to th case, and through it to the upper half of the needle and the uninsulate terminal.

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[1] For many years, Lord Kelvin's quadrant electrometer has been available for measuring electrostatically small *continuous* P.D.s; but when so arranged (the needle having an independent charge) th instrument is incapable of measuring *alternating* P.D.s.

If we suppose that the terminals of the instrument are connected to a source of E.M.F., then the needles and octants become charged as shown in fig. 33, and it is at once evident that a deflection of the needle will result.

The instrument is shown in fig. 32 with the lower part of the case removed. This is provided with four mica windows arranged at the extremities of two mutually perpendicular diameters, and by this means the octants, needle, and mirror may be observed without having to remove the case, and the ray of light is allowed to enter and leave the instrument by one of the mica windows.

The usual lamp-and-scale method of reading the deflections is used in connection with this instrument, and the mirror is clearly seen in fig. 32 above the octants.

All electrostatic instruments have to be screened from external electrostatic disturbances, and for this purpose they are nearly completely enclosed in a metallic case. Some opening must, however, be left for reading the scale, and this part of the case is made of glass. It has been found that a very slight amount of friction, such as that obtained by wiping the glass with a handkerchief, is capable of electrifying the glass to an extent which is sufficient to seriously affect the readings of the instrument.[1] In order to eliminate this source of disturbance, Prof. Ayrton and Mr. Mather have devised a transparent *conducting* varnish with which the glass may be coated, and thereby rendered as efficient a screen as one made of metal.

Resistance of Voltmeters and Temperature Error.—One consideration which determines the amount of resistance to be given to a voltmeter whose action depends on the passage of a *current* is that the introduction of the instrument into the circuit should not materially affect the previously existing conditions of the circuit—*i.e.*, that the total resistance of the part of the circuit included between the two points across which the P.D. is to be measured should not be materially affected by connecting up the voltmeter. Now the change in the total resistance between the two points will be made as small as possible by making the resistance of the voltmeter *as high as possible*.

Suppose the voltmeter resistance to be denoted by r, and the P.D. across its terminals by V; then the power absorbed by the instrument is V^2/r. Hence for a given value of V the power wasted in heating the instrument will be reduced by increasing r. This furnishes another reason for giving voltmeters a high resistance.

In the case of all voltmeters except those of the electrostatic type, the deflection is produced by the *current*, and since this latter depends on the P.D. and the *resistance* of the instrument, it follows that if the latter is, under all conditions, to give the same deflection with the same P.D. across its terminals, its resistance must be constant, and must not be affected by either temperature changes in the surroundings or temperature changes brought about by the heating effect of the current. In order to secure this condition, the bulk of the resistance of a voltmeter usually consists of a resistance coil wound with a wire having very small temperature coefficient (German silver, platinoid, manganin &c.), while the *working coil* consists of copper wire, the use of whic enables this coil to be made much lighter and more compact than i the high-resistance and low temperature coefficient material wer employed throughout.

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[1] This difficulty is by no means confined to electrostatic instruments; some other delicate instruments are equally liable to be affected by electrification of their glass covers.

The resistance of the voltmeter circuit cannot be made absolutel constant at all temperatures, and any error arising from temperatur changes is referred to as the *temperature* or *heating error*.

It is obvious that no such error exists in the case of ammeters.

Hot-Wire Voltmeters. Hot-wire voltmeters may be constructed o the same principle as hot-wire ammeters. Descriptions of such instruments will be found in the section on alternate current measurements. In common wit electrostatic instruments, hot-wire voltmeter possess the property of reading correctly o alternate current circuits when calibrated wit continuous currents.

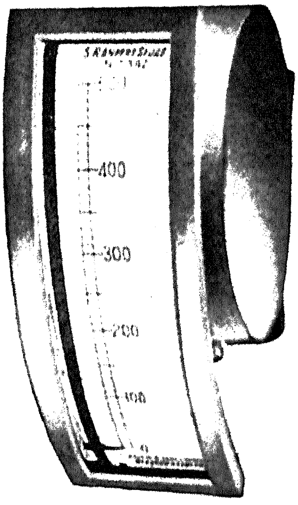

Fig. 34.—Kelvin's S.R. Edgewise Voltmeter

Electrodynamic and Electromagnetic Voltmeters. All the instruments under this hea which have already been described as ammeter may be wound as voltmeters. Fine wire woul be used in winding the working coil, and large additional resistance having a negligibl temperature coefficient would be connected i series with this coil. The Weston instrumen for example, illustrated in fig. 27, could b transformed into a voltmeter by removing th shunt across its terminals, and substituting f it a very high resistance connected *in seri* with the working coil.

Lord Kelvin's S.R. Type Voltmeter. A form of switchboard voltmete known as the "edgewise" pattern[1] (a form of construction first introduced by Mr. Evershed), one example of which is illustrated in fig. 3 has recently come much into favour on account of the small spac which it takes up on a switchboard. The instrument shown in fig. 3 is of the S.R. (siphon recorder) or moving-coil type, and is shown i fig. 35 with the outer case removed. The magnets are circular i shape. The moving coil and phosphor-bronze hair-spring are clearl seen in fig. 35, and other details of construction will be readily unde stood by an inspection of this figure.

Calibration of Voltmeters.—Voltmeters may be calibrated by comparison with a standard instrument, or by the potentiometer metho It is needless to remark that in comparing the readings of two vol meters these instruments should be connected *in parallel*, and not i series as would be the case with ammeters. A Kelvin Centi-amper Balance connected in series with a high resistance, and one of Lor

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[1] Ammeters of a similar pattern are also largely used.

Kelvin's electrostatic voltmeters, are frequently used as standard instruments.

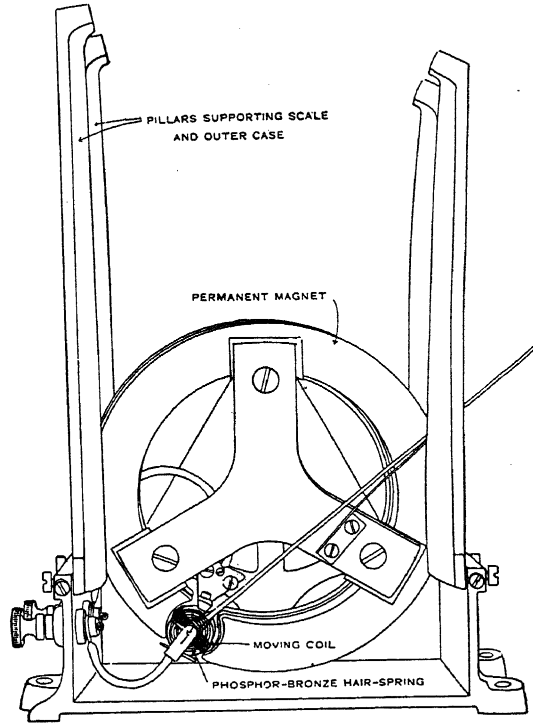
PILLARS SUPPORTING SCALE AND OUTER CASE
PERMANENT MAGNET
POINTER
MOVING COIL
PHOSPHOR-BRONZE HAIR-SPRING

Fig. 35.—Kelvin's S.R. Edgewise Voltmeter

The Potentiometer.—The principle underlying the potentiometer method was originally introduced by Poggendorff, and is illustrated in fig. 36. It is essentially a method for determining the ratio of two E.M.F.s or P.D.s. Let E be a source of constant E.M.F. (a battery of secondary cells) which maintains a constant current in the conductor A B, and let this conductor be so arranged that contact may be established at any point C of it (by means of a slider), the value of the resistance A C being accurately known. If while the current is flowing along A B we arrange a branch circuit between the points A and C, including a galvanometer G in this circuit, then the P.D. which exists between the points A and C will also produce a current in the branch circuit from A to C, and this current will be indicated by the galvanometer. Suppose next that an E.M.F. E_s is introduced into the branch circuit, the direction of this E.M.F. being from

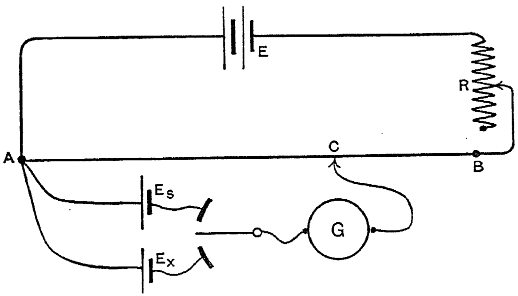

Fig. 36

C to A, *i.e.* in opposition to the P.D. existing between those two points. If we suppose that this E.M.F. is steadily increased, then the effect will be to steadily decrease the current in the branch circuit, and for a certain value of E_S—when it becomes just equal to the P.D. between A and C—the current will vanish, and the galvanometer will show no deflection. We then say that E_S *balances* the P.D. across A C. If the value of E_S be still further increased, a current will flow from C to A, the galvanometer deflection being reversed.

Instead of supposing that E_S is varied while the P.D. across A C remains constant, let us now take E_S to have a definite fixed value, and to represent the known E.M.F. of a standard cell. Balance may in this case be obtained by varying the P.D. across A C. This may be done in two ways: either by shifting the point of connection C along the wire A B while the current in A B remains constant, or else by keeping C fixed and altering the current along A B by means of an adjustable resistance R placed outside A B. Let us suppose that by either of these two methods balance has been obtained: then we know the value of the P.D. across A C, for this must equal the E.M.F. E_S of the standard cell. Again, if r_s be the resistance of A C corresponding to a state of balance, then the current along A C (or along A B) is given by E_S/r_s. Let, next, an unknown E.M.F. or P.D. of amount E_X be substituted for E_S[1] (this may be quickly done by means of the two-way switch shown in the diagram), and let balance be once more obtained *by shifting the point* C, *the current along* A B *remaining constant.* If r_x is the resistance of A C corresponding to balance in this second case, then E_X/r_x gives the current along A B, and since this has the same value as before, we must have $E_X/r_x = E_S/r_s$, whence $E_X = \frac{r_x}{r_s} E_S$. We thus have a method of obtaining an unknown E.M.F. in terms of a known one, and the arrangement shown diagrammatically in fig. 36 for doing this is known as a *potentiometer.*

A simple but crude and not very accurate form of potentiometer is one in which A B is represented by a bare wire of high-resistivity material stretched along a graduated scale, the diameter of the wire being as uniform as possible, so that the resistance of any portion of it may be assumed to be proportional to its length, and hence the ratio r_x/r_s equal to that of the corresponding two lengths of the potentiometer wire; these lengths may be read off directly on the scale. The point C would correspond to a knife-edge contact attached to a suitable slider.

The main objection to such an arrangement is the want of uniformity in the wire, and its liability to become accidentally damaged or destroyed. For these reasons, in all accurate forms of potentiometer,[2] the slide-wire either forms only a small fraction of the total resistance corresponding to A B, or else is entirely absent.

A good though not very convenient form of potentiometer may always be improvised by using two resistance-boxes, one to represent A C, the other C B, and, balance having been obtained with the standard cell and the value of A C noted, again obtaining balance with the unknown E.M.F. or P.D., by varying the resistances A C and C B in such a manner as to keep their sum constant. This is done by plugging an equivalent resistance in C B when a certain resistance in A B has been unplugged, and *vice versa.*

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[1] Care being taken to connect it up the right way—in opposition to the P.D. across A C.

[2] The best-known forms are those manufactured by Messrs. Crompton & Co., Ltd., Messrs. Elliott Brothers, and Messrs. Nalder Bros. & Co.

It will be noticed that when balance has been obtained, the standard cell is not required to give any *current.* If it is allowed to send large currents, its E.M.F. is lowered, and an error introduced into the measurement. In order to prevent the possibility of large currents passing through the standard cell, it is advisable, during the early stages of the measurement, to introduce a large resistance into the galvanometer and cell circuit, and then to cut this out as balance is approached.

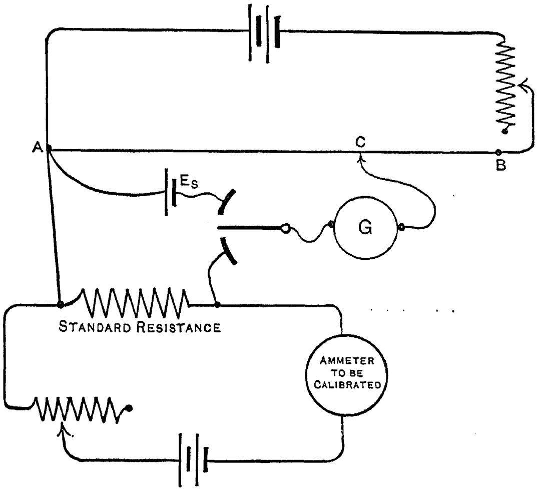

Fig. 37

The potentiometer itself as usually constructed is suitable for the measurement of only relatively small E.M.F.s —not exceeding about 1.5 volt. In combination with a standard resistance, it may be employed for the calibration of ammeters as already mentioned, the standard resistance taking the place of E_x in fig. 36 (E_x would be represented by the P.D. across the standard resistance). The complete diagram of connections is given in fig. 37.

The range of the instrument may be enormously extended, so that it may be used for the measurement of very high P.D.s, by the simple device of using a small, accurately-*known* fraction, of the unknown high P.D., and balancing this on the potentiometer.

Subdivision of P.D. by Means of Resistance—Calibration of Voltmeters.— A known fraction of an unknown P.D. is easily obtained by introducing between the two points across which the P.D. exists a resistance of known value, and using a known fraction of the total resistance. The P.D. across this section of the resistance then corresponds to the same fraction of the total P.D. This arrangement enables us to calibrate high-reading voltmeters by means of the potentiometer.

The diagram of connections is shown in fig. 38. A resistance r is connected in parallel with the voltmeter, so that the P.D. across this resistance is the same as that which exists between the voltmeter terminals. The P.D. across r_1, which is a known fraction of r, is then balanced on the potentiometer; if the value corresponding to balance is E_x volts, then the true value of the voltmeter reading is $E_x\, r/r_1$.

Recording Ammeters and Voltmeters.—In many cases it is desirable to have a continuous record of the current in a circuit, or of the P.D. across its terminals, and in such cases *recording* instruments are employed. The electrical and magnetic arrangements of such an instrument present no peculiarities and are similar to those used in ordinary indicating instruments. But the simple pointer is replaced by a tracing point or pen which traces out a record on a sheet of paper moved at a constant rate by clockwork in a direction which is at right angles to the motion of the tracing pen. One such form of recording instrument will be found described in a later chapter.

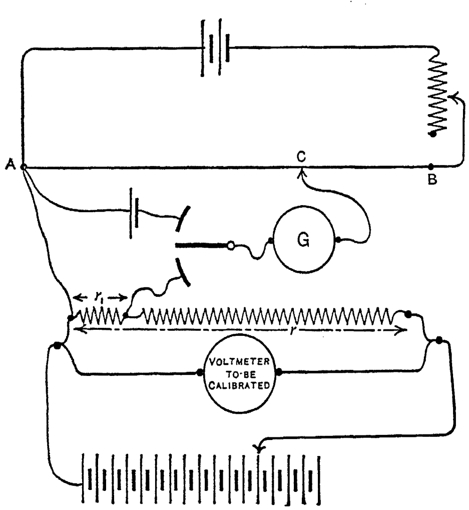

Fig. 38

Short-Scale Instruments.—Electrical energy is generally supplied at a constant P.D. and many of the voltmeters found in central stations are intended to enable the switchboard attendants or engine-drivers to maintain a constant P.D. between certain points. It is evident that under these conditions only a limited portion of the scale is used, readings which are either much higher or much lower than the normal P.D. not being required. It is an obvious advantage to make the working part of the scale as open as possible, and to suppress the remainder. This may be done in a variety of ways. For instance, in the case of a moving-coil instrument the field in which the coil moves may be made of variable strength by making the air-gap of variable length, the shortest length, and hence most intense field, corresponding to the normal reading; a given change in the current through the coil at the normal reading will then produce a much larger displacement of the coil across the field than at a reading either lower or higher than the normal.

Although short-scale instruments are mainly represented by voltmeters, the construction is equally applicable to ammeters, should such be required.

CHAPTER VI

WATTMETERS AND SUPPLY METERS

Wattmeters.—The power spent in any part of a circuit is, as we have seen, measured by the product of the P.D. into the current. This power is, in the most general case, used up in two ways: first, in producing *heat* in the part of the circuit considered; if r stand for the resistance of this part, and i for the current, then the rate of heat production is in every case given by $r i^2$ (not unfrequently, this is the only way in which the power is employed, and if so, $r i^2$ becomes identical with $V i$, where V is the P.D. across the part of the circuit considered); secondly, the power may be used in effecting some transformation of electrical energy into a form other than heat—*e.g.* mechanical or chemical energy. It is important to bear in mind that the second kind of energy transformation is *always* accompanied by the production of a counter-E.M.F. (or opposing E.M.F.), and that the corresponding rate of transformation, or power, is given by $E i$, where E is the counter-E.M.F. Assuming the existence of such a counter-E.M.F., the total power is given by $r i^2 + E i = V i$.

Instruments for measuring power are termed *wattmeters*. In the case of continuous-current circuits, if a voltmeter and ammeter are available, a simple multiplication of the volts and amperes gives us the power in watts. A wattmeter would, therefore, in this case be superfluous, although it might sometimes be convenient to replace the volt- and am-meter by a single instrument. In the case of alternating-current circuits, however, the *mean* power is not in general given by the product of the volts and amperes, and the wattmeter in this case performs a much more complicated mathematical operation than a simple multiplication. Hence, although wattmeters are sometimes used in connection with continuous currents, their chief use is on alternating-current circuits.

A wattmeter is essentially a combination of a volt- and am-meter. It is provided with two windings, one being a high-resistance winding, which represents the voltmeter part of the instrument; and the other a low-resistance winding, which corresponds to the ammeter part. Four terminals are frequently provided, one pair to each winding; in some cases, however, there are only three terminals (two large and one small), one of these corresponding to the junction of the two windings.

The high-resistance winding is frequently spoken of as the *pressure coil*, and the low-resistance one as the *current coil*, of the instrument. As in the case of voltmeters, the high-resistance winding consists of two parts, one being the working coil of copper wire, and the other, by far the larger portion of the resistance, a winding of high-resistivity, low-temperature-coefficient material, arranged so that it produces no appreciable magnetic effect (wound *non-inductively*, as it is termed).

Practically every form of wattmeter is an instrument of the *electro-dynamic* type. The thick-wire or current coil is fixed, and produces a field proportional to the current. In this field is placed the movable pressure coil, which carries a current proportional to the P.D. Hence for any given relative positions of the two coils, the couple acting at any instant on the movable coil is proportional to the product of the P.D. and current at that instant, *i.e.* to the instantaneous power. If the power fluctuates rapidly, as it does in alternating-current circuits, the mean couple acting on the movable coil will be proportional to the mean power.

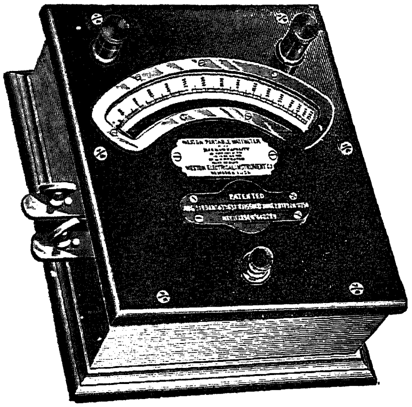
Fig. 39.—Weston Wattmeter

As a typical example of a good wattmeter, we shall consider the Weston Wattmeter, the general appearance of which is shown in fig. 39. The two heavy strips of copper seen projecting to the left of fig. 37 are the ends of the current coil, while the small terminals at the top of the case are in connection with the ends of the pressure circuit. The method of arranging the connections when using the wattmeter is shown in fig. 40, where T_c, T_c' are the terminals of the current coil, and T_p, T_p' those of the pressure circuit. The coiled line L represents that part of the circuit in which power is being measured. The case of the instrument is made of mahogany,[1] with the exception of the top, which is of ebonite, and which carries the coils. Fig. 41 shows the instrument with the top unscrewed and laid upside down on the case, so as to exhibit clearly the current and pressure coils. The controlling couple is furnished by the usual phosphor-bronze hair-springs.

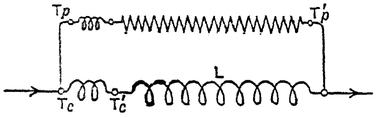

Fig. 40

Electric Supply Meters — Quantity and Energy Meters.—A form of electrical measuring instrument which is used in larger numbers than any other, is the instrument which is connected to the wiring of a consumer of electrical energy for the purpose of measuring the total amount of energy supplied to him. From the electrical point of view such instruments may be divided into two large classes, known as *quantity* meters (coulomb-meters, or integrating ammeters) and *energy*-meters (watt-hour meters, or integrating wattmeters).

Electrical energy is supposed to be supplied to a consumer at a constant, or approximately constant, P.D. Now, if the P.D. be assumed constant, it is evident that the power will be simply proportional to the current, and hence the energy to the quantity of electricity (in coulombs or ampere-hours) supplied. On the assumption of constancy of P.D., an instrument capable of measuring the quantity of electricity

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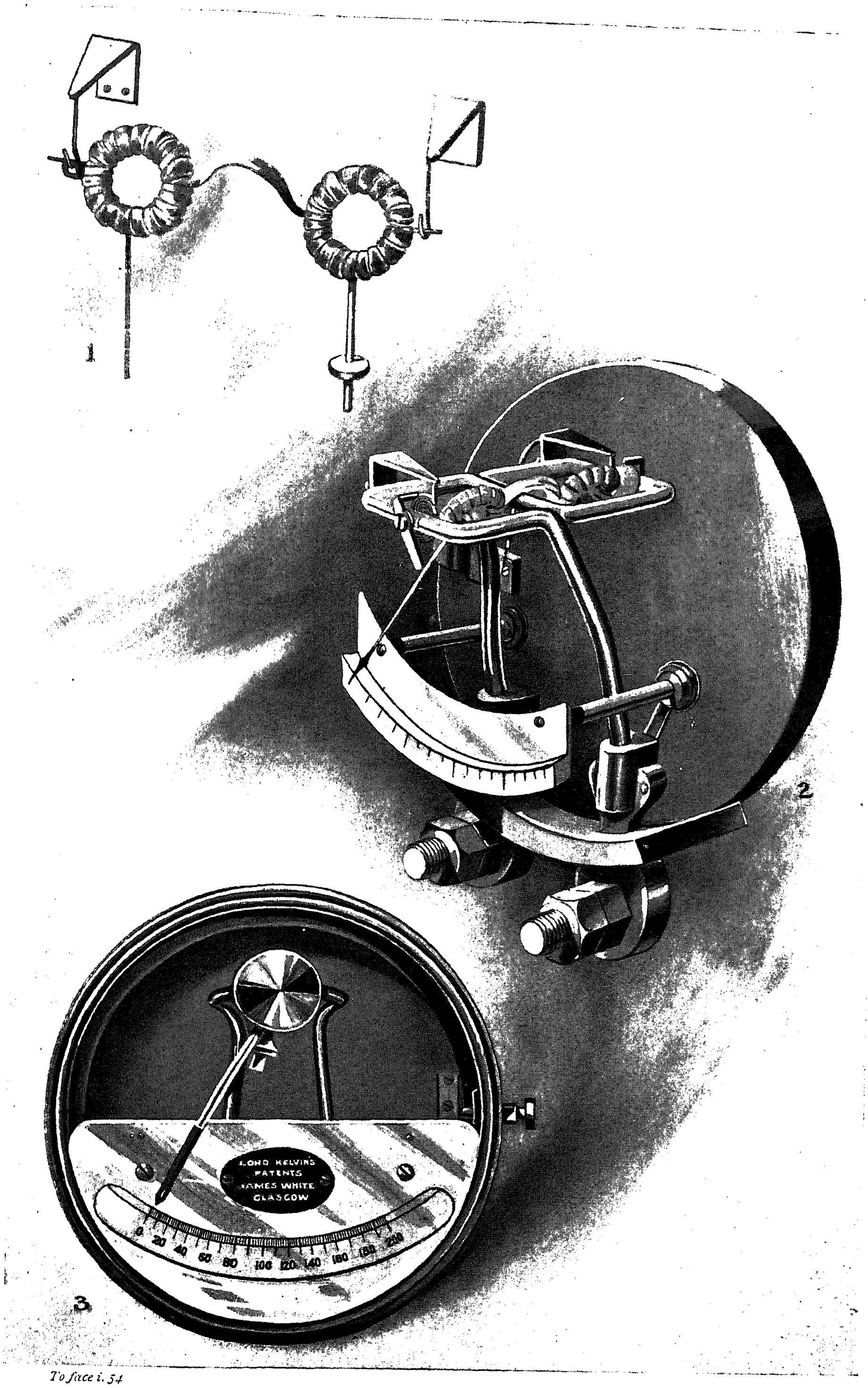
[1] In the case of all wattmeters intended for use with alternating currents, it is extremely important to avoid the use of metallic cases or metallic supports for the coils, as the currents induced in such masses of metal may seriously affect the readings of the instrument.



To face i. 54

1, Kelvin Engine-Room Wattmeter (moving system); 2, Kelvin Engine-Room Wattmeter (case removed); 3, Kelvin Engine-Room Wattmeter

SOME MEASURING INSTRUMENTS.—III

supplied may be used as an energy meter. Many forms of such *quantity* meters are in use.

A more satisfactory form of meter is, however, one capable of measuring correctly the actual energy, and not merely the quantity.[1] Such a meter is called an energy meter.

The mechanism of an electric supply meter is so arranged as to make the meter read directly in Board of Trade units (kilowatt-hours), either on a series of dials—resembling those used in a gas meter—or on some other suitably-arranged scale.

The fundamental law which a thoroughly satisfactory meter must fulfil is that the rate of registration at any instant should be proportional to the current in the case of quantity meters, and to the power in the case of energy meters; and the great difficulty which manufacturers have always had to face is the construction, at a reasonable cost, of meters capable of obeying this law with sufficient accuracy.

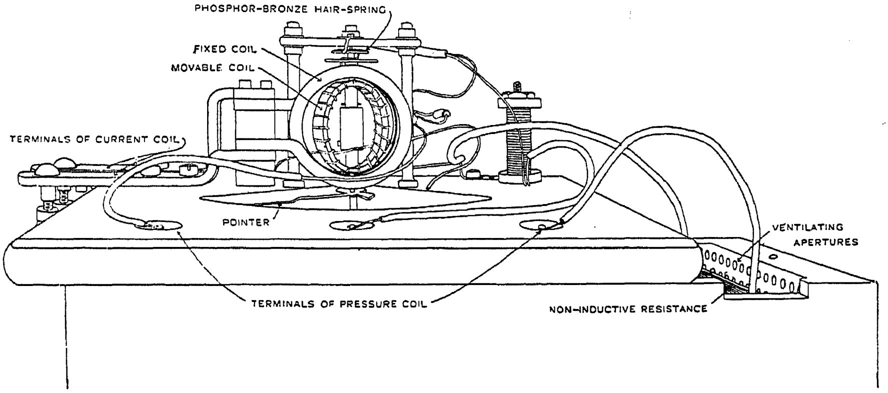

PHOSPHOR-BRONZE HAIR-SPRING
FIXED COIL
MOVABLE COIL
TERMINALS OF CURRENT COIL
POINTER
VENTILATING APERTURES
TERMINALS OF PRESSURE COIL
NON-INDUCTIVE RESISTANCE

Fig. 41.—Weston Wattmeter

Electrolytic Meters.—Numerous forms of electrolytic quantity meters have been devised. Edison used copper and, at a later stage, zinc voltameters; by weighing the cathode plates after the meter had been in use for some time, the amount of copper or zinc deposited could be determined, and from this the quantity of electricity supplied calculated. Such meters, however, are not direct-reading, and involve the troublesome operations of collecting and weighing the plates.

Two direct-reading electrolytic quantity meters are in use at the present time. One of these, the Bastian meter, consists of a glass vessel fitted with platinum electrodes which reach down to the bottom of the vessel. The vessel is filled with acidulated water, and a layer of oil is floated on the top of the water to prevent loss by evaporation. The current passing through the meter decomposes the water, and the quantity of electricity supplied is directly proportional to the amount of water decomposed, and may be read off on a suitably-arranged vertical scale.

The main defect of the Bastian meter is its large counter-E.M.F.

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[1] If the P.D. at which a consumer is supplied falls below the specified value, a quantity meter will register an amount which is in excess of the actual energy received by the consumer.

(about 1.5 volt), and consequent large absorption of power; hence it is only suitable for small currents.

In the Wright electrolytic meter, illustrated in figs. 42 and 43,[1] a double iodide of mercury and potassium is used as the electrolyte, and the counter-E.M.F. is so small as to be negligible. D is a reservoir filled with mercury, and connected with a flat glass vessel whose lower part is provided with an annular groove A C containing mercury. This circle of mercury forms the anode, and as the mercury is used up by going into solution, its level is maintained by the flow, at intervals, of additional mercury from the reservoir D, the place of this mercury being taken by the electrolyte, which forces its way past the narrow neck connecting the two vessels when the mercury level in A C falls slightly. The entire vertical glass tube seen below A C is filled with the electrolyte. The conical cathode B is made of iridium foil, and the mercury which collects on it drops into the funnel F at the top of the siphon tube E, and gradually fills this tube. When the tube has been completely filled (which corresponds to 100 units), the mercury automatically siphons out and falls to the bottom of the outer tube, as shown at G in the figures. The current which produces electolysis is only a small fraction of the total current, the bulk of which passes through the manganin shunt R. An iron wire resistance r_1 is connected in series with the voltameter, the value of r_1 being so adjusted as to make the total resistance of the voltameter circuit independent of temperature changes. When all the mercury contained in the reservoir D has been used up, the meter is easily reset by tipping it up and allowing the mercury to flow back into G.

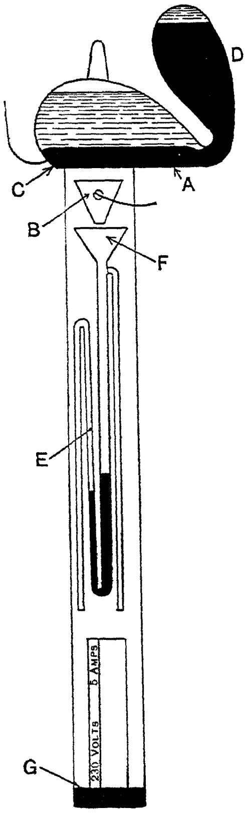

Fig. 42.—Wright Electrolytic Meter

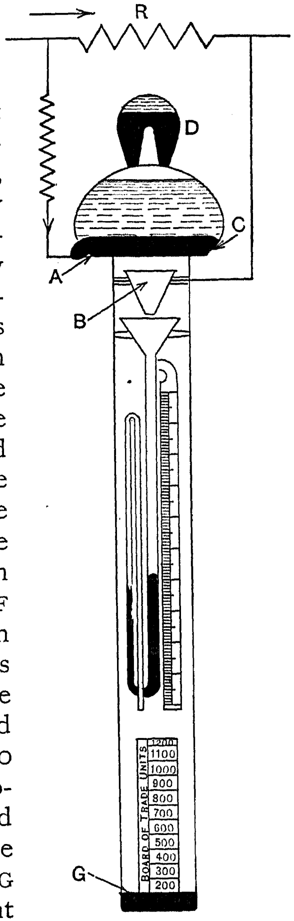

Fig. 43.—Wright Electrolytic Meter

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[1] From the *Journal of the Institution of Electrical Engineers*, by permission of the Council of the Institution. In order to prevent any mercury from being shaken down the measuring tube from the annular anode by mechanical vibration, a guard cylinder of platinum gauze is fixed into the glass lip which forms the inner boundary of the annulus.

Clock Meters. The general principle underlying the construction of this type of meter is that of having two clocks differentially geared, and making use of the magnetic effect of a current for accelerating one of them and retarding the other. The modern form of the Aron clock meter, which is an energy meter, will be found described in a later chapter.

Motor Meters—Ferranti Meter. This is a very numerous class, and includes a number of well known types. The examples which we have selected for a detailed description are the Ferranti, and the Chamberlain and Hookham meters. Both of these are quantity meters.

The Ferranti meter is illustrated in fig. 44, and consists of an iron-clad electromagnet whose core S P is of steel, and which produces a vertical field in the space enclosed by the iron ring I R, which is insulated along its entire circumference from the top and bottom pole-pieces. This space is filled with mercury, forming a mercury bath M B in which is immersed a light fan F mounted on a vertical spindle geared, by means of a worm W at its upper end, with the counting mechanism. The mercury is in contact with the ring I R, but is insulated from the magnet core, with the exception of a circular space C at its centre. The current enters the instrument by the terminal T +, flows along the core S P, reaching the cup C, and thence spreading out radially towards the ring I R. From this ring it flows round the magnetizing coil C C, and leaves the instrument by the terminal T −. The electromagnet is so designed that within the working range of the instrument the field which it produces may be taken to be proportional to the current. In this field is placed a conductor (mercury) which carries the same current, in a direction at right angles to the field. The mercury is thus set in rotation, carrying the fan F round with it, and the driving torque which acts on it is proportional to the square of the current. The retarding torque is due to fluid friction, and it has been found possible to construct the instrument so that within

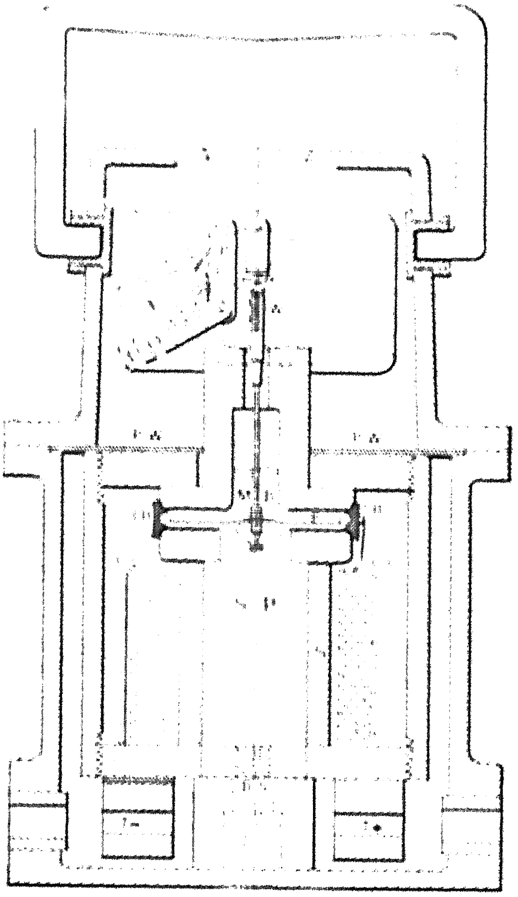

Fig. 44. Ferranti Meter.

its working range the resisting torque due to fluid friction is proportional to the square of the speed. Thus we have (since driving torque = resisting torque when a steady state has been reached) the current proportional to the speed, and hence the quantity to the number of revolutions.

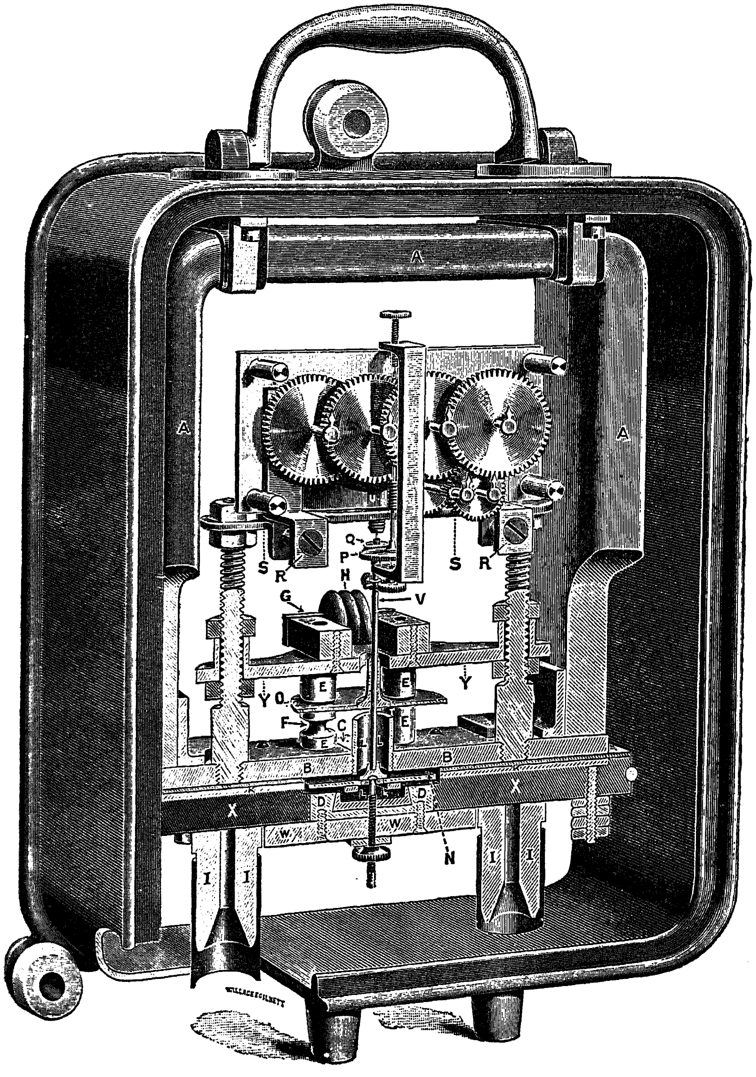

Fig. 45.—Chamberlain and Hookham Meter

A *steel* pole-piece or core S P is used in order that the permanent magnetism may enable the fan to start against the friction of the counting mechanism when the current is very feeble. The soft-iron tube S surrounding the steel core is intended to prevent the field due to the residual magnetism from reaching too high a value should too large a current be accidentally sent through the meter—it having been found that in this case, in the early forms of meter not fitted with the soft-iron tube, the excessively powerful residual magnetism caused the meter to run too fast with the smaller currents.

Chamberlain and Hookham Meter.—This form of motor meter is shown, partly in section, in fig. 45. A A is a powerful permanent magnet to whose poles are attached the soft-iron pole-pieces B B, separated from each other in the middle by the insertion of a brass distance-piece C. Below these pole pieces is placed a piece of soft iron D D, so that some of the magnetic lines pass downwards on one side and upwards on the other across the gap which separates D D from B B. In this gap is placed the horizontal copper disc N, which forms the "motor disc", the remainder of the space being filled up with mercury. The remaining magnetic lines pass up two cylindrical pole-pieces E on one side, across iron bridge-pieces G, and then down two pole-pieces E on the other side. In so doing, the lines have to cross the air-gaps interposed in their path half way up each pole piece. Into these air-gaps is introduced the "brake disc" O, which is made of a low temperature-coefficient alloy, and which is mounted on the same vertical spindle as the motor disc. This spindle is geared with the counting mechanism.

The current flows along the left-hand copper strip K, enters the mercury, and then flows through the motor disc in a diametral direction, leaving it by the right-hand copper strip K. The magnetic field in which the disc is placed causes the latter to rotate—the speed of rotation being, even at full load, slow, and hence the friction of the mercury inconsiderable. Since the field is constant, the driving torque acting on the motor disc will be simply proportional to the current. The retarding torque is provided by the currents induced in the brake disc on account of its motion across a non-uniform magnetic field, and since the intensity of this field remains constant, the retarding torque is simply proportional to the speed. Hence the speed varies in direct proportion to the current.

The mercury friction cannot, however, be altogether neglected, and in order to balance it a compensating coil H is wound round the upper iron bridge-piece; this coil is supplied with the main current, and weakens the field in which the brake disc is placed, thereby tending to make it run faster and thus counteracting the effect of fluid friction.

CHAPTER VII

MAGNETIC MEASUREMENTS

Magnetic Intensity and Magnetic Induction — Susceptibility and Permeability.—We have already, in Chapter I, considered some of the more important magnetic units. It will now be necessary to introduce a number of additional units, which are indispensable in connection with the testing of the magnetic qualities of iron and steel.

One of the terms which we have frequently had occasion to use is magnetic force or magnetic intensity, and we have defined this as the force which would act on a unit pole if placed at the point considered, the assumption being tacitly made that the point lies in an air-space. Let us now suppose that this restriction is removed, and that we consider a point inside a mass of iron. If we attempt to determine the force acting on a unit pole by scooping out a cavity in the region immediately surrounding the point, and introducing our unit pole into this region, then we find that the force depends on the shape of the cavity, and varies from a very small to a very large value, according to the shape. Unless, therefore, we specially restrict the meaning of the term "magnetic force" or "magnetic intensity", this term ceases to have any definite meaning. It has been assigned a perfectly definite meaning by specifying the manner in which the cavity surrounding the given point is to be shaped. We agree, namely, to restrict the term to denote that particular value of the force acting on the unit pole which is obtained by forming the walls of the cavity so that they are everywhere *along the direction of magnetization* of the iron. The main effect of this arrangement is the absence of magnetic polarity on the bounding surfaces of the mass of iron which immediately surrounds the point—in other words, the effect due to the magnetization of the mass of iron immediately surrounding the point is eliminated. The magnetic force so obtained is usually denoted by **H**.

Let us now take the other extreme, and suppose the cavity so shaped that the development of magnetic polarity on its bounding surfaces is as great as possible. This will be obtained by making the walls everywhere *perpendicular to the direction of magnetization.* Let the magnetic polarity per unit of area of the wall be **I**.[1] Then the ratio **I**/**H**, generally denoted by κ, is defined to be the *susceptibility* of the material. We thus have

$$\kappa = \frac{\mathbf{I}}{\mathbf{H}}, \text{ or } \mathbf{I} = \kappa\, \mathbf{H}.$$

Let us now introduce our unit pole into this second cavity, and determine the force which acts on it. This force is defined to be the *magnetic induction* at the point considered, and is denoted by **B**. The ratio **B**/**H**, denoted by μ, is called the *permeability* of the material. Thus

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}, \text{ and } \mathbf{B} = \mu\, \mathbf{H}.$$

Instead of saying that the magnetic induction has the value **B** at a given point, we sometimes say that there are **B** lines of magnetic induction per unit of area of a plane drawn at right angles to the direction of magnetization at that point.

If we take any surface bounded by a closed curve, divide it up into a large number of small elements of area, multiply each little area by the value of the magnetic induction in a direction perpendicular to the area,[1] and then take the sum of all the products so formed, then this sum gives us what is termed the *total number of magnetic lines* crossing the surface, or the *total magnetic flux* through it. For this reason **B** is sometimes spoken of as the magnetic *flux density*, since it corresponds to the flux per unit of area.

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[1] This, it will be remembered (see Chapter I), is identical with intensity of magnetization, or magnetic moment per unit volume.

We may now obtain a relation connecting **H**, **I**, and **B**. The force which acts on the unit pole in the second case (walls perpendicular to direction of magnetization), and which we denote by **B**, may be regarded as made up of two amounts: (1) the force **H**, which exists independently of magnetic polarity in the immediate neighbourhood of the point; (2) the additional amount due to the development of polarity on the walls of the cavity in the second case. In order to find this additional amount, we notice that the pole-strength per unit of area being **I**, and the lines of force going straight across the cavity, the number of these lines per unit of area is $4\pi\mathbf{I}$, since each pole gives rise to 4π lines. But the number of lines of force per unit of area is the same as the force acting on the unit pole. Hence, this latter amounts to $4\pi\mathbf{I}$. Thus the total force acting on the unit pole **B** is the sum of **H**, the force existing quite apart from the presence of any magnetic material in the neighbourhood of the point considered, and $4\pi\mathbf{I}$, which is the additional amount contributed by the development of magnetic polarity on the walls of the cavity. We thus have

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}.$$

From this we at once deduce the relation

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} = 1 + 4\pi\frac{\mathbf{I}}{\mathbf{H}},$$

$$\text{or } \mu = 1 + 4\pi\kappa.$$

Stress between Two Magnetized Surfaces.—Imagine two plane magnetized surfaces separated by a short air-gap, the intensity of magnetization being **I** and the magnetic induction **B**. Then a unit pole introduced into the gap would experience a force of **B** dynes. This force may be regarded as due partly to the north and partly to the south pole-face, the amount contributed by each being one-half of the total, *i.e.* $\mathbf{B}/2$. Now imagine the unit pole placed *on* one of the surfaces. Then this surface will exert no force on the pole in a direction at right angles to the surface, so that the total force on a unit pole which is *on* either surface is that due to the other surface only, viz. $\mathbf{B}/2$. But the pole-strength of either surface per sq. cm. is **I**. Hence the pull per sq. cm. of the surface is $\frac{1}{2}\mathbf{B}\mathbf{I}$. If **H** be negligible in comparison with **B**, then $\mathbf{I} = \frac{\mathbf{B}}{4\pi}$, and the stress between the two magnetized surfaces, per sq. cm. of area, becomes

$$\frac{\mathbf{B}^2}{8\pi}.$$

---

[1] It must be remembered that magnetic induction is a quantity possessing *direction* as well as magnitude.

Relation connecting B and H—Magnetization Curves.—If we take a piece of iron which is in a non-magnetic condition to start with, and apply to it a known magnetic force **H** whose value is steadily increased, measuring **B** (by any of the methods to be explained presently) at the same time; then, on plotting a curve connecting values of **B** and **H**, we find that this curve, as shown in fig. 46, may be described roughly as consisting of three approximately-straight-line portions united by curves. Such a curve is called a **B–H** or *magnetization curve.* The upper bend in the curve is spoken of as the *knee*, and for any point lying well beyond the knee, and corresponding to the region in which a relatively large increase of **H** produces only a relatively small increase in **B**, the iron is said to be *saturated.*

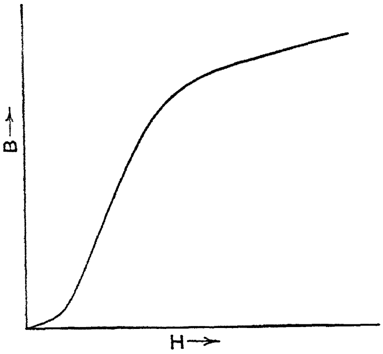

Fig. 46

A knowledge of the exact relation connecting **B** and **H** is of prime importance in all problems concerning the design of electromagnets, and in the following table we give the data from which the reader may plot the **B–H** curves for (*a*) cast iron; (*b*) wrought iron of good quality; (*c*) dynamo-magnet cast steel:—

H.	B.		
	Cast Iron.	Wrought Iron.	Cast Steel.
5	1900	9000	8150
10	3000	12200	12100
15	3900	13600	14000
20	4550	14450	15000
25	5100	15050	15700
30	5500	15500	16200
35	5870	15800	16500
40	6180	16100	16750
45	6450	16300	16900
50	6700	16500	17140
60	7150	16800	17450
70	7530	17000	17750
80	7900	17220	18000
90	8250	17400	18200
100	8570	17580	18400
125	9200	17930	18900

The Magnetic Circuit—Calculation of Ampere-Turns Required to Produce Given Magnetic Flux.—Every line of magnetic induction forms a closed loop or curve, and the assemblage of the paths followed by the magnetic lines is spoken of as the *magnetic circuit.* In order to produce a given magnetic flux in a circuit a definite M.M.F. must be applied to it, and by analogy to an electric circuit the ratio $\frac{\text{M.M.F.}}{\text{magnetic flux}}$ is termed the magnetic *reluctance* of the circuit.

We shall now briefly indicate how the ampere-turns necessary to produce a given flux may be approximately calculated. Let us suppose that we proceed around a closed loop which is a line of induction, and that the value of the magnetic flux through every cross-section of the circuit is given. Then by dividing the flux by the cross-section we obtain the value of **B** in every part of the circuit. In the most general case, in passing along any line of induction we find that **B** varies continuously from point to point. In order to simplify the problem it is usual to divide the line of induction into a number of parts, and to assume that **B** has a constant value along each part. Then if the values of **H** corresponding to those of **B** be determined by reference to the **B H** curves, the sum of the products obtained by multiplying each value of **H** by the corresponding length of the curve gives the M.M.F. Finally, .8 time the M.M.F.[1] gives the ampere-turns which must be provided.

A numerical example will make the matter clear. Let it be required to find the ampere-turns necessary to produce a flux of 40,000 C.G.S. lines across the space between the parallel plane faces of the pole-pieces P P in fig. 47, the area of each pole-face being 8 sq. cms., the pole-pieces being of cast iron, and the sum of the lengths of the portions A B and C D of the dotted line of induction amounting to 10 cms. Let the length of the air-gap between the pole-pieces be .5 cm. Further, let the exciting coil be wound on the cylindrical core M, made of wrought iron, 4.5 cms. long and 4 sq. cms. in cross-section.

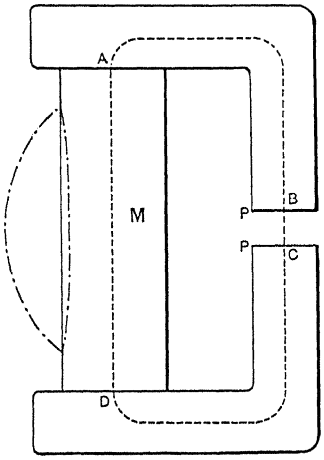

Fig. 47

Of the lines produced by the exciting coil at the middle cross-section of the core M, only a certain fraction will pass across the plane faces of the pole-pieces, a large number *leaking* out into the surrounding air-space, forming closed loops of induction such as the one indicated by the chain-dotted curve in the figure. Before we can solve the problem we must know what fraction of the total lines produced in M will cross the air-gap, where they are wanted. The ratio of the maximum flux through M to the useful flux across P P is termed the *leakage coefficient* of the magnetic circuit. Let us assume that in the case considered its value is 1.3. We further assume that instead of a uniformly distributed leakage, we have to deal with a leakage which takes place suddenly along the edges of the pole-pieces. This is tantamount to assuming that **B** is constant along each of the portions D A, A B, B C, and C D of the magnetic circuit.

Now consider each of these portions separately, and find the corresponding values of **H**. In the air-gap $\mathbf{H} = \mathbf{B} = \frac{40,000}{8} = 5000$. The flux through the core M and the pole-pieces is, on account of leakage, $1.3 \times 40,000 = 52,000$. Hence, **B** in the pole-pieces $= \frac{52,000}{8} = 6500$. Referring to the **B–H** curve for cast iron, we find the corresponding value of **H** to be 46. Next, in the wrought-iron core we have $\mathbf{B} = \frac{52,000}{4} = 13,000$. A reference to the curve for this material gives $\mathbf{H} = 12.5$. The total M.M.F. round the circuit is thus

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[1] See Chapter I.

$$\underbrace{5000 \times .5}_{\text{Air-gap.}} + \underbrace{46 \times 10}_{\text{Cast-iron pole-pieces.}} + \underbrace{12.5 \times 4.5}_{\text{Wrought-iron core.}} = \underbrace{3016}_{\text{Total.}} \text{ approximately.}$$

The corresponding ampere-turns are $.8 \times 3016 = 2413$ approximately.

It will be noticed that by far the largest portion of the M.M.F. is employed in maintaining the flux across the air-gap. Such is frequently the case in practice.

Calculation of Winding to Produce Given Number of Ampere-Turns.—A coil may be required to produce a given number of ampere-turns either with a given exciting current, or with a given P.D. across its terminals. In the former case, the number of turns is at once obtained by dividing the ampere-turns by the amperes. Thus, in the numerical example just considered, if the exciting current is given as 10 amperes, the turns which must be wound on the coil are 241. The *size* of the wire under these conditions does not, obviously, affect the result, and is determined solely by considerations regarding the permissible rise of temperature.[1]

Let us next suppose that V, the P.D. at which the excitation is to be produced, is given. In order to calculate the winding, we have to make an assumption regarding the probable *depth* of winding, and from this to find the *mean* length of a turn.[2] Let the length of turn so determined be l, and let the size of wire be such that the resistance of this length is r. If we denote the current by i, and the number of turns by S, then, since the total resistance of the coil is Sr, we have $i = \frac{\mathrm{V}}{\mathrm{S}r}$, or $\kappa = \frac{\mathrm{V}}{\mathrm{S}i}$; *i.e.*,

$$\text{resistance of mean turn} = \frac{\text{P.D.}}{\text{ampere-turns}}\,.$$

Knowing the length l of the turn, we find the resistance of the wire per unit of length, r/l; a reference to wire tables then gives the *size* of wire required (in such tables the resistances per 1000 feet, or 1000 yards, of the various gauges at specified temperatures are usually given).

It will be noticed that in this second case (when the P.D. is given) the ampere-turns depend on the *size* of wire used, and are (so long as the length of the mean turn is not appreciably altered) independent of the number of turns, or the amount of wire used; this latter is determined solely by considerations regarding heating.

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[1] An old-established but not very scientific "rule-of-thumb" is to allow a current-density in the wire of 1000 amperes to every square inch of cross-section.

[2] In electromagnets of moderate size, the depth of winding lies between 1 and 3 inches.

Experimental Determination of B–H Curve — Ballistic Ring Method.— By far the most satisfactory method of finding the relation which connects **B** and **H** is to use the sample of the material to be tested in the form of a ring, such as the one shown in fig. 14. A convenient size is one having an external diameter of about 6 inches, a radial depth of $\frac{3}{4}$ inch, and an axial length of 1 inch, the section being rectangular (it would be an advantage to arrange the area of the cross-section to correspond to an exact number of sq. cms.). This ring is wound with some 100 or 200 turns of fine silk-covered copper wire, carefully insulated; this winding, which forms the secondary or exploring coil, need not be uniformly distributed, but may be concentrated over a small portion of the ring. Another, *uniformly distributed* winding, consisting of several layers, and forming the primary or magnetizing coil, is then wound round the ring.

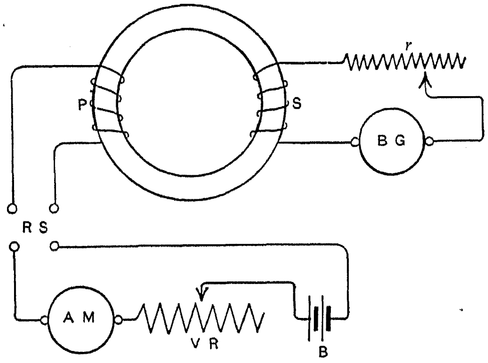

Fig. 48

The arrangement of connections is shown in fig. 48, where P and S are the primary and secondary coils,[1] R S a reversing switch, A M an ammeter for measuring the exciting current, V R a variable resistance, and B the battery (of secondary cells) supplying the current. The secondary coil is joined in series with a resistance r of convenient amount and a *ballistic* galvanometer B G—*i.e.*, a galvanometer having a long period of vibration and subject to as little damping as possible.[2]

The problem is to find **H**, and the corresponding value of **B**. The former is at once found by the method already explained in Chapter III. If i = exciting current in amperes, S_1 = number of primary turns. and l = mean circumference of ring, then the mean value of **H** over the cross-section of the ring is given by

$$\mathbf{H} = \frac{1.257 \, S_1 i}{l}.$$

In order to find **B**, we observe the throw of the galvanometer obtained on reversing the primary current. If a = cross-section of ring in sq. cms., then the change in the magnetic flux through each turn of the secondary coil on reversal amounts to $2\,\mathbf{B}\,a$. If we suppose that the reversal takes t secs., then the mean rate of change of flux—which is numerically equal to the mean induced E.M.F. in C.G.S. units—amounts to $2\,\mathbf{B}\,a/t$, and the total mean E.M.F. (in C.G.S. units) in the secondary coil is $2\,\mathbf{B}\,S_2 a/t$, where S_2 = number of turns in secondary coil. If now we suppose that the total resistance of the secondary circuit (including secondary coil, galvanometer, and the extra resistance r) amounts to r_2, then the mean value of the induced secondary current is $\frac{2\,\mathbf{B}\,S_2\,a}{t\,r_2}$, and as this current lasts t secs., the total quantity q discharged through the galvanometer amounts to

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[1] For the sake of clearness in the diagram, P is shown concentrated over a portion of the ring; in reality, it is *uniformly* distributed.

[2] The most convenient form of ballistic galvanometer for carrying out such tests is a moving-coil galvanometer of the *undamped* type (*i.e.* without any metallic frame supporting the coil).

$$q = \frac{2\,\mathbf{B}\,S_2\,a}{r_2},$$

whence

$$\mathbf{B} = \frac{r_2}{2\,S_2\,a}\,q \quad \ldots\ldots \text{(1)}$$

All the quantities on the right-hand side are known except q. The final step, then, is to find the connection between q and the corresponding throw of the galvanometer, *i.e.* to calibrate the galvanometer.

There are several ways of doing this. The most satisfactory is to use a *standard solenoid* (some 4 feet long and 3 inches in diameter), uniformly wound and having a small exploring or secondary coil *inside* it at the middle. When all the readings have been taken with the sample ring, the standard solenoid is substituted for it, and the throws obtained on the galvanometer scale by reversing a number of gradually increasing primary currents are noted. The corresponding quantities may now, however, be easily calculated. For since **H** has the same numerical value as **B** in this case (there being no iron core), the flux through each turn of the secondary coil is given by

$$\frac{1.257\,S_1'\,i_1\,a_2}{l_1},$$

where S_1' = turns in solenoid, i_1 = current in amperes, a_2 = area, in sq. cms., of one turn of secondary coil, l_1 = length, in cms., of solenoid (see Chapter III).

By reasoning similar to that employed above in connection with the ring, we find for the quantity discharged through the galvanometer when a primary current of i_1 amperes is reversed the value

$$\frac{2 \times 1.257\,S_1'\,S_2'\,i_1\,a_2}{l_1\,r_2},$$

where S_2' = number of turns in secondary coil of solenoid. Everything in this expression being known, we can calculate the quantity.

A curve may now be plotted giving the relation connecting galvanometer throw with quantity, and this *calibration curve* may be used for determining the values of q corresponding to the various throws obtained in the experiment with the sample ring. Formula (1) may then be used for calculating **B**.[1]

Traction Methods—S. P. Thompson's Permeameter.—The ballistic-ring method just described, although requiring a considerable amount of time and experimental skill, is the most reliable method as yet devised for the magnetic testing of iron or steel. A rough *workshop* method, in which the value of **B** is calculated from the pull required to separate two magnetized surfaces originally in contact, has been devised by Prof. S. P. Thompson, and will now be briefly described.

The instrument is shown in fig. 49, and is termed by its inventor a "permeameter". The specimen is in the form of a cylindrical rod, which passes through one side of a massive rectangular block of iron, technically termed a "yoke", into a magnetizing solenoid, the lower faced end of the rod butting against the faced inner surface of the yoke. The cross-sectional area of the yoke being very much larger than that of the rod, **B** inside the yoke will have a small value, and **H** in the yoke will be very small in comparison with its value inside the rod. We may therefore neglect the work done in carrying a unit pole along a line of induction inside the yoke, and assume that the M.M.F. consists entirely of the work done in conveying the unit pole inside the rod from end to end of the solenoid. On this assumption, $\mathbf{H} = 1.257\, Si/l$, where S = turns in solenoid, i = current in amperes, l = distance between inner faces of yoke.

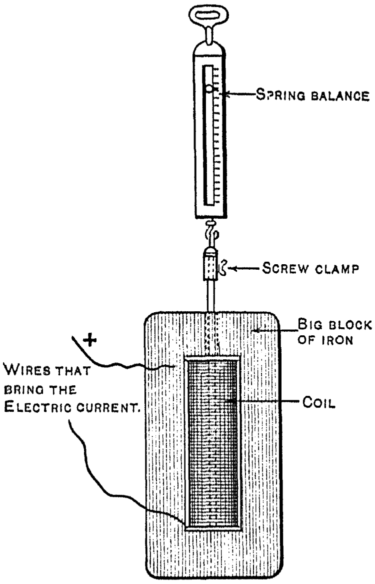

Fig. 49.—Thompson's Permeameter[2]

The pull required to detach the rod from the yoke having been found, we divide this pull by the area of contact (or area of cross-section of the rod), thus obtaining the stress per unit of area. If this stress be expressed in dynes per sq. cm., then, as we have already shown, its value is $\mathbf{B}^2/8\pi$, and we have a means of calculating **B**.

Residual Magnetism—Coercive Force—Hysteresis.—When a piece of iron is subjected to the action of a magnetic force which is subsequently withdrawn, the induction does not disappear, and in general a very large fraction of it is retained by the specimen. The curve connecting **B** and **H** is, in fact, for descending values of **H** totally different from the curve for ascending values. This is shown in fig. 51, where O P B is the ascending, and B C the descending branch of the curve.

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[1] It must be particularly noted that all the quantities on the right-hand side of (1) are in C.G.S. units. So that if the resistance of the secondary circuit is, *e.g.*, given in ohms, then before substituting in the formula it must be multiplied by 10^9.

[2] From Prof. Ewing's *Magnetic Induction in Iron and Other Metals.* By permission.

The induction O C which remains in the specimen is termed *residual induction*, and its value depends, for a given specimen, on the maximum induction previously reached by it. Thus, if the maximum induction had been carried up to the point D in fig. 50, the corresponding residual induction would have been represented by O E. The property possessed by the magnetic materials of retaining a certain amount of magnetization after the withdrawal of the magnetic force is sometimes spoken of as *retentiveness.* In order to wipe out the residual induction, a definite magnetic force O F must be applied in the reverse direction, and this force—which will, of course, vary with the residual induction —is termed the *coercive force* for that induction.

If in fig. 50 we consider any value of **H**—such as O G—then corresponding to this value of **H** we get a value of **B** which is represented by G H if the specimen is thoroughly demagnetized to start with, and **H** is simply increased from zero to O G. But we get the very much larger value G K if **H** is first increased to O M, and then brought down to O G; and the still larger value G L if it is increased to O N, and then reduced to O G. In fact, corresponding to any value of **H** we have an *infinite number* of values of **B**, depending on how the particular value of **H** has been arrived at. There is thus no definite connection between these two quantities, unless we specify the manner in which the changes in **H** and **B** take place. When we speak of the "**B–H** curve", we always mean the particular curve obtained when the specimen is thoroughly demagnetized to start with,[1] and when **H** is *steadily increased*, and never allowed first to increase, and then to decrease, or *vice versa.*

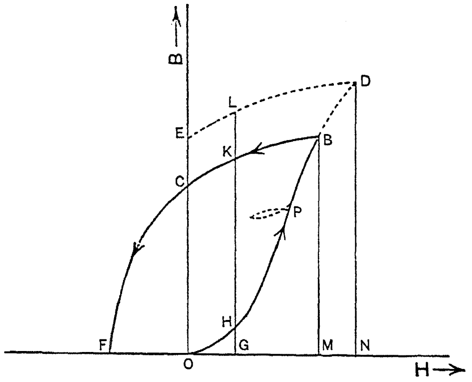

Fig. 50

If, for example, when the point P (fig. 50) is reached we first decrease **H** and then restore it to its original value, the curve connecting **B** and **H** will trace out the loop shown by the dotted line, and when **H** has regained its original value, **B** will have increased by a small amount.

The facts which we have been considering may be summed up by saying that a magnetic material always tends to retain its magnetic state—the changes in the magnetic force being always ahead, as it were (not as regards time, but as regards phase), of those in the induction. The term *hysteresis*, introduced by Prof. Ewing, is used to describe this species of magnetic inertia.

Hysteresis Loops and their Experimental Determination.—If a piece of iron be subjected to a series of cyclical changes in the magnetic force, consisting of repeated applications of it in opposite directions, the magnetic force increasing continuously from a zero to a fixed maximum value, then the curve connecting **B** and **H** will take the form of a closed loop, as shown in fig. 51, which consists of an ascending and a descending branch, the two branches enclosing a certain area. Such a closed loop, representing a complete cycle of magnetic operations, is termed a *hysteresis loop*.

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[1] Such demagnetization is effected by subjecting the specimen to the action of a magnetic force which undergoes rapid reversals while its amount slowly diminishes and ultimately vanishes.

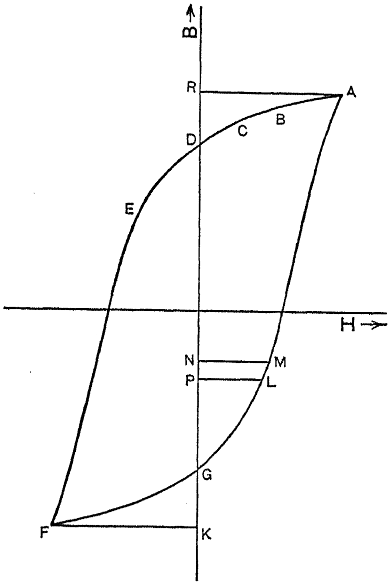

Fig. 51

The experimental determination of the hysteresis loop corresponding to a given maximum value of **B** or **H** is a matter of extreme importance, and we shall now explain how this determination may be carried out.

The loop being symmetrical, it will be completely determined by finding either of its two branches. The most convenient method is always to start from the corner or peak of the loop, A in fig. 51, and to observe the throws of a ballistic galvanometer obtained on passing from A to a number of points B, C, D, &c., on the descending branch, the calculation of the changes in **B** and **H** being carried out as already explained in connection with the **B**–**H** curve.[1] In order to pass from A to B, for example, all that is necessary is to suddenly reduce the current by an amount corresponding to the value of **H** at that point; and this may be easily done by suddenly unplugging resistance in the primary circuit. By varying this resistance, a series of points between A and D may be obtained. In order to reach a point (such as E) which lies lower down, to the left of the **B**-axis, a *reversal* of current is necessary, but the reversed current must have a value much smaller than the maximum corresponding to the points A or F. The operation of passing from A to E involves, then, a sudden increase in the resistance of the primary circuit, accompanied by a reversal.

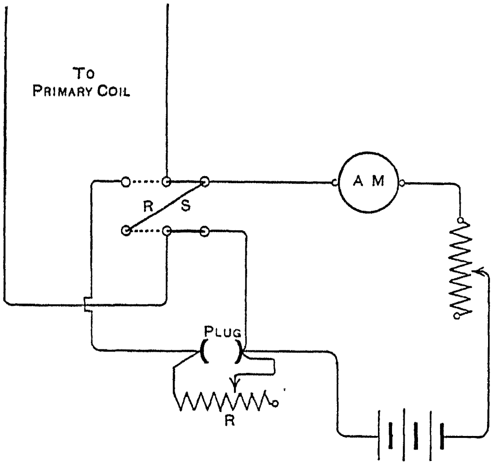

Fig. 52

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[1] The specimen is supposed to be in the form of a ring, with primary and secondary coils arranged as explained above.

In order to be able to effect these changes conveniently, a reversing switch may be used which is connected to a variable resistance R as shown in fig. 52—an arrangement due to Prof. Ewing. R S is the reversing switch, the dotted lines showing the change of connections due to throwing over the switch. It will be noticed that when the resistance R is plugged up, throwing over the switch from right to left produces a simple reversal; but if some of the resistance is unplugged, the introduction of this into the primary circuit accompanies the act of reversal.

Between every two readings, R is plugged up, and the switch worked to and fro a number of times, so as to produce a series of simple reversals and thereby restore the iron to a steady cyclic state.

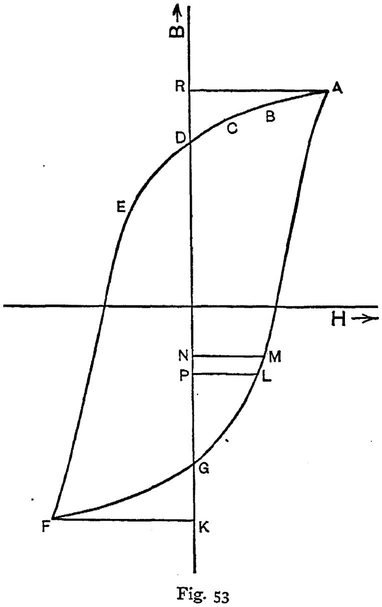

Fig. 53

While taking readings corresponding to points lying between A and D in fig. 51, the switch is kept in the left-hand position, the reduction of current being obtained by suddenly unplugging any desired amount of resistance. For points lying beyond D, such as E, the switch would be in the right-hand position, and the point E would be reached by throwing it over to the left.

Energy Dissipated by Hysteresis—Steinmetz's Law.—When a mass of iron is carried through a rapid succession of magnetic cycles, it is found to grow hot. The development of heat is due to the dissipation of part of the energy spent in producing the magnetization, and it may be shown that the amount of energy dissipated in every cubic cm. of the iron during a complete magnetic cycle is directly proportional to the area of the corresponding hysteresis loop.

Referring to fig. 53, let us suppose that at a certain stage the point L has been reached, and that **H** increases from P L to N M, the increase being supposed small, and the mean value of the current, *in C.G.S. units*, during this change being i. Let b = N P be the corresponding change in **B**. If a = cross-section of the ring in sq. cms., then the total change of magnetic flux through each turn of the primary coil is ba, and if this takes place during t secs., the average induced E.M.F. is $\mathrm{S}\,b\,a/t$, where S = number of turns in primary coil. Since the current is increasing, the induced E.M.F. opposes it, and the power spent in maintaining the current against this E.M.F. is $i\,\mathrm{S}\,b\,a/t$; the power being exerted during t secs., the total *energy* spent in producing the change of flux is $i\,\mathrm{S}\,b\,a$. Now if l = mean circumference of ring, then the value of **H** corresponding to i is $\mathbf{H} = \dfrac{4\pi \mathrm{S}\,i}{l}$, so that $\mathrm{S}\,i = \dfrac{1}{4\pi}\mathbf{H}\,l$. Substituting this value of $S i$ in the expression just obtained for the energy, we find that (fig. 53)

$$\text{energy spent in passing from L to M} = \frac{1}{4\pi} b\mathbf{H} l a.$$

But since la = total volume of iron in cubic cms., we see that in passing from L to M we communicate to every cubic cm. of the iron an amount of energy equal to $\frac{1}{4\pi} b\ \mathbf{H} = \frac{1}{4\pi} \times$ area of strip PLMN.

Hence if we pass from G to A, the total energy given to the iron will be $\frac{1}{4\pi}$ times the area GAR.

Now in going back from A to D, the current is decreasing; the induced E.M.F. will help to maintain it, and the ring in giving up its magnetism will act as a generator of energy. The amount of the stored energy so returned (by each cubic cm. of the iron) is given by $\frac{1}{4\pi}$ time the area RAD. The remainder, $\frac{1}{4\pi}$ time the area GAD, is not given up. Similarly, as the cycle is completed by passing from D through E and F to G, a further amount of stored energy, represented by $\frac{1}{4\pi}$ time the area DFG, is not returned. Hence during a complete cycle there is an amount of the stored energy, per cubic cm. of the iron, represented by $\frac{1}{4\pi}$ *time the area of the hysteresis loop*,[1] which is not returned, and which is used up in producing heat

We thus see that $\frac{1}{4\pi}$ time the area of a hysteresis loop gives the energy (in ergs) dissipated in every unit volume of the material subjected to the magnetic cycle. The area of the loop will, of course, depend on the maximum value of the induction. As a result of a very large number of measurements, Steinmetz found that if $\mathbf{B}$ = maximum induction, then the energy dissipated, in ergs per cubic cm. per cycle, could be represented by the formula

$$\eta\ \mathbf{B}^{1.6},$$

where η is a constant for a given material, called the *hysteretic coefficient*, and has a value varying from .001 to .003 for soft iron.

The result that for a given material the hysteresis loss varies as the 1.6th power of the maximum induction is known as *Steinmetz's Law*. This law only holds within a certain range of induction, and is not correct for either very low or very high inductions.

Ewing's Hysteresis Tester.—If we attempt to rotate a well-laminated mass of iron in a magnetic field, then although, on account of the lamination, there will be no appreciable resistance due to induced eddy currents.

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[1] This energy will be expressed in C.G.S. units, *i.e.* ergs.

yet a considerable resistance will be encountered by reason of the dissipation of energy by hysteresis. The resistance to the motion will increase in proportion to the energy dissipated per cubic cm. per cycle.

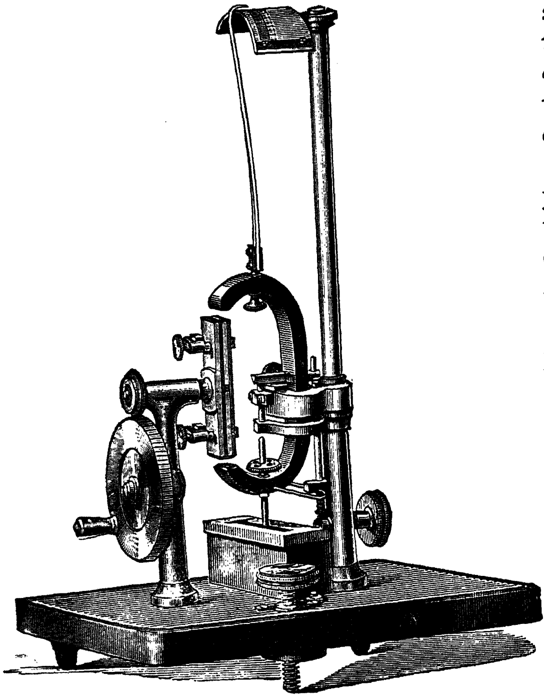

Fig. 54.—Ewing's Hysteresis Tester[1]

A highly ingenious yet simple instrument for the comparative testing of the hysteresis loss in specimens of sheet iron or steel has been constructed on this principle by Prof. Ewing, and is shown in fig. 54. Between the poles of a permanent magnet supported on a horizontal knife-edge and provided with a pointer which moves over a scale is rotated the specimen to be tested, which takes the form of a few strips of the sheet-metal held in a suitable clamp. The greater the hysteresis loss the larger will be the angle through which the pointer is deflected.

CHAPTER VIII

PRIMARY BATTERIES

Simple Voltaic Cell and its Defects.—Before the introduction of the dynamo, the only practicable method of producing electric currents was by means of primary cells. Since the advent of the dynamo, however, primary cells have become much less important, and in many cases where until recently they were largely employed, they are now being superseded by secondary cells. There is little doubt, however, that for certain purposes—such as electric bells, &c.—they will always continue to be used. Further, we have the important class known as standard cells, which form the commonly employed standards of E.M.F., and have for this reason been very carefully studied. In view of these facts, no account of the subject of electric measurements would be complete without a reference to primary cells.

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[1] From Prof. Ewing's *Magnetic Induction in Iron and Other Metals.* By permission.

A typical form of rudimentary or simple voltaic cell consists of a plate of copper and one of zinc in dilute sulphuric acid. One great objection to this form is the waste of zinc taking place during the time that the cell is not in use, since ordinary commercial zinc is readily attacked by sulphuric acid. In order to remedy this defect, the practice of amalgamating the zinc, or coating it with a thin layer of mercury, was introduced in 1828 by Kemp. The coating of amalgam is found to afford a very effective protection to the zinc. The readiness with which ordinary zinc is attacked by the acid is due to impurities in the zinc; these impurities when in contact with the acid form little voltaic cells with the zinc, and since the circuit of such cells is closed through the contact of the impurity with the zinc, a local current flows from the impurity to the zinc through the point of contact, and from the zinc back to the impurity through the acid, causing the zinc to go into solution. For this reason the waste of zinc which goes on when there is no current being taken out of the cell is generally referred to as "local action".

There is, however, another very serious defect in the simple voltaic cell under consideration. As soon as the cell is allowed to send a current, the sulphuric acid is split up by it into its constituent ions—H_2 and SO_4—the former appearing at the copper plate, the latter at the zinc plate, which it attacks, forming $Zn\,SO_4$. The hydrogen which appears at the copper plate partly escapes, but some of it adheres to the plate, and gives rise to an effect known as "polarization", which consists in a lowering of the E.M.F. of the cell. A plate of copper coated with hydrogen bubbles does not, in fact, behave like an ordinary copper plate. A simple experiment is sufficient to show the difference. For if into a vessel filled with dilute sulphuric acid we introduce a plate of zinc and two newly-cleaned plates of copper, then on connecting a galvanometer between the two copper plates, no appreciable current will, in general, be found to flow, showing that there is no difference between them. But if one of the copper plates be connected for a time to the zinc plate through an external resistance, and a deposition of small bubbles of hydrogen be thereby produced, then on introducing a galvanometer between the two copper plates a current will be found to flow round the external circuit from the unused plate to the one coated with hydrogen. The hydrogen, then, introduces an opposing E.M.F. into the circuit of the cell, and as a consequence there is a rapid fall in the current. In order to overcome this "polarization" of the cell, some means must be adopted for preventing the deposition of the hydrogen on the cathode plate. The various forms of primary cell differ from each other mainly in the means adopted for counteracting polarization. We shall here describe only the more important types of primary cell.

The Daniell Cell and its Modifications.—In this form of cell, which is shown in fig. 55, polarization is prevented by the use of two electrolytes—copper sulphate and sulphuric acid—separated from each other by a porous pot (of unglazed earthenware). The cylindrical plate of zinc and dilute sulphuric acid[1] are contained in the porous pot, while the external containing vessel is filled with a saturated solution of copper sulphate, the copper plate surrounding the porous pot.

When a current passes through the cell, $S\,O_4$ appears at the zinc plate, and causes it to go into solution. The H_2 which is split off from the last molecule of $H_2\,S\,O_4$ before the $Cu\,S\,O_4$ is reached attacks this latter, re-forming $H_2\,S\,O_4$, and the action is transmitted through a chain of $Cu\,S\,O_4$ molecules, until the last molecule is reached, whose Cu is deposited in the form of metallic copper on the copper plate. The final result, then, is the solution of Zn and the deposition of copper on the copper plate. At the same time the outer liquid gets poorer in $Cu\,S\,O_4$, and in order to supply the deficiency it is usual to have a supply of $Cu\,S\,O_4$ crystals in the outer vessel, so as to maintain the solution saturated.

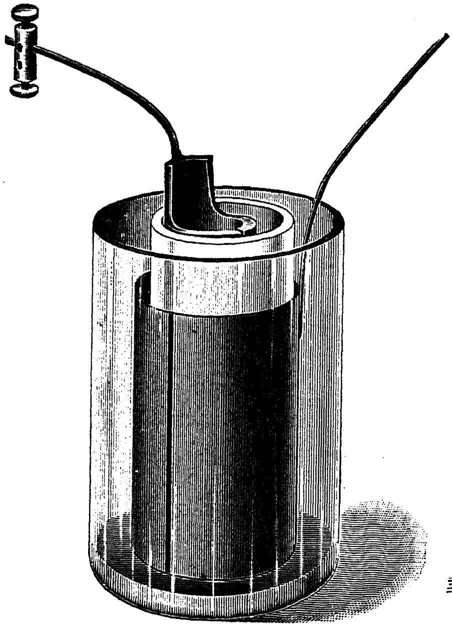

Fig. 55.—Daniell Cell

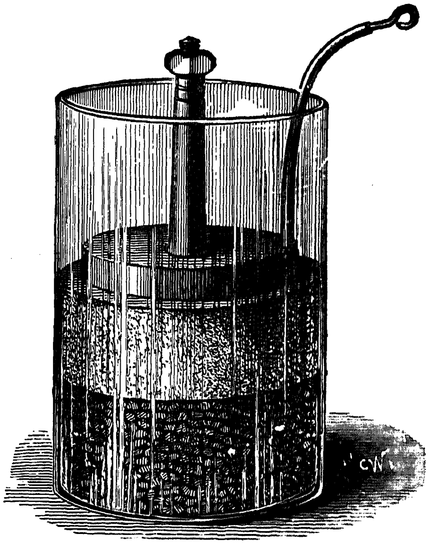

Fig. 56.—Minotto Cell

The substance, then, which prevents polarization—or, as it is technically termed, the *depolarizer*—in the Daniell cell is a liquid—copper sulphate.

Numerous modifications of this cell have been devised. One of the most important of these is the *Minotto* cell, shown in fig. 56. It consists of a cylindrical glass jar, with a disc of copper (to which is riveted a connecting wire) at the bottom of the jar, a layer of crystals of $Cu\,S\,O_4$ above the disc, then a layer of blotting-paper or cloth, followed by a thick layer of saw-dust or clean river sand, another thickness of blotting-paper, and finally a disc of zinc with a column supporting the terminal. The cell is, to start with, filled with water slightly above the level of the disc of zinc. This cell is largely used in India for telegraph circuits.

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[1] This gradually becomes contaminated with $Zn\,SO_4$ during use.

The Bichromate Cell.—This is shown in fig. 57, which represents the form known as the "porous-pot" variety of this cell. The zinc and sulphuric acid are contained in a porous pot, and outside this is a block of carbon in a solution of chromic acid. This latter is frequently prepared by introducing into the outer vessel a solution of potassium bichromate—$K_2Cr_2O_7$—and adding to it sulphuric acid, when the following reaction takes place:—

$$K_2Cr_2O_7 + H_2SO_4 = K_2SO_4 + 2CrO_3 + H_2O.$$

The chromic acid forms the depolarizer, the hydrogen, which would otherwise appear at the carbon plate, being attacked by the CrO_3, according to the equation

$$2CrO_3 + 3H_2 + 3H_2SO_4 = Cr_2(SO_4)_3 + 6H_2O.$$

The zinc in the type of cell illustrated in fig. 57 is made with an expanded base which rests on a layer of mercury at the bottom of the porous pot. This mercury, by creeping up the sides of the zinc, keeps it well amalgamated.

Another form of this cell is also made, in which no porous pot is used. In this form, arrangements must be provided for lifting the zinc out of the liquid when the cell is not in use, in order to prevent the formation of an insoluble chromium salt on the surface of the zinc.

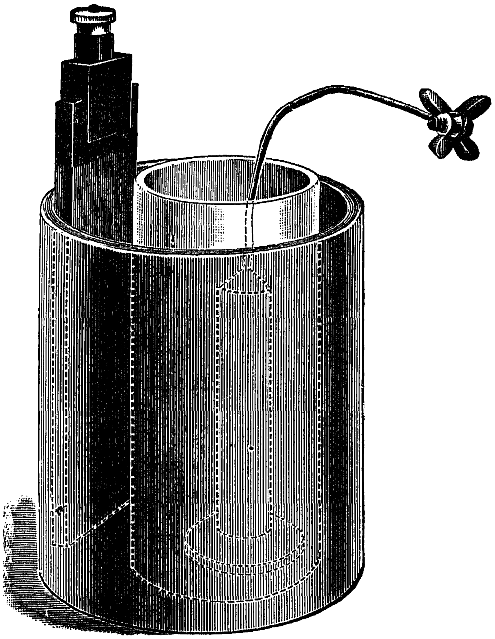

Fig. 57.—Bichromate Cell

The Leclanché Cell and its Modifications. — One form of Leclanché cell, known as the porous-pot variety, is shown in fig. 58; while fig. 59 shows the "agglomerate block" type. In each case the zinc is in the form of a cylindrical rod, and the other plate consists of a block of carbon. Further, the liquid used is in each case a solution of sal-ammoniac, and the depolarizer is manganese dioxide (MnO_2). The only difference between the two types consists in the method of applying the MnO_2, which is an example of a *solid* depolarizer. In the porous-pot form, the space inside the porous pot around the carbon block is tightly packed with a mixture of granulated MnO_2 and carbon, and the top of the porous pot is sealed, a small opening being left for the escape gases. In the agglomerate pattern, the carbon plate is held

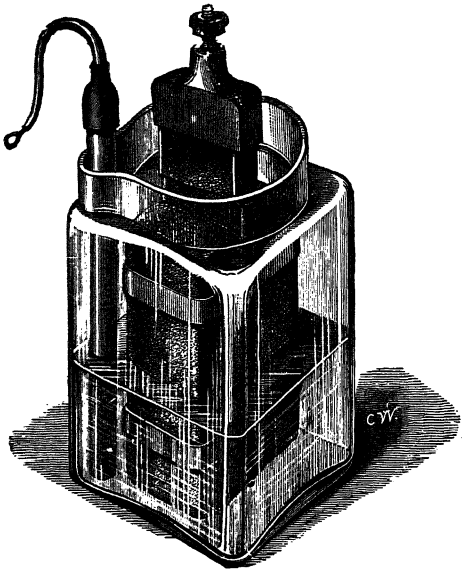

Fig. 58.—Leclanché Cell

between two blocks, formed under great pressure, of a mixture of MnO_2 and carbon.

When the cell is in use the zinc dissolves, forming $ZnCl_2$, and the NH_4 —which is the cation of an NH_4Cl molecule —is acted on by the MnO_2, according to the equation

$$2NH_4 + 2MnO_2 = Mn_2O_3 + H_2O + NH_3.$$

Various other forms of Leclanché cell have been devised, and the so-called "dry cells" are simple modifications of it.

The Leclanché cell, if used for intermittent work, is the least troublesome of all primary cells, and will work for months without requiring any attention. It is very largely used for bell and telephone work and testing purposes.

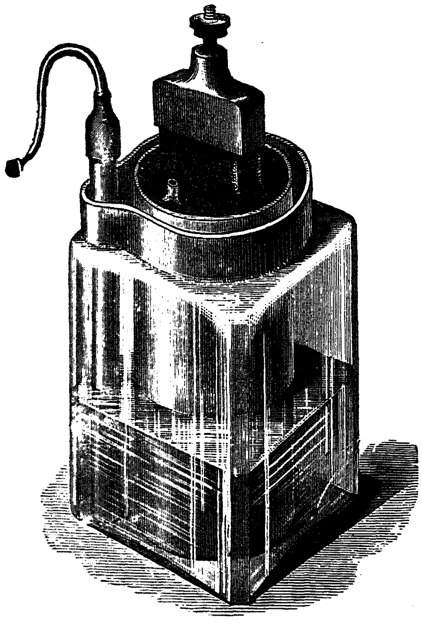

Fig. 59.—Leclanché "Agglomerate Block"

The Edison-Lalande Cell.—One of the most successful forms of primary cell for the production of relatively large currents is the Edison-Lalande cell. The electrolyte used is alkaline, consisting of a strong solution of caustic potash. This is contained in a cylindrical glass vessel, from the cover of which are suspended a central plate of zinc, and two blocks, one on either side of the zinc plate, of compressed finely-ground copper oxide. When a current passes through the cell, potassium zincate is formed, according to the equation

$$CuO + 2KOH + Zn = Cu + H_2O + Zn(OK)_2,$$

the copper oxide being reduced to metallic copper.

Standard Cells—The Clark Cell.—The construction of the cell which has been chosen by the Board of Trade as the practical standard of E.M.F., and which is known as the Clark cell, will be understood by reference to fig. 60, which shows (full size) the type recommended by the Board of Trade. The containing vessel consists of a glass tube, and contact is made with the mercury by means of a platinum wire sealed into a glass tube, as shown. Above the mercury is a paste consisting of mercurous sulphate and a saturated solution of zinc sulphate. The insoluble mercurous sulphate, which acts as the depolarizer, settles down on the top of the mercury, leaving a layer of clear solution of zinc sulphate above it. In order to maintain the solution saturated at all temperatures, a few crystals of the salt are added, and these may be generally seen resting on the top of the mercurous sulphate.

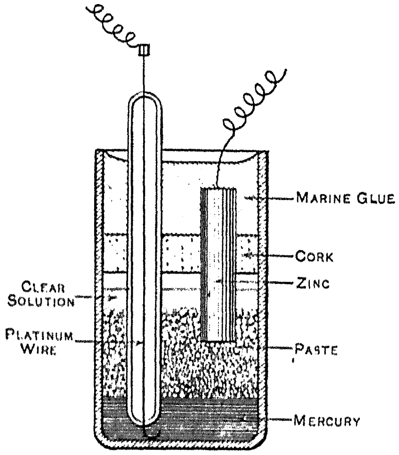

Fig. 60.—Clark Cell[1]

The cell constructed as described possesses several disadvantages, and numerous modifications of it have been suggested and used. In order to render it more portable, Dr. Muirhead dispenses with the large mass of mercury at the bottom of the cell, and for it substitutes a flattened spiral of platinum wire, which has been amalgamated by heating to redness and plunging into mercury; the small amount of mercury retained by the spiral is quite sufficient, and this arrangement renders the cell much more convenient to handle.

The E.M.F. of a Clark cell at 15° C. is 1.434 volt, and the E.M.F. at any other temperature t° C. is given by

$$1.434 \left\{ 1 - .00077\,(t - 15) \right\}.$$

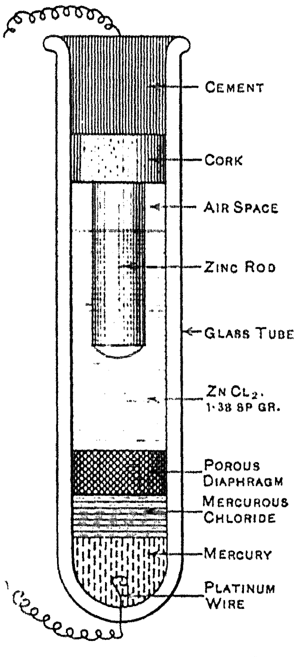

Fig. 61.—Hibbert's Standard Cell[1]

Hibbert's Standard Cell.—The variations in the E.M.F. of a Clark cell with temperature constitute a serious disadvantage, and a cell whose E.M.F. is practically unaffected by temperature changes would be much more convenient to use. One such cell, originally suggested by Helmholtz, but first put into a practical form by Mr. W. Hibbert, is shown in fig. 61. It differs from the Clark cell in that the chlorides of zinc and mercury are used instead of the sulphates. The change in the E.M.F. amounts to only .000085 volt per degree, and a further advantage of this cell is that it may be constructed to give exactly 1 volt.

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[1] From *Primary Batteries*, by W. R. Cooper. ("The Electrician" Printing and Publishing Co., Ltd.) By permission.

2. Alternating-Current Measurements

CHAPTER I

INTRODUCTORY

Definition.—The alternating current, as its name implies, is one which, instead of flowing always in one direction, flows alternately in opposite directions round a circuit. This general definition will clearly include a great many kinds of alternating currents, in which the rate of change both in magnitude and direction may be very different; the variation may be sudden or gradual, uniform or irregular, and, strictly speaking, each kind of alternating current requires separate treatment. The laws governing the flow of such a current through a circuit depend, to a very large extent, on the rate at which it changes, and although we may have exactly the same total quantity of electricity flowing first in one direction and then in the other, the potential difference that will be required to send it through a circuit will be very different for currents having different gradations of change.

The Wave Form.—Let us suppose that we have a current which at a time 0 has zero value; at a time .001 second later on has a value of 5 amps.; at a time .002 second later on has a value of 8 amps.; and so on. We may plot a curve showing the relation between current and time as in 4 (fig. 62); this curve will tell us all we want to know about the current, and is called the wave form of the current; it shows: (1) the time that is occupied by one complete alternation; (2) the way in which the current varies; (3) the greatest value the current will attain. A number of such wave forms of current and potential difference, taken from actual circuits, are given in fig. 62, and it will at once be seen how various are the wave shapes that the current may take.

Frequency.—The general definition given above does not make any stipulation as to the number of changes of direction of current which take place in a given interval of time; the change in direction may be made once in a hundredth of a second, once a second, or once an hour; each of these, according to our definition, is an alternating current. The alternating currents, however, which are usually spoken of as such, are those in which the change occurs in a comparatively short interval of time, in most cases less than one-tenth of a second.

The reversal, too, may be, according to definition, either regular or irregular, but in actual practice the rate of alternation is always constant, or very nearly so. Currents in which it is very variable are rarely met with, and the study of their behaviour is so complicated that in an article of this length it is impossible to more than mention their existence. It is important to bear in mind, however, that the relationships worked out refer only to currents in which the rate of alternation is constant.

The "periodic time" of an alternating current is defined as the interval that elapses between two consecutive changes of direction of the current in the same sense, *i.e.* either between two changes from a positive to a negative current, or between two changes from a negative to a positive.

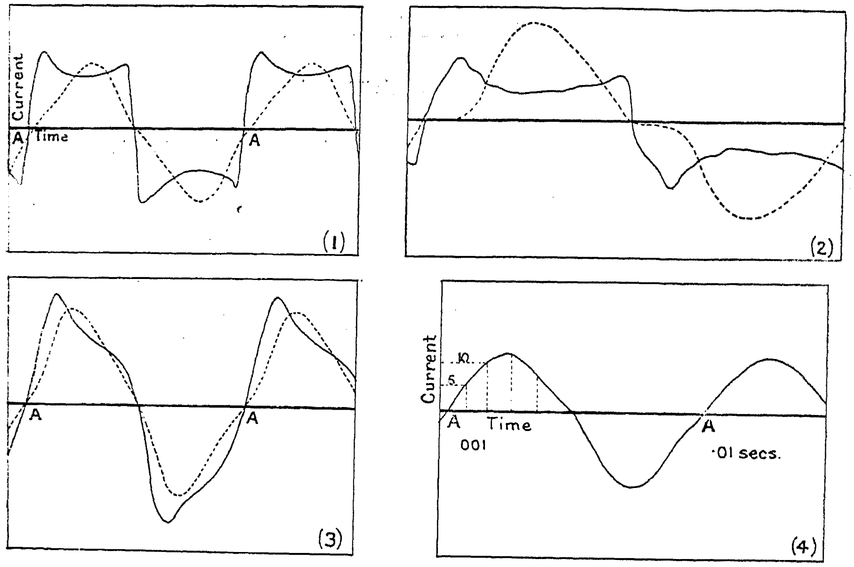

Fig. 62

In fig. 62 it is represented by the time corresponding to the length A A. The reciprocal of this quantity represents the number of alternations that occur in a second, and this number is called the frequency of the alternating current. The frequencies most commonly used at the present day vary from 133 to 15 alternations per second. The former frequency is used in some (a rapidly diminishing number) of the American lighting installations, and the latter in power transmissions. In this country, and on the Continent, the tendency nowadays is to use a frequency of between 40 and 50 alternations per second where both light and power are required, and a frequency of 25 for power alone.

In order to measure experimentally the frequency of an alternating current several arrangements have been devised. The simplest is one in which the free end of a steel strip capable of vibration is placed in front of a coil with laminated iron core, which carries alternating current. The variations in the attracting force of the coil causes the strip to vibrate at a frequency depending on that of the alternating current flowing through the coil; the position of the point of support of the clamped end of the strip is then varied until the note given out by it is as loud as possible; in other words, until the strip resounds. When this adjustment has been made, the frequency at which the strip is vibrating is known from its length and cross-section. The motion of the point of support is transferred by a rack and pinion to a pointer moving over a fixed scale, and the position of this pointer thus indirectly determines the frequency at which the strip is vibrating when in a condition of resonance. The scale may be graduated directly in frequencies.

Delineation of the Wave Form.—The shape of the wave of the alternating current has a very important influence on the way in which that current will behave, and it is important therefore to consider how that shape may be determined. Since the frequency of the alternating current employed in this country is usually fairly high, about 50 a second, it is clear that in order to obtain an idea of the way in which such a current is changing, an instrument which will move with great rapidity will be required. Such instruments have, however, been devised, and can be used to delineate wave forms with the greatest accuracy. The instrument which is most used in this country is one which was originally suggested by Prof. Blondel, but which has since been brought to great perfection by Mr. Duddell. It consists essentially of two strips of metal *s s* (fig. 63), made of phosphor-bronze, which are stretched by means of a spring-balance applied to a pulley P, between the poles of a powerful magnet N S. The circuit is so arranged that the alternating current passes up one strip and down the other.

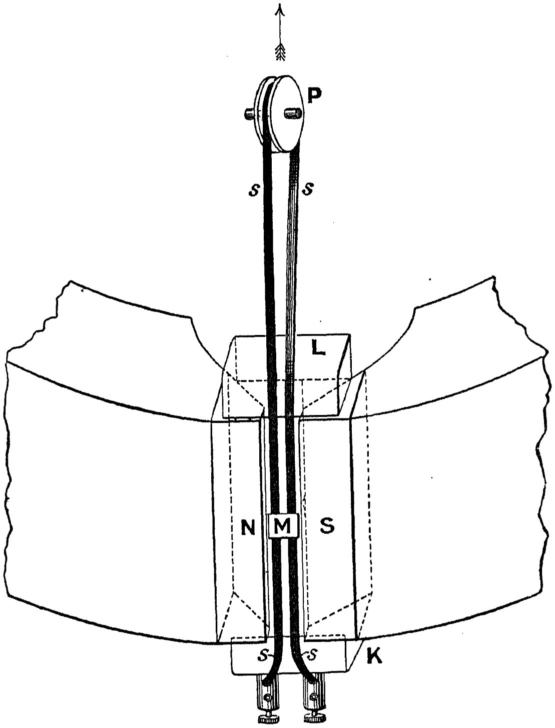

Fig. 63.—Duddell Oscillograph[1]

When any current is sent through this arrangement both of the strips will be deflected, as is well known from the fact that a conductor carrying a current, when placed in a magnetic field, tends to move in a direction perpendicular both to itself and to the direction of the field, but, since the current flows up one strip and down the other, the deflections will be in opposite directions. In order to indicate the deflection of these strips a little mirror M is stuck on to them, so that when the strips are deflected the mirror will turn, and the direction in which it will turn will depend on the direction of the current flowing in the strips. Such, in its barest outline, is the instrument known as the Duddell Oscillograph. A general view is shown in fig. 64. The conditions that it has to fulfil, in order that it may serve as an *accurate* recorder of the current, require that a special construction of the parts should be adopted. In the first place, the moving parts must be made of such a size that they will be able to follow the rapidly changing forces that are made to act upon them. In order to illustrate this point we may consider first a large ocean steamship which is sailing in a rough sea. Such a ship has a definite period of vibration of its own, that is, if tilted from its stable position it will oscillate about it at a given slow rate. This large boat, when subjected to a series of disturbing influences rapidly varying, such, for instance, as would be felt in a "choppy" sea, will be scarcely affected, partly on account of its large mass, but mostly because it has no time to respond to the rapidly varying impulses it receives. The same ship in a sea with a strong ground-swell, where the impulses are

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[1] Figs. 63, 64, and 65 are taken from the *Journal of the Institute of Electrical Engineers*, by permission.

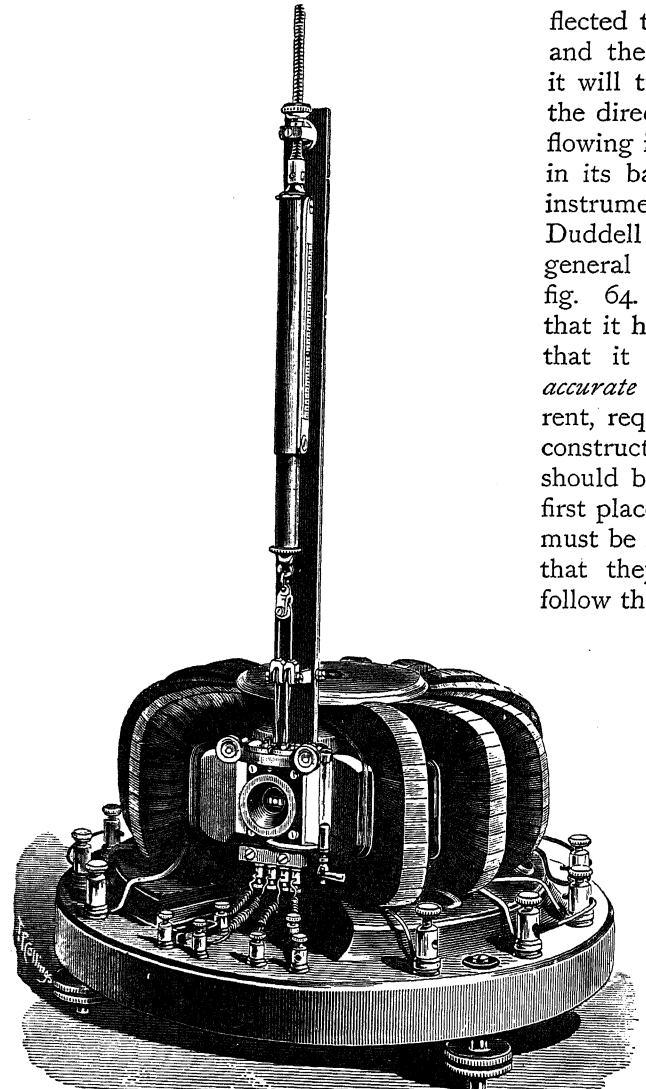

Fig. 64.—Duddell Oscillograph (general view)

given at much longer intervals, will be very much more deflected, and will "roll", although the actual strength of the impulses it receives may be no greater than those it felt in the "choppy" sea. A small rowing-boat, on the other hand, will respond to any sort of impulse, whether it be due to a choppy or a rolling sea, the amount of deflection being simply dependent on the magnitude of the impulse received. It will, of course, at the same time be subjected to a translational motion, but that for the moment may be left out of consideration. The same general mechanical principle holds for the strips of an oscillograph; they too, if they are deflected, will oscillate about their steady position at a given frequency; the amount of deflection that will be produced, however, will depend on the rapidity of the impulses given, *i.e.* it will depend on whether the strips are able to follow the forces. If the strips are to follow very rapidly changing forces they must be able to move with great rapidity, or, in other words, their "free periodic time" must be very small. In an actual oscillograph, such as we are considering, the strips, when deflected, will complete a back-and-forth movement in one three-thousandth of a second, and with this instrument it is easy to record quite accurately the wave form of an alternating current with a frequency of 100 ~ per second.

It is not sufficient, however, that the moving parts be small and that their free periodic time be small, since, even with these conditions perfectly fulfilled, the instrument will still not record accurately unless the movement be "damped".

To use an electrical illustration: if the needle of an ordinary Thomson galvanometer be deflected by a current passing through it, it will not come at once to its steady reading, but will oscillate about it, coming to the actual position only after a considerable interval of time. If the needle be enclosed in a small air space, so that a great deal of air friction is introduced, the oscillation will be damped down until finally it disappears, and the needle comes just to its proper deflection without first overshooting the mark. In the Duddell oscillograph this damping is effected by enclosing the moving strips in a space filled with castor-oil; this oil has great viscosity, and damps effectively the movement of the strips, so that the instrument will be deflected by its proper amount without any preliminary oscillation. When both of these conditions are fulfilled the instrument will be deflected at any moment by an amount proportional to the actual value of the current flowing through it at that moment.

Supposing that we have an instrument in which these conditions are fulfilled, the difficulty that now arises is that, although we know that the instrument is being deflected by a varying amount, it is impossible to distinguish the various deflections; when the alternating current is sent through the strips all that is seen is a band of light reflected from the mirror on the screen. In order to actually see the wave some means must be devised of spreading the curve out; if the deflection be horizontal, the spot of light must be moved vertically so that it is not in the same vertical position for every deflection. This is effected in a variety of ways, the simplest of which is by the use of a revolving mirror. The spot of light, when it has been deflected, is not viewed directly, but its reflection in a rapidly turning mirror is observed. In the case above taken, in which the motion of the spot is horizontal, the mirror is turned round a horizontal axis, so that an up or down vertical motion is given to the image of the spot as seen in the mirror. The actual arrangement is illustrated in plan in fig. 65 S being the position of the observed spot, and R the revolving mirror.

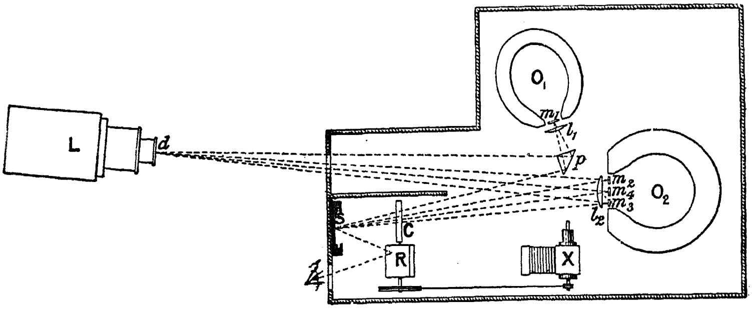

Fig. 65.—Plan of Duddell Oscillograph

Other methods of producing "time" motion have been employed, but it is not necessary in a brief description to mention them all. Other oscillographs, too, have been devised, based on entirely different principles, the most successful (next to the Duddell) being perhaps the Blondel instrument, in which the moving part is a small soft-iron needle, which is deflected by a coil of wire carrying the alternating current. The fact that it depends for its working on a magnetic action, instead of on an electromagnetic, necessarily entails a diminished accuracy for all waves that are not of a certain shape—a shape, however, to which, in a great many cases, a close approximation is made in practice.

CHAPTER II

THE SINE WAVE AND ITS REPRESENTATION

The Sine Wave.—The wave shape referred to in the last chapter is one the assumption of which greatly simplifies the study of certain alternating-current problems, and for many purposes does not lead to serious error. In the course of the next few paragraphs, therefore, which deal with the laws governing the flow of current through different kinds of circuit, this shape will be assumed. A very simple method may be adopted for drawing the wave. Take a line OC (fig. 66), and draw it of such a length that it represents to some scale a given current, and assume that it is rotating round a point O at a constant speed, a speed such that one complete revolution is made in a time corresponding to one complete alternation of the current we are considering. Suppose that the line starts from the horizontal position at time 0, at which time the current is zero, and after a given interval has moved round to a position O 1. From 1 draw a line parallel to O C to cut a vertical line drawn from 1 on the time scale, the intercept on this vertical line will represent to the proper scale the magnitude of the current after the time taken for the line to move from 0 C to O 1 (in the figure this time is represented on the scale of time by the length 0 1). This process may be repeated for a number of different positions of the line O C and a complete wave form drawn.

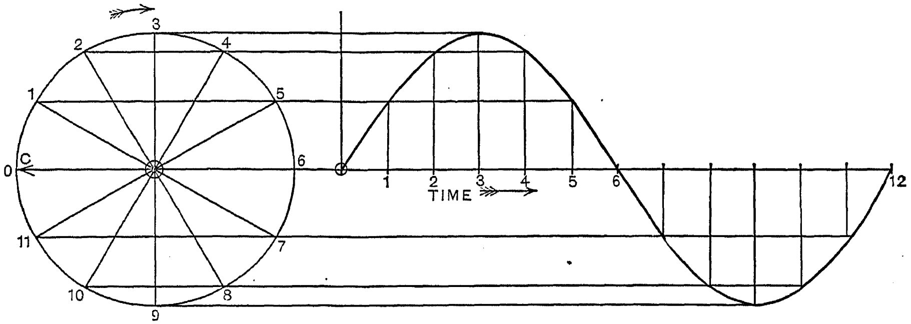

Fig. 66

The magnitude of the current flowing at any instant is determined by the length of the shadow or orthogonal projection of the moving line on a vertical plane at that instant, and such a wave is called a "Sine" wave, because the value of the current is proportional to the sine of the angle turned through by the rotating line (from its original position 0 C). The length of the line O C will clearly represent the maximum current that will flow through the circuit. Drawing the wave in the way described above, it is clear that an alternating current of this shape is completely determined when the length, speed of rotation, and direction at some instant of the corresponding line or vector, as it is sometimes called, is known.

Summation and Subtraction of Vectors.—Since for many alternating-current calculations it is exceedingly convenient to use vectors to represent alternating currents, the laws of addition and subtraction applying to them may be briefly considered. Two vectors may be added together or subtracted from one another just as two numbers may, but whereas in the addition and subtraction of numbers it is sufficient to know what the magnitudes of the numbers are, with vectors it is necessary also to know their direction. The method of adding one vector to another is exactly the same as that used in adding forces, or (to use the more ordinary term) in determining the resultant of two forces. If there are two coplanar forces acting on a body they may be represented in magnitude and direction by two lines O F_1 O F_2 (fig. 67). Suppose that there

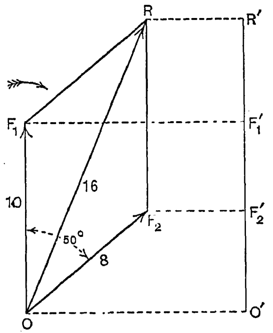

Fig. 67

is a force of 10 lbs. along OF_1, we may represent it by a line 10 mm. long (one millimetre being taken to represent one pound), and a force of 8 lbs. along OF_2, we may represent it by a line 8 mm. long. To determine the resultant or vector sum of the two, a parallelogram must be drawn having OF_1 and OF_2 as adjacent sides. The diagonal of this parallelogram will then represent in direction and magnitude the resultant force, which in this case is 16 lbs., since the diagonal OR is 16 mm. long.

The method that is adopted for determining the sum of two forces is a special case of the general method which may be employed to determine the sum of any two quantities that may be represented by vectors, a vector being a line of given length in a given direction. We may suppose, for instance, that the two lines OF_1 OF_2 represent alternating currents of the same frequency flowing in the two branches of the circuit in fig. 68, and we wish to determine the sum of the two currents, *i.e.* the current flowing in R. Exactly the same construction is employed. We may, for example, suppose that in F_1 a current whose maximum value is 10 amps. is flowing, and which is represented at any instant by the projection or shadow of the line OF_1, 10 mm. in length, 1 mm. corresponding to 1 ampere. In the same way we may suppose that the value of the alternating current in the branch F_2 *at the same instant* may be represented by the projection of the line OF_2, 8 mm. in length. To determine the sum (or resultant) complete the parallelogram, and the current at the instant under consideration will be given by the shadow or projection of the line OR, which in this case is 16 mm. long. Since the frequencies of the currents in the branches F_1 and F_2 are the same, the two vectors will always be at the same angle with each other, and the resultant OR will be fixed relatively to OF_1 and OF_2, so that the value of the alternating current in R will be determined at any instant by the length of the projection of OR, where OR is a line of constant length revolving at the same speed as OF_1. The maximum value of the current in R in this case will be 16 amperes.

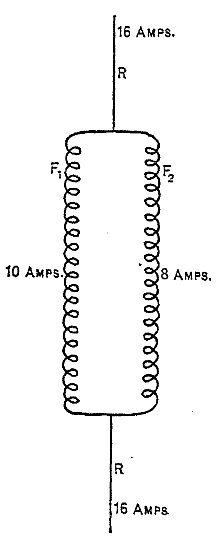

Fig. 68

Phase Difference.—The value of the resultant current determined in the above way will clearly depend not only on the actual magnitudes of OF_1 and OF_2, but on the angle between them. The angle between the vectors is defined as the "angle of phase difference" between the alternating currents. In the case considered above, the angle of phase difference is about 60°, and supposing the vectors to be going round in the same direction as the hands of a clock, the alternating current in F_1 is said to lag by an angle of 60° behind the current in F_2 (see fig. 67), while the resultant lags by about 28° behind F_2, and leads by an angle of 32° on F_1. The current in F_1 will not reach its positive maximum value until some time after that in F_2 (see fig. 69), a time corresponding to the angle F_2OF_1. Since one complete revolution through 360° corresponds to the time (T) occupied by one alternation, the difference in the times at which F_1 F_2 are a maximum is $\frac{(F_2OF_1)}{360}$ T; or, since $T = \frac{1}{f}$, it is $= \frac{(F_2OF_1)}{360f}$; or, more generally, expressing angles in radians, time difference $= \frac{\phi}{2\pi f}$, where ϕ is the angle of lag of one behind the other. The difference in time between the moment at which the current OF_1 reaches its maximum and at which OR reaches its maximum may be determined in the same way.

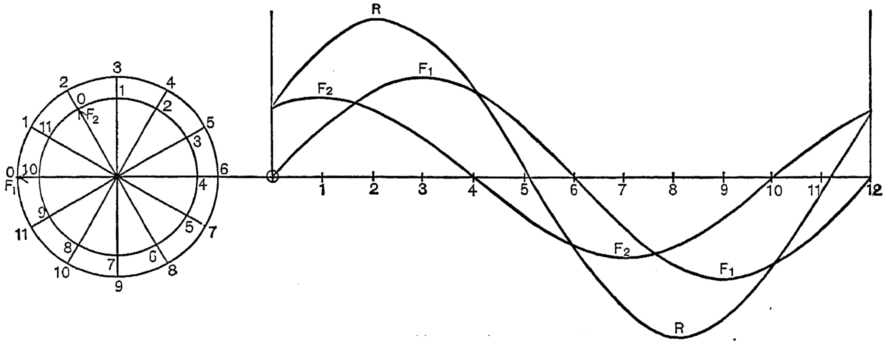

Fig. 69

If the two vectors be in the same direction, *i.e.* if the currents be in phase, the resultant current will be simply the sum of the two, in this case 18 amperes. If the currents be 180° out of phase, the vectors will be opposite, and the resultant current will be the difference between them, in this case 2 amperes. The actual value of the resultant for varying angles of phase difference will be anything between these two limits. The construction that has been described above may be very simply proved. The value of the actual current *at any instant* in R is the sum of the currents *at that instant* in F_1 and F_2, so that the vector which represents the resultant current must be of such a length that its orthogonal projection or shadow on a vertical line is equal to the sum of the projection of the vectors representing the currents in F_1 and F_2. From fig. 67 it is clear that the length of the projection or shadow $O'R'$ cast by OR will be equal to the sum of the lengths of the shadows $O'F_1'$ and $F_1'R'$ cast by OF_1 and F_1R, but F_1R is equal and parallel to OF_2, so that its projection $F_1'R'$ is equal to $O'F_2'$ that of OF_2. Hence

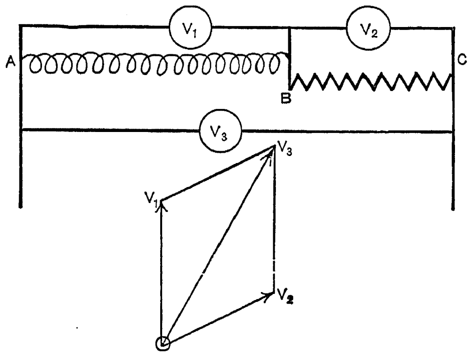

Fig. 70

the length of the projection of O R is equal to the sum of the projections of O F_1 and O F_2, and hence O R will represent the resultant current flowing in the circuit considered.

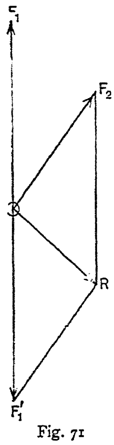

Fig. 71

All that has been said above with reference to the representation of alternating currents by vectors applies to alternating potential differences, and the potential difference between two points A and C (fig. 70) in a circuit may be found by adding vectorially the potential differences that exist between A and B and between B and C.

The method of subtracting alternating currents one from the other is a very simple process. Subtraction is negative addition, and if we wish to subtract a vector O F_1 from O F_2 (fig. 71), the method employed is to add O F_2 to ($-$ O F_1). $-$ O F_1 is O F_1 in the opposite sense, O F'_1 in the figure, and the difference between O F_2 and O F_1 is obtained by adding O F_2 to O F'_1. Performing this operation, we have a resultant O R representing the difference between the two vectors. In just the same way alternating potential differences, represented by vectors, may be subtracted from one another.

CHAPTER III

SELF-INDUCTION

When dealing with the flow of direct or continuous currents through any circuit it is sufficient to know what is the resistance of the circuit, in order to determine the strength of the current that will flow when a given potential difference is applied to the terminals. With alternating currents, however, other properties of the circuit have to be taken into account. The magnitude of the alternating current that will flow depends not only on the resistance but on the shape of the circuit, *i.e.* on the way in which the parts are arranged relatively to each other. In order to explain this phenomenon it will be necessary in the first place to consider the more simple case of a continuous-current circuit which is made and broken.

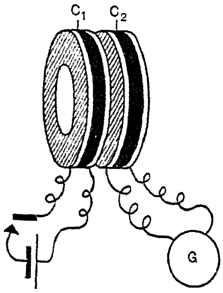

Fig. 72

From Faraday's experiments on electromagnetic induction we know that if two coils of wire C_1, C_2, are held near one another in parallel planes, as in fig. 72, and if a current be started through one of the coils, it will induce an E.M.F. in the other coil. And further, from Lenz's Law we know that the direction of the current that will tend to flow in the second coil will be opposite to that in which the current is flowing in the first coil. When the current in the first coil is broken the effect will be reversed. The explanation given by Faraday is that when the current is started in the first coil, lines of force are produced which thread the second coil; an E.M.F. is induced in the second coil which is equal to the rate of change of number of lines of force threaded by that coil. When the current is steady there is no change in the number of lines of force, and therefore no electromotive force is induced; when the current is broken, the number of lines, instead of increasing, is diminishing, and therefore the direction of the electromotive force is reversed.

From Faraday's experiments it is known that this effect will be increased if the coils are brought closer together. Suppose, now, that the two coils are brought very close together, that the two are made from the two strands of a twin flexible wire, for example, the effect on the one coil will be all the stronger when a current is started through the other. Finally, suppose that the insulation between the two coils is done away with, and that we are left with only a single coil of wire. Exactly the same effect will occur; when the current is started in the coil there will be an opposing E.M.F. induced, which will be measured by the rate of change of number of lines of force threading the coil. When the current is stopped, there will be an E.M.F. induced which will be in such a direction as to tend to keep the current flowing. These electromotive forces are said to be due to the self-induction of the circuit, and the coefficient of self-induction is determined by the magnitude of the back E.M.F. induced when the current is changing at a given rate. The unit of self-induction called the henry is defined as follows:—

A coil of wire is said to have a coefficient of self-induction of 1 henry if the back electromotive induced, when the current is changing at the rate of 1 ampere per second, be 1 volt. The effect produced by self-induction when a current is started through a coil is to retard the flow of the current, which, instead of rising at once to its steady value as determined from Ohm's Law, increases gradually, and theoretically only attains its steady value after an infinite time. For example, the time taken by the current to reach nine-tenths of its full value round the shunt coils of a dynamo, which has a self-induction coefficient of 100 henrys and a resistance of 10 ohms, is 23 seconds. It is not necessary to deal with the subject in detail here, as the rise of current in coils having iron cores is greatly complicated by the fact that iron is a material with a variable permeability, so that the coefficient of self-induction of such a coil is not a constant quantity, but depends on the state of magnetization of the iron. The electromotive force due to self-induction, which is induced when an inductive circuit, such as the shunt circuit of a dynamo, is broken, is of considerable practical importance. The magnitude of the potential difference is determined in the same way as when the current is increasing, but in this case the potential difference will be in such a direction as to tend to keep the current flowing in the same direction as formerly. The actual voltage induced is obtained by multiplying the coefficient of self-induction of the circuit by the rate of change of current flowing through it; or, expressing it as an equation, $E = -L\frac{di}{dt}$, since the

E.M.F. of self-induction acts against the current when the latter is increasing. As an example, consider the shunt circuit taken above witl a resistance of 10 ohms and a self-induction of 100 henrys. Suppos that by means of a quick-break switch a current of 10 amps. is destroyed in one-tenth of a second. The rate of change of current wil be 100 amps. per sec., and the E.M.F. induced will be 100 × 10 = 10,000 volts, a pressure 100 times as great as that used to send steady current through the coils. In practice, of course, the curren could never be broken as quickly as has been assumed, as an arc woul be formed at the switch contacts, but the calculation is sufficient t show the great danger of too suddenly breaking a highly inductive circuit

So far we have been considering only the making and breaking o direct currents, but it will at once be clear that, with an alternating curren

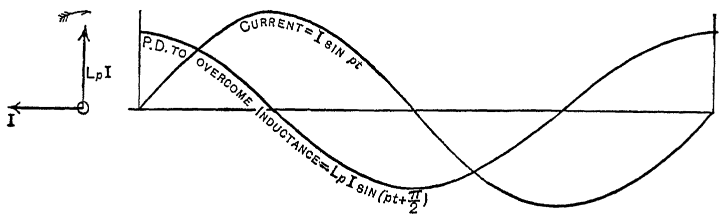

Fig. 73

which is continually varying, there will be a continuously varying self-induced E.M.F., always tending to prevent change in flow of curren With the sine wave described above it is found[1] that there is an alternating E.M.F. due to self-induction when the current is flowing, whic is also a sine wave (if the self-induction be constant), and whose maximu value is L p I, where L is the self-induction coefficient, p is 2π × frequency, and I is the maximum value of the current flowing, and moreover that this E.M.F. is 90° out of phase with the current. The reaso of this phase difference will at once be clear when it is considered that a its maximum value the alternating-current wave is flat, so that during very short time the change in current is small, while when the curren is zero the rate of change is a maximum, *i.e.* the slope of the curren curve is a maximum (see figs. 66 and 69).

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[1] Assume that the current is equal to I sin pt at time t, the current at time $t + \delta t$ is I sin $p\,(t + \delta t)$.

$$\begin{aligned}
\text{Change in current in time } \delta t &= \mathrm{I}\sin p\,(t+\delta t) - \mathrm{I}\sin pt. \\
\text{Rate of change} &= \frac{\mathrm{I}\,(\sin p\,(t+\delta t) - \sin pt)}{\delta t} \\
&= \frac{\mathrm{I}\,(\sin pt \cos \delta t + \cos pt \sin p\delta t - \sin pt)}{\delta t}. \\
\text{But } \cos p\delta t &= 1, \text{ and } \sin p\delta t = p\delta t \text{ if } p\delta t \text{ be small.} \\
\text{Hence rate of change} &= \frac{\mathrm{I}\,(\sin pt + p\delta t \cos pt - \sin pt)}{\delta t} \\
&= \mathrm{I}p\cos pt, \\
\text{and self-induced E.M.F.} &= -\mathrm{LI}p\cos pt = -\mathrm{LI}p\sin\left(\frac{\pi}{2} + pt\right).
\end{aligned}$$

Its maximum value = LpI, and it lags by 90° behind the current, as stated above. The E.M.F. th must be applied to overcome this self-induced E.M.F. is $+\mathrm{L}p\mathrm{I}\sin\left(\frac{\pi}{2} + pt\right)$, and leads by 9 on the current.

We may now draw the waves (fig. 73) which give us the current, and the potential difference required to send that current through a given self-induction. The potential-difference curve is zero when the current is a maximum, and a maximum when the current is zero. It is interesting also to notice that in this case, during one quarter wave the current is flowing in the same direction as the applied potential difference—as it would do in a continuous-current circuit, while during the next quarter wave the current is flowing against the potential difference—as it would, for instance, in a storage cell during charging. This is a consideration which becomes of great importance when dealing with the expenditure of power in alternate-current circuits. If we denote the current by a vector I, the potential difference required to overcome self-induction will be given by a vector of length corresponding to L p I perpendicular to I. In practice it is impossible to have a coil with a large coefficient of self-induction without at the same time having a certain amount of resistance; the resistance may be very low, but it is never quite negligible. We must, therefore, when considering the flow of alternating currents through most circuits, take into account their resistance. Just as for direct currents the value of the current I flowing through a resistance R when a potential difference E is applied is $I = \frac{E}{R}$, or $E = I R$, so for alternating currents, and at any instant the value of the potential difference required to send the alternating current through the resistance is given by the above equation.

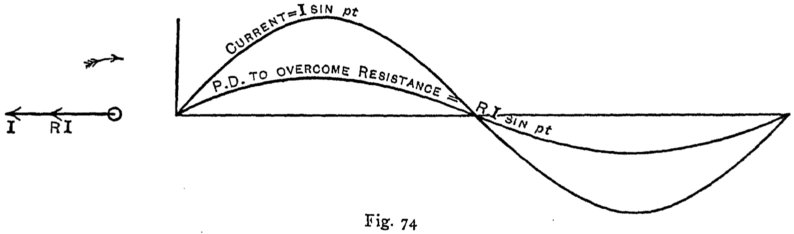

Fig. 74

Working in the same way as before, assuming that we have a given current flowing, the potential difference[1] that must be applied to overcome resistance is I × R, so that if the alternating current be determined by a vector of length (to a given scale) = I, then the potential difference required will be represented by a vector in the same direction, that is, in phase with the current (see fig. 74).

If we have a circuit, then, without self-induction and with a certain amount of resistance — a non-inductive resistance as it is called — we shall have the current at any instant determined simply by Ohm's Law, the wave of current being in phase with the wave of applied potential difference. In the more general case stated above, where we have both self-induction and resistance, we must take into account both the potential differences, *i.e.* that necessary to overcome inductance and also that necessary to overcome resistance. The determination of the total potential difference that must be applied to send the given current is very simply made by adding these two vectors.

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[1] Assume that the current wave is represented by I sin pt, the potential difference wave to overcome resistance is R I sin pt.

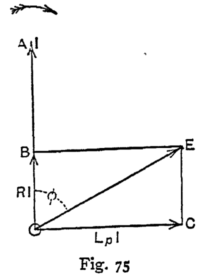

Fig. 75

Let O A in fig. 75 represent the current flowing through a coil having resistance R and self-induction coefficient L. Let f be the frequency (and $p = 2\pi f$). The "resistance" potential difference will be given by a vector O B along O A, and the self-induction potential difference will be given by a vector O C leading by 90°, *i.e.* perpendicular to O A, the resultant O E represents the total potential difference necessary to send the given current. From the figure it is clear that

$$(\mathrm{OE})^2 = R^2 I^2 + L^2 p^2 I^2,$$

or if E denote the length of O E to the proper scale of volts,

$$E^2 = I^2 (R^2 + L^2 p^2),$$

$$\text{and } I = \frac{E}{\sqrt{R^2 + L^2 p^2}}.$$

L p is called the "reactance" of the circuit. $\sqrt{R^2 + L^2 p^2}$ is called the "impedance" of the circuit. To take an actual case, suppose that there is a current of 10 amps. flowing through a coil of resistance 1 ohm self-induction = .01 henry and that the frequency = 16 ~ per sec., so that p = 100. Reactance = L p = 1 ohm.

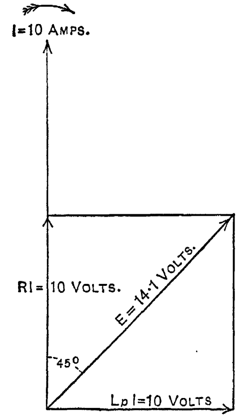

Fig. 76

Then maximum resistance P.D. = 10 volts,
maximum inductance P.D. = 10 volts.

Summing these two the total potential difference = 14.1 volts (see fig. 76), and in order to send this current such a potential difference would have to be applied. There is, however, another point of great interest to be determined from the above figure—namely, the lag of current behind applied potential difference. The vectors of current and potential difference are not in the same direction. The angle between them is easily determined in the above figure.

$$\tan\phi = \frac{BE}{OB} = \frac{LpI}{RI} = \frac{Lp}{R} = \frac{\text{reactance}}{\text{resistance}}.$$

In the special case taken L p = 1.0 ω R = 1.0 ω,

so that $\tan\phi = 1$ and $\phi = 45°$,

or the current lags in phase by 45° behind the applied potential difference. The special cases considered above, where we have circuits (1) with large inductance and small resistance, and (2) with large resistance and small inductance, at once come out from the above equations. In the first we have $\tan \phi = \frac{L p}{R}$ and $L p$, the reactance, is very large compared with the resistance, $\tan \phi = \infty$ very nearly, so that $\phi = \frac{\pi}{2}$ and the current lags by 90°. In the second case, $\tan \phi = \frac{L p}{R} = 0$ (nearly), and $\phi = 0$, the current and potential difference are in phase. All practical cases lie between these two limits.

CHAPTER IV

CAPACITY

With direct currents the amount of current flowing when a potential difference is applied is dependent solely on the resistance of the circuit, and if this resistance be very high no appreciable current will flow. With alternating currents, however, this is no longer true, as it is possible for an alternating current to flow through a circuit which is divided at some point by insulating material. In other words, the flow of alternating current depends on the "capacity" of a circuit, and such a current can flow through a condenser.

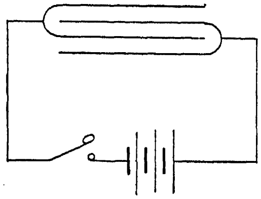
Fig. 77

We will, in the first instance, consider the simple case of the making and breaking of a condenser circuit with a source of continuous current. Let us suppose that, in the first instance, the condenser (usually made up of a number of sheets of tin-foil connected alternately with each other, and separated by sheets of mica or paraffined paper) is to be charged. When the circuit (fig. 77) is made there is a sudden rush of current into the condenser; the "quantity" Q of electricity that will flow will depend on its capacity C, and on the E.M.F. E of the battery. It is determined by the equation $Q = C E$.

When the full difference in potential has been attained all flow of current will cease. When the circuit is broken again the condenser will still remain charged, but if the battery be removed and the condenser terminals connected there will be a corresponding rush of electricity, flowing this time in the opposite direction, which is called the discharge current of the condenser; the amount, neglecting leakage, will be the same as that originally sent into the condenser on charge, *i.e.* $Q = C E$.

The process of charging a condenser is analogous to the filling up of a vessel with water, or perhaps more closely to the production of a difference in level between two vessels connected by a pipe. In order to obtain the necessary difference in level a certain amount of water is forced by some means from one vessel into the other. Suppose, for example, that we have two tubes, such as are shown in fig. 78, in the junction pipe between which a flexible membrane R is placed; it is possible to produce a difference in level in the two pipes by depressing one of the columns A by means of a piston, distorting the rubber membrane R in the junction to some such position as that shown by the dotted line, thus forcing a certain amount of water into the other tube B. In this case, although the tubes are actually separated so that no current of water can pass, there will be a small current down A and up B. If the pressure were applied to B the current would, of course, flow in the opposite direction. Further, if the pressure were an alternating one, being first applied to A, then to B, and so on, there would be an alternating flow of water in the pipes, first in the one direction then in the other. The amount of flow would clearly depend on the pressure applied at A and B, the size of the membrane R, and the kind of material of which it is made. We may now apply this analogy to explain the electrical case of an alternating current flowing through a condenser.

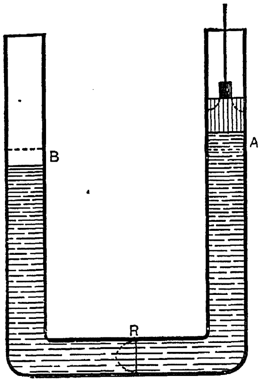

Fig. 78

When an E.M.F. is applied to the condenser a strain is produced in the dielectric or insulating material between the coatings (which is exactly analogous to the strain in the rubber), and a current will flow into the condenser on one side and out at the other; when the strain is removed or reversed an opposite flow of current occurs. The amount of current that will flow will depend on the size or capacity of the condenser, and in this term is included the effect produced by the kind of dielectric that is employed in its construction.

It will next be necessary to consider the amount of current that will flow through such a condenser. If q be the quantity on either coating at any instant, we know that $q = Ce$ with the above notation; the current that flows is given by the rate at which the quantity on the condenser plates is changing, in ordinary units 1 ampere corresponds to a flow of 1 coulomb per sec.

Hence $i = + \frac{dq}{dt} = + \frac{d(Ce)}{dt}$. Now the capacity C is a constant quantity, so that $i = + C\frac{de}{dt}$, or the current is proportional to the rate of change of potential difference. It will be noticed that this equation is exactly similar in form to that found for an inductive circuit, but that in this case the quantities current and potential difference are interchanged.[1] Representing the relationship vectorially, we have the condition in fig. 79. We have, so far, considered a circuit which has been assumed to have no resistance, but just as in the case of an inductive circuit this is a physical impossibility, and there must always be a certain amount of resistance to be overcome. This may be allowed for in exactly the same way as was done in the inductive case (fig. 75). Drawing a vector diagram

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[1] Assuming that an alternating sine wave of potential difference is employed, we know that at any

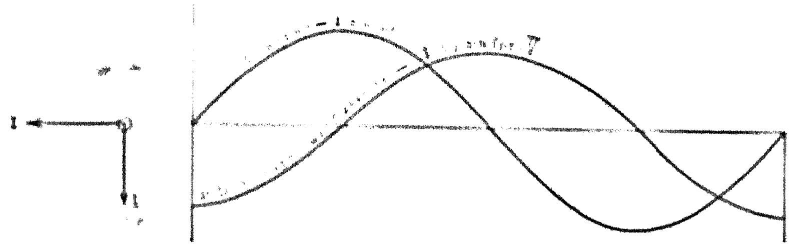
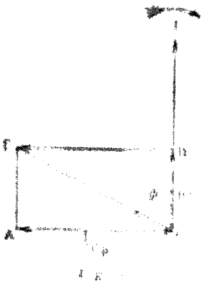


Fig. 79.

(fig. 80) for a circuit possessing both resistance and capacity, we have first a vector in phase with the current whose length, to some suitable scale, is R I, and next a vector at right angles to this lagging behind the current, whose length, to the same scale, is $\frac{I}{Cp}$ and summing these two vectors by forming a parallelogram we have as a resultant a vector O E, which represents the total potential difference that must be applied to the circuit to send the current through it.



The magnitude of O E is easily found from the geometry of the figure.

$$OE^2 = (OB)^2 + EB^2 = RI^2 + \left(\frac{I}{Cp}\right)^2,$$

$$\text{or} \qquad E = I\sqrt{R^2 + \left(\frac{1}{Cp}\right)^2}.$$

The equation will also clearly determine the current I, flowing in a circuit to which a given potential difference is applied.

The potential difference will lag behind the current by an angle ϕ, which is easily determined from the figure

$$\tan\phi = \frac{EB}{OB} = \frac{1}{RCp}.$$

Since $e = E \sin pt$, and as in the inductive case $\frac{de}{dt} = Ep \cos pt$, so that at any instant $i = ECp$ [illegible] $i = ECp \sin\left(pt + \frac{\pi}{2}\right)$, and the current leads by 90° on the applied potential difference.

The maximum value of the current flowing when the potential difference, whose maximum value is E, is applied is $I = ECp$.

Where C = capacity of condenser in farads (the unit of capacity),
p = 2π × frequency,
E being expressed in volts and I in amperes.

To take an actual case, suppose that we have a condenser of [illegible] microfarads capacity, since a microfarad is one millionth of a farad (capacity = [illegible] farad), to which an alternating potential difference whose maximum value is [illegible] volts is applied, and that the frequency is [illegible], so that p = [illegible], then the current = [illegible] amps. In many cases the wave of current flowing in a condenser is not a sufficiently close approximation to a sine wave to allow of this simple calculation, since condensers tend to magnify irregularities occurring in the wave of applied potential difference.

As before, to take an actual case (fig. 81), suppose that we have in series, with a capacity of .001 farad, a resistance of 10 ohms, and that through this circuit a current of 10 amps. is flowing. Let the frequency be 16, so that $p = 100$.

$$RI = 10 \times 10 = 100.$$

$$\frac{I}{Cp} = \frac{10}{.001 \times 100} = 100.$$

Drawing the vector diagram, we find the necessary potential difference to send this current is 141.4 volts. The angle of lag of potential difference behind current is given by

$$\tan\phi = \frac{EB}{OB} = \frac{100}{100} = 1, \text{ and } \phi = 45^\circ.$$

The current leads, therefore, on the applied potential difference by 45°, and the latter must be equal to 141.4 volts.

Condensers in their ordinary form are not very extensively employed in practice, though they are applied to certain uses, notably in the starting of single-phase motors. The reason for their non-use is chiefly their cost of manufacture, though another very important point is the difficulty of making them thoroughly reliable. In other forms, however, they are frequently met with. A concentric cable forms a condenser of considerable capacity, the outer sheath forming one plate and the core the other, so that when an alternating potential difference is applied to concentric mains, a certain current will flow through the dielectric of the cable.[1] The first, or perhaps the first well-known case, in which these dielectric or condenser currents produced appreciable effect was at Deptford, in the high-tension installation arranged there to supply Westminster. The transmission in this case was at 10,000 volts, and it was noticed that the amount of current sent into the cable was quite large when there was no "load" on the circuit. The current was, of course, flowing across the dielectric of the concentric cable, which in this case was nearly 10 miles long, and had a total capacity of over 30 microfarads.

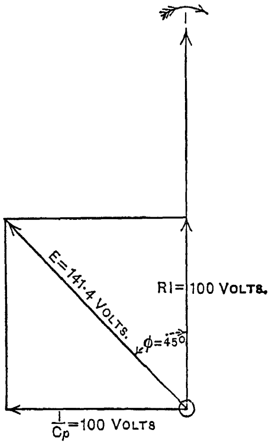


Fig. 81

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[1] This also involves a certain straining of the dielectric, which is never a "perfect" solid, so that the condenser current will have a tendency to heat up the insulating material. In some cases the heating effects produced by these dielectric strains cause a temperature rise in the cable greater than that due to the energy wasted in the resistance of the cable.

CHAPTER V

ELECTRICAL RESONANCE

Capacity and Self-Induction Combined.—It has been shown in the above paragraphs that when an alternating potential difference is applied to an inductive circuit a lagging current is produced, while that when the same pressure is applied to a capacity, a leading current is sent. It now remains to consider the more general case of combined capacity and self-induction. Let us suppose that the circuit is that shown in fig. 82, in which we have a self-induction of L henrys, and a resistance of R ohms in series with a capacity of C farads. Let us suppose that a current of I amperes flows through the circuit. Then we may determine, by aid of the diagrams described above, the potential difference necessary to produce this current. Let O I (fig. 83) be the vector of current, then the potential difference required to send the current through the inductance is at right angles to this, leading on it by 90°. The magnitude of this vector is $LpI = OA$. In order to send the current through the capacity a potential difference $= \frac{I}{Cp}$ is necessary, but this potential difference lags by 90° behind the current, and is represented by O C in the figure. The two potential-difference vectors, therefore, required to send the current through the inductance and capacity are directly opposed to one another, and the resultant of the two will be obtained by subtracting one from the other. In the figure this length is O D. We still have to take into consideration the potential difference necessary to overcome the resistance of the circuit. This as before will be represented by a vector O B equal in length to R I to the proper volt scale, and in phase with O I.

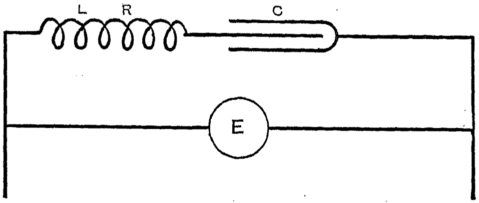

Fig. 82

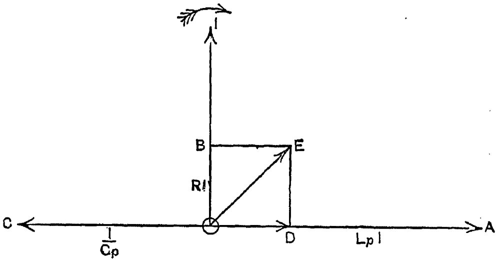

Fig. 83

The resultant of all these vectors is obtained by forming a parallelogram of which O B and O D are adjacent sides, the diagonal O E will represent the total pressure that must be applied to the circuit in order to send the given current through it.

The length of O E is easily determined.

O D is the difference between O A and O C $= I\left(Lp - \frac{1}{Cp}\right)$, and $O\,B$ is $= R\,I$, so that

$$(O\,E)^2 = (O\,B)^2 + (O\,D)^2 = I^2\left(R^2 + \left(Lp - \frac{1}{Cp}\right)^2\right),$$

$$\text{or } I = \frac{E}{\sqrt{R^2 + \left(Lp - \frac{1}{Cp}\right)^2}}.$$

The angle of lag or lead of the current is also readily found

$$\text{since } \tan\phi = \frac{B\,E}{O\,B} = \frac{I\left(Lp - \frac{1}{Cp}\right)}{R\,I} = \frac{Lp - \frac{1}{Cp}}{R}.$$

Electrical Resonance.—Various very interesting conclusions may be drawn from this equation and from the above figures. The most obvious is, that if $Lp = \frac{1}{Cp}$ the current flowing will be the same as if there were no self-induction or capacity present, its value being given simply by the equation $I = \frac{E}{R}$, and in this case the angle of lag $= 0$. When this is the case there is said to be a condition of resonance in the circuit. The fulfilment of the condition in any particular case will depend on the frequency of the applied pressure. Since $p = 2\pi \times$ frequency, we have, when the condition is satisfied—

$$Lp = \frac{1}{Cp} \text{ or } p^2 = \frac{1}{LC} = (2\pi)^2 f^2,$$

so that

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

The frequency calculated in this way from the electrical constants of a circuit is said to be the "natural frequency" of the circuit.

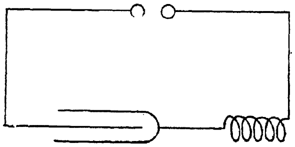

Fig. 84

An electrical circuit may, in fact, be regarded as analogous to a pendulum. If the pendulum bob be displaced by any means, it will oscillate at a given frequency, dependent simply on the dimensions of the pendulum; and in an electrical circuit, when the electric stability of the circuit is disturbed, electric currents will flow backwards and forwards at a given rate dependent on the electrical dimensions (*i.e.* capacity and self-induction) of the circuit. One way in which this disturbance may be produced is by the aid of an electric spark. If a potential difference be produced between two knobs connected to a circuit as shown in fig. 84, the electric stability of the circuit is disturbed, so that when the spark passes, surgings will pass across the spark gap at a fixed frequency, thus giving rise at the spark gap to the well-known electric waves now so familiar from their employment in wireless telegraphy. The *frequency* of these waves is determined simply by the capacity and self-induction of the circuit, but it is important to bear in mind that the *resistance* of the circuit also affects their production, for, when the resistance (which is the electrical analogue to friction in the pendulum case) exceeds a certain value electric oscillations cannot be produced by a spark. The actual equation limiting the value of the resistance is $R^2 > \frac{4L}{C}$.

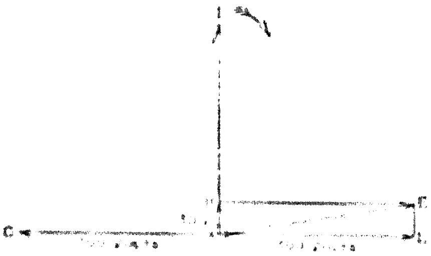

Fig. 85.

Returning now to the consideration of the circuit to which an alternating potential difference of given frequency is applied, it is important to notice the potential differences that may exist between the terminals of the self induction and the capacity. Suppose that we have a coil with a self induction of .1 henry and a resistance of 1 ohm in series with a capacity of 1000 microfarads, and that a current having a maximum value of 10 amperes is flowing through this system, the frequency being $\frac{100}{2\pi}$ per second.

The maximum value of the potential difference (OL, fig. 85) to overcome inductance

$$LpI = 10 \times .1 \times 100 = 100 \text{ volts},$$

and this is 90° ahead of the current. The potential difference to overcome resistance OR = 1 × 10 = 10 volts, so that the total potential difference between the terminals of the coil = $\sqrt{10^2 + 100^2}$ = 100.5 volts. The potential difference required to send the current through the capacity OC = $\frac{I}{Cp} = \frac{10 \times 10^6}{1000 \times 100}$ = 100 volts, and this lags by 90° behind the current. This will represent the potential difference between the capacity terminals.

To determine the total potential difference we must add all three vectors. Since OL and OC are each = 100 volts their vectorial sum is zero, and the total potential difference is that necessary to overcome the resistance of the coil (OR in the figure) = 10 volts. So that, although we have in this case only 10 volts applied to the circuit, we have over a hundred volts between the terminals, both of the inductance coil and of the capacity (fig. 86).

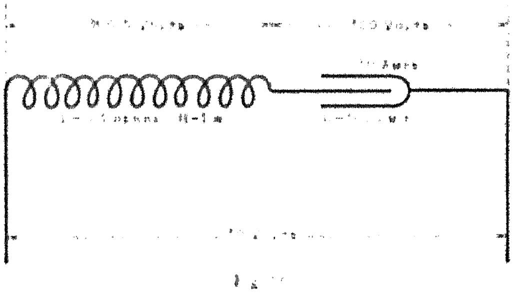

Fig. 86.

Generally, the current flowing is not known, but only the total potential difference applied. When this is the case, the current may be at once determined from the equation given above:

$$I = \frac{E}{\sqrt{R^2 + \left(Lp - \frac{1}{Cp}\right)^2}},$$

and then the whole problem can be easily worked out by the above method.

It need hardly be pointed out that the fact that much higher potential differences can be obtained in alternating-current circuits than are actually applied to them should lead to considerable caution in their design, since it is quite an easy matter to get on an ordinary 100-volt lighting circuit pressures of many hundreds of volts by a suitable arrangement of choking coils and condensers. In testing sheathed cable for dielectric strength, this phenomenon can be made use of to obtain the necessary high pressure without the use of a high-tension generator, for, since the cable and its sheath form a capacity, all that is necessary is to put in series with it a coil of suitable inductance and apply to the whole the ordinary pressure.

The effect described above was first observed by Mr. Ferranti when working the installation at Deptford to which reference has already been made. In his case the alternators themselves which were supplying the current possessed a certain amount of inductance, and in series with them was the long concentric cable, having a capacity of about 30 microfarads. The effect which he observed has therefore sometimes been called the "Ferranti effect".

CHAPTER VI

ALTERNATING-CURRENT AMMETERS AND VOLTMETERS

Measurement of Alternating Currents.—The only method that has been adopted so far for determining the strength of an alternating current has been the estimation of the maximum value that the current attains in either direction. In practice, however, this determination is attended with considerable difficulty, necessitating the use of some such instrument as the oscillograph.

Ordinary instruments can be used for measuring alternating currents *provided the direction of deflection of the needle is not dependent on the direction in which the current is flowing through the instrument.* For example, in an ordinary incandescent lamp the effect of the current in one half-wave will be to produce a heating effect, and such an effect will also be produced by the second half-wave when the current is flowing in the opposite direction, so that the lamp filament will be heated and the lamp will glow; the brightness of the lamp will serve as a very rough indication of the strength of the current flowing. Instruments in which the deflection of the pointer is produced by the extension of a wire, due to the heating effect produced, can therefore be used just as well for alternating as for direct currents. A second type of instrument is that based on the interaction of two coils both carrying the current. Such an instrument is the Siemens dynamometer, and this can be used as well for alternating as for direct current. A third type of instrument of the same character is that which consists essentially of a solenoid, into which a core of iron is sucked by the magnetic action of the current.

We must next consider what is actually measured when an alternating current is sent through, say, a hot-wire instrument. The deflection of such an instrument will clearly not give us the maximum value of the current—since the heating effect is not so great as if the maximum current had been flowing the whole time through the wire—it will give a reading of some quantity less than the maximum.

The actual deflection is determined in a very simple way. The rate of heat production at a given time is proportional to the square of the current flowing at that instant. In a short interval dt, at which the current is i amp., the heat produced in a resistance $R = R\,i^2\,dt$ joules; a little later on the current will have a different strength, say i_1, and the heat produced during a second interval dt will be $R\,i_1^2\,dt$, and so on. The total heat produced in the time T corresponding to one complete wave will be $= R\,i_1^2\,dt_1 + R\,i_2^2\,dt_2 + R\,i_3^2\,dt_3 + \ldots$, where the intervals dt_1, dt_2 are supposed to be very short, and the sum of them is equal to T. The deflection of the instrument will depend on the value of this quantity. Let us suppose that the actual deflection is the same as that produced by a direct current of strength I. If this direct current had been flowing through the instrument the heat produced in a time T would have been $= R\,I^2\,T$, but since the deflections are the same with the direct and alternating current we know that the heat produced in the two cases must be the same, and we have the equation, since R is a constant multiplier on both sides—

$$I^2\,T = i_1^2\,dt_1 + i_2^2\,dt_2 + i_3^2\,dt_3 + \ldots$$

$$\text{and } I = \sqrt{\frac{i^2\,dt_1 + i_2^2\,dt_2 + i_3^2\,dt_3 + \ldots}{T}}$$

$$\text{The quantity} \quad \frac{i^2\,dt_1 + i_2^2\,dt_2 + i_3^2\,dt_3 \ldots}{T}$$

is the mean square of the current, and the actual deflection produced is proportional to the square root of this quantity, or to the square root of the mean square of the alternating current. The quantity actually measured is called the Root Mean Square or R.M.S. value of the current. Instead of using the letters R.M.S. the word "effective" is sometimes employed, the strength of an alternating current being said to be so many "effective" amperes; but R.M.S. is a much better term, as it gives an idea of the actual quantity measured, and is no more trouble to write. When the alternating current is strictly sinusoidal the R.M.S. current is a certain fixed proportion of the maximum current, the actual ratio being

$$\frac{\text{R.M.S. current}}{\text{maximum current}} = \frac{1}{\sqrt{2}} = 0.707$$

As stated above, instruments for measuring alternating currents can be divided into three classes: (1) Hot-wire Instruments; (2) Dynamometer Instruments; (3) Iron-core Instruments.

We will consider, in the first instance, a few types of hot-wire instruments. The principle of their action is exceedingly simple. The current to be measured flows through the wire, heats it, and causes it to expand, and the amount of expansion serves as a measure of the current flowing. When the current is a very large one a shunt is used in parallel with the wire, so that only a small fraction of the total current flows through it. It is important to remember that the fraction of the total current flowing through the hot wire is not a constant quantity, unless both the shunt and wire heat up to the same temperature and are of the same material. In most cases the shunt is arranged to become less heated than the wire, so that with the materials employed a smaller fraction of the total current flows through the wire for large currents than for small. Such an instrument is therefore only accurate for the particular shunt for which it is calibrated. A hot-wire instrument cannot safely be shunted to decrease its sensibility, nor can the shunt be removed to increase its sensibility, and the instrument used for measurement unless a calibration is performed *throughout the whole scale.*

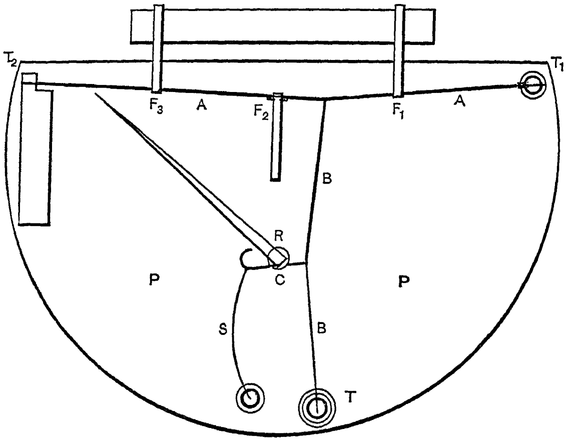

Fig. 87

One of the most popular instruments of this class is that made by Messrs. Johnson & Phillips, an instrument originally devised by Messrs. Hartmann & Braun (fig. 87). In this a platinum-silver wire A A is stretched between two blocks T_1 T_2, one of which is adjustable. To the centre of the platinum-silver wire is attached a much thinner one B B of phosphor-bronze, which is fixed at its other end to a terminal T; to the phosphor-bronze wire is attached a piece of cocoon fibre C, which is wrapped round a grooved metal roller R, to which the pointer is fixed, the other end terminating in an eyelet attachment to a flat steel spring S. When the current flows through the wire the sag increases, thus enabling the phosphor-bronze wire to become more deflected. The increased deflection of the latter is measured by the movement of the thin cocoon fibre, which turns the pointer. The whole of the hot-wire movement is mounted on a compensation plate P P, whose coefficient of expansion is the same as that of the wire.

The wire A A is divided into either two or four equal parts, arranged electrically in parallel with one another by means of silver-foil connecting-strips, the object being to divide up the current passing through the measuring wire and thus to diminish its effective resistance (fig. 88). With a two-segment wire the current enters at the centre and leaves at the ends, as shown in the figure, while with a four-segment wire the current enters at two points a quarter of its length from either end, and leaves at the centre and ends. By this arrangement 4 or 5 amps. may be passed through the wires with a drop of potential of less than ¼ volt. A thin metal plate is fixed near the wire to protect it from any disturbance due to air draughts. The shunts used with these instruments when designed for measuring large currents are made of constantin, a substance which has a negligible temperature coefficient, so that the indications of the instruments are independent of temperature variation.

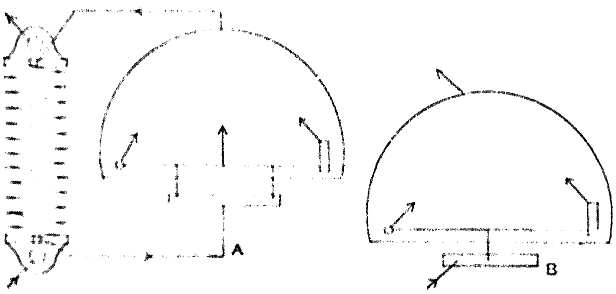

Fig. 88. A, Diagram of Connections in an Ammeter with the Wire divided up into Four Segments. B, Connections with a Two-Segment Wire

The instrument comes to its proper reading somewhat slowly owing to the time taken by the wire to attain a steady temperature; for many purposes this is rather an advantage, and for the measurement of steady alternating currents considerable accuracy can be attained. A damping device is attached to the spindle of the needle, consisting of an aluminium disc (see fig. 89) moving between the poles of a strong magnet, which is said to assist the working of the instrument, but as the motion of the needle is slow under ordinary conditions the effect produced is not very great. When the current is suddenly switched off, however, the damping prevents the needle from swinging back too quickly, thus obviating any tendency of the pulley, to which the pointer is attached, to slip.

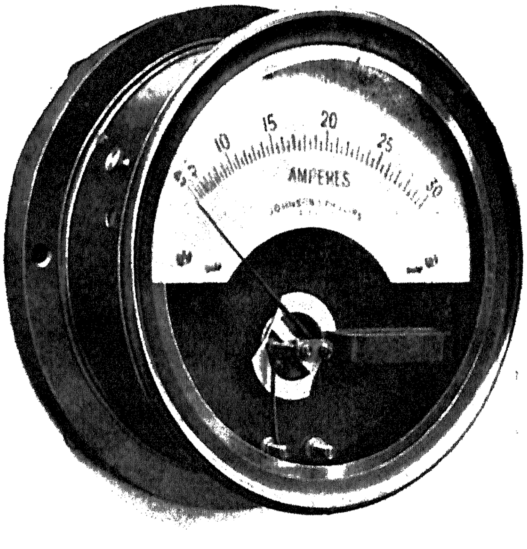

Fig. 89.—Johnson & Phillips Ammeter

Messrs. Hartmann & Braun have devised a recording ammeter (fig. 90) working in very much the same way. The pointer is replaced by a very light aluminium wheel, nearly 5 inches in diameter, round which is wrapped a thin fibre. Two instruments are employed, one placed at the top and the other at the bottom of the case, the same cord passing round both wheels; the pen, which moves up and down between slides, is attached to the cord, and as the wheels revolve, the pen moves up and down between its guide-rods, making a trace on the drum placed at the centre of the instrument. The drum is kept revolving by clockwork as in nearly all other recorders.

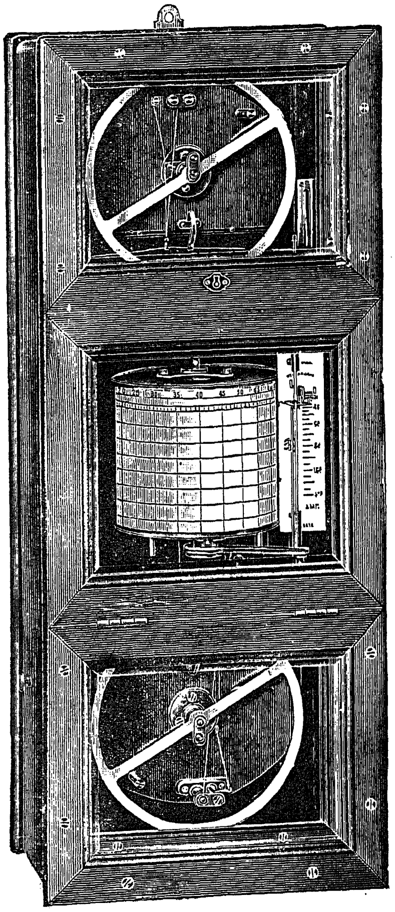

Fig. 90.—Hartmann & Braun Recording Ammeter

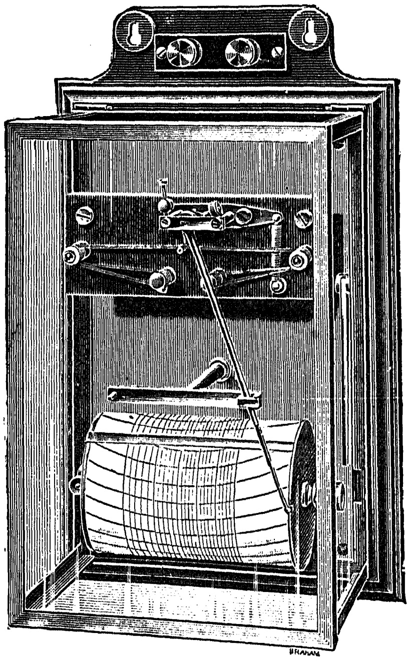

Fig. 91.—Pitkin Recording Hot-Wire Ammeter

Another type of recording hot-wire ammeter is that manufactured by Messrs. Pitkin (fig. 91). The pointer of this instrument is fitted at its end with a pen containing writing fluid, and this pen moves over the surface of paper wrapped round a revolving drum, thus tracing out on the paper a curve which records the value of the current flowing through the instrument at any particular time. In this instrument the pen moves in a circular arc, and the pointer must therefore be made very flexible, so as to enable the pen to touch the paper in all positions. The method of actuating the pointer is very simple. The current to be measured (or a small fraction of it) passes through a number of stretched horizontal wires in parallel, which extend when they become heated by the current.

The wires are kept taut by means of the pin which serves to impart to the pointer the necessary motion when the wires are heated by the current, the balance of the moving system being so adjusted that there is always a slight upward pressure of the pin on the wires. The pointer system is pivoted between jewels on a framework of special material, such that the coefficient of expansion is as nearly as possible the same as that of the wires.

The second type of instruments is that depending on electromagnetic action. The oldest instrument of this form is the Siemens dynamometer, which consists of two coils connected in series with each other, one fixed and the other (placed at right angles to the first) suspended by a silk thread, connection being made to it through two mercury cups into which its ends dip. The motion of this coil is controlled by a spiral spring, one end of which is attached to the coil, and the other to a torsion head, which can be moved round by hand. The amount of motion of the torsion head is measured by a scale attached to the top of the instrument. A pointer is fixed to the moving coil, which rests opposite a fixed mark when no current is flowing through the instrument. This mark is so adjusted that when the pointer is in the given position the coils are mutually perpendicular. The action of the instrument depends on the mutual attraction and repulsion of the currents in the fixed and moving coils; a more detailed description of it is given in "Continuous-Current Measurements". Taking a section of the instrument (fig. 92), and denoting wires in which the current is flowing from front to back by a cross, and wires in which current is flowing from back to front by a dot, it is clear that the forces acting on the moving coil will be as shown in (*a*), attraction between A and D, and C and B, where the currents are in the same direction, and repulsion between A and C and B and D, where the directions of the current are opposite. The moving coil will therefore tend to go round in a clockwise direction.

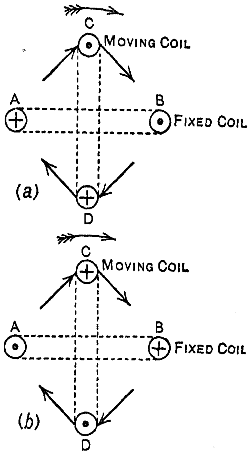

Fig. 92

If the direction of flow of the current be reversed in these coils (*b*), the forces acting on the moving coil will be in the same direction as before (clockwise). The magnitude of the force acting will depend on the product of the currents flowing through the fixed and moving coils, and will therefore, since the same current flows through both coils, be proportional to the square of the current.

When an alternating current flows through the coils the mean force acting will be proportional to the mean square of the current, and such an instrument will therefore measure the mean square value of the current.

These instruments are little used in practice for ordinary measurements on account of the trouble involved in adjusting the torsion head. taking the square root of the reading, &c., but for laboratory standards they are valuable.

The constant may easily be determined absolutely for direct current by a suitable copper voltameter, and the instrument may then be used to determine absolutely the value of an alternating current. It is important to see, however, that there are no metal rings or other conductors in which eddy currents may be induced, as, if this is the case, the accuracy of the instrument will be seriously impaired.

Another instrument which is also usually employed as a standard is the Kelvin balance (fig. 25). This consists essentially of two coils of wire attached to the ends of a suspended lever. Each of these coils or rings is actuated by two fixed rings, the connections being so arranged that the currents through the six coils tend to pull one of the moving rings up and the other down. For use on alternating-current circuits the rings are made up of a stranded wire rope, in which each component is carefully insulated from the others. By this arrangement the instrument is made accurate both for alternating and direct currents, and can be used for alternating measurements after having been calibrated on a direct-current circuit. A detailed description of this instrument is given in "Continuous-Current Measurements".

Instruments of the third type are much used, chiefly because of their simplicity of construction and consequent cheapness.

The typical instrument of this class consists simply of a solenoid carrying the current, into which is attracted a thin iron wire. The attraction of this wire is the deflecting force, while the control is usually gravity, the motion of the pointer having the effect of producing a want of balance in the rotating system. Instruments of this type were devised by Messrs. Dolivo Dobrowolsky and Ferranti. Their disadvantage is, that from the nature of their construction they must depend on the frequency of the current applied to them. The thin wire is subjected to an alternating magnetization, and this causes eddy currents to be induced in it, the magnitude of the eddy currents depending on the square of the frequency. By using a very fine wire this error has been practically eliminated, and instruments are now made which are accurate to within 1 per cent for any frequency up to 80 ∼ per second.

Another instrument similar in character to the above is that manufactured by Messrs. Everett & Edgcumbe. The moving part consists of a thin flat disc of iron eccentrically pivoted, which is sucked up into a flat shaped coil. The control in these instruments is either "gravity" or "spring".

Voltmeters.—We have been considering up to the present the measurement of alternating currents; the instruments that are used for measuring alternating potential differences are, however, very similar in their action. We may divide the various classes of instruments into four groups: (1) Hot-wire, (2) Dynamometer, (3) Iron-core, (4) Electrostatic.

Hot-wire voltmeters are generally used for measuring low alternating pressures. They require for their working a certain amount of current, as a rule from $\frac{1}{8}$ to $\frac{1}{4}$ ampere, otherwise the heating effect is not large enough to produce a deflecting force of sufficient magnitude. Since the amount of current necessary is independent of the pressure that is being measured, it will be obvious that for high voltages the amount of energy expended must be considerable. Take, for example, an instrument designed to measure 2000 volts. The resistance would be arranged so as to give a current of $\frac{1}{4}$ ampere, we will suppose, and this would mean an expenditure of $\frac{1}{4} \times 2000$ watts, or $\frac{1}{2}$ kilowatt. Although devices are employed to reduce this amount of energy, such as by the use of transformers, it is generally better to use some other type (preferably electrostatic) for high pressures. Among the best-known hot-wire instruments is that manufactured by Messrs. Hartmann & Braun, which is exactly similar in principle to the hot-wire ammeter before described. A thinner wire, however, is used, and the current passed from end to end of it; in series with this wire is a suitable resistance; the current required for this instrument is, at full pressure, .2 ampere. Similarly, the recording instruments above described are adapted for use as voltmeters by the substitution of thinner wires and resistances in series.

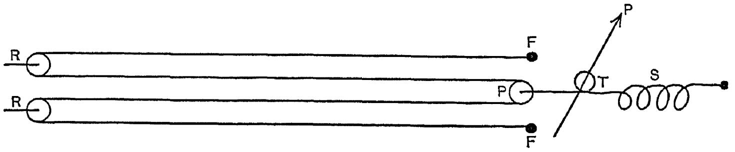

Fig. 93

There is one hot-wire voltmeter which has never been adapted for use as an ammeter, and which was the first of its kind manufactured. It was devised by Major Cardew, and is called the Cardew voltmeter. The working part consists essentially of a long platinum-silver wire, which passes four times up and down a long tube attached to the instrument. Both ends of the wire F F (fig. 93) are rigidly fixed, the whole being stretched on pulleys. The pulleys R R are attached by rigid supports to the end of the tube, and the extension of the wire is measured by the motion of the pulley P. To this pulley a very fine wire is attached, which is wrapped round a grooved pulley T and attached at its other end to a spring S. When the wire stretches under the influence of the current the spring S takes up the extension, and the amount of movement is recorded by the motion of the pointer P. In the actual instrument (fig. 94) the motion is magnified by a pair of spur-wheels of 5 : 1 ratio, one of which is rigidly fixed to the pulley T and the other attached to the pointer spindle, the wheels being kept in gear by a hair-spring.

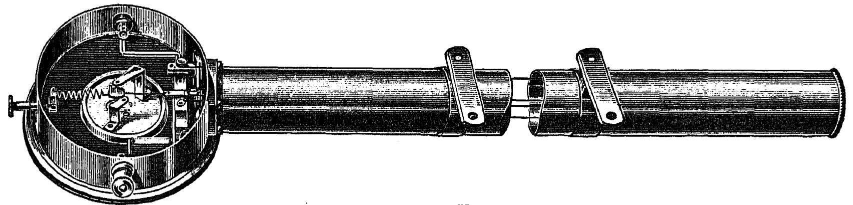

Fig. 94.—Cardew Voltmeter

These instruments are very dead-beat in their action and are very sensitive. Resistances are sometimes supplied to enable the instruments to be used at a higher voltage, but, as explained above, this is a very wasteful method of measuring pressure.

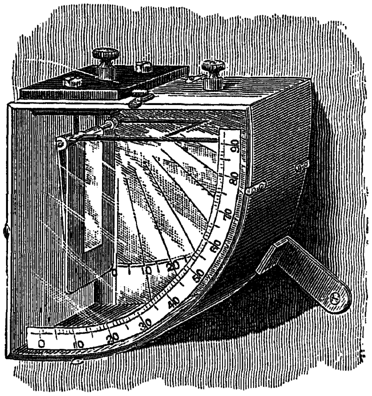

Fig. 95.—Kelvin Gold-Leaf Gauge

Coming to the second-group instruments, *i.e.* those of the Siemens dynamometer type, these are not much used; the Kelvin balance is, however, adapted to this purpose, the coils of the instrument, instead of being wound with heavy cable, are made of fine wire, but otherwise the mechanism differs in no way from that of the ordinary current balance.

Instruments of the third or iron-core type are employed frequently as voltmeters, the only modification being that the coils instead of being wound with thick are wound with a number of turns of fine wire.

It is very necessary, in designing voltmeters of the dynamometer and iron-core types (since the instruments depend for their working on the production of a magnetic field, and hence upon having a certain inductance), to see that the resistance of the coils is very large compared with this inductance. The current flowing in the

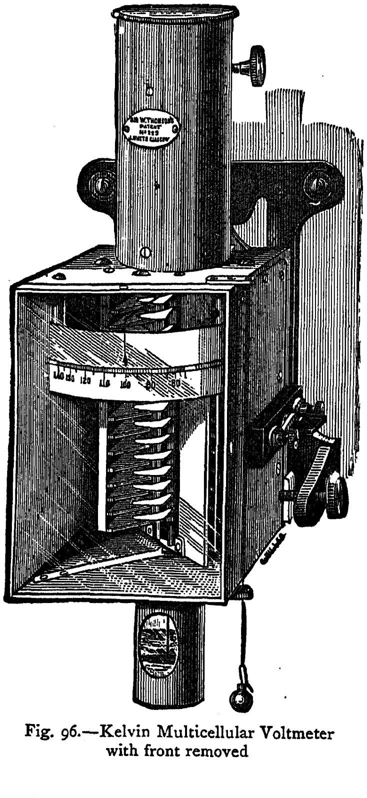

Fig. 96.—Kelvin Multicellular Voltmeter with front removed

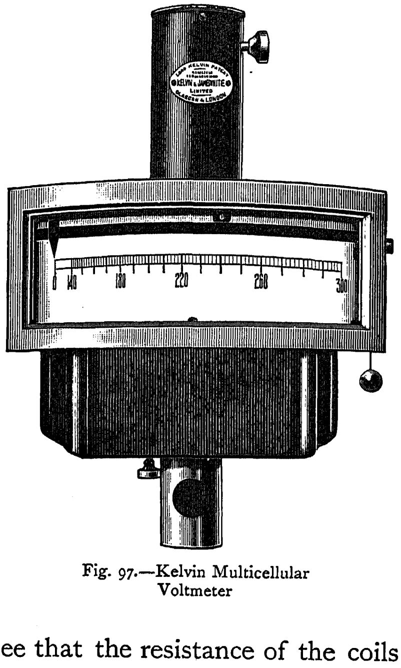

Fig. 97.—Kelvin Multicellular Voltmeter

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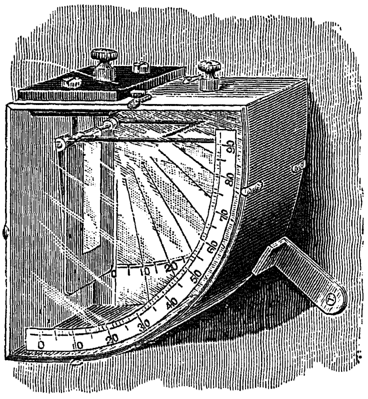

Fig. 95.—Kelvin Gold-Leaf Gauge

to be used at a higher explained above, this is method of measuring pre

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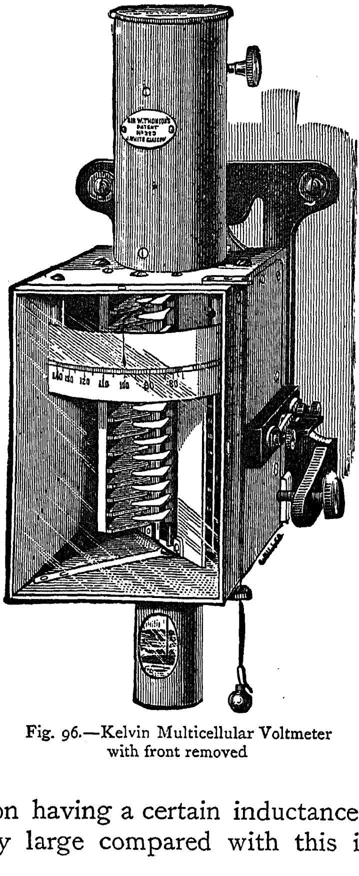
SIR W. THOMSON'S PATENT No 283 WHITE GLASGOW
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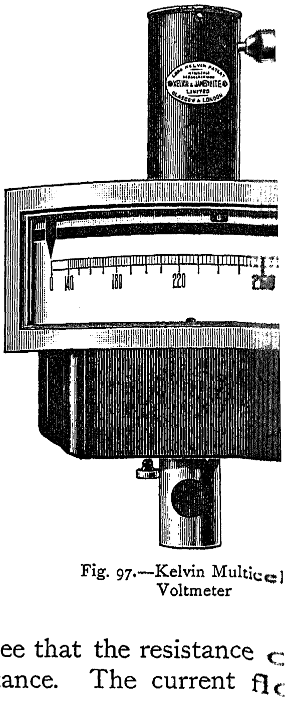
Fig. 96.—Kelvin Multicellular Voltmeter with front removed


KELVIN & JAMES WHITE LIMITED GLASGOW & LONDON
140
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220
Fig. 97.—Kelvin Multice Voltmeter

upon having a certain inductance), to see that the resistance c very large compared with this inductance. The current fl coils is what produces the deflecting force, and therefore, unless the above condition is fulfilled, the deflection of the instrument, for the same potential difference, will depend upon the frequency. It is also necessary, as in direct-current instruments of this type, to use, for the coils, wire which has a small resistance temperature coefficient.

The best instruments for measuring ordinary voltages are, however, those of the electrostatic type. They differ from all the others described in that they require little, if any, current and no power to actuate them. They depend for their action on the known principle that when two bodies are charged to different potentials they attract one another. The simplest instrument of this type is the gold leaf gauge adapted by Lord Kelvin. It consists of a single thin sheet of gold-leaf fastened along its top edge to a metal plate (fig. 95). The leaf and plate are mounted in a quadrantal box with a glass front, being insulated from the case by ebonite supports. One terminal is placed in electrical connection with the gold-leaf, the other with the case, and when there is a potential difference between these the gold leaf is drawn upwards toward the top of the case, the amount of deflection being observed on a scale engraved both on the face and back of the instrument. This gauge is, of course, not accurate, and will only measure pressures of over 1500 volts. A more sensitive instrument is the multicellular voltmeter (figs. 96, 97), also devised by Lord Kelvin. In this case the moving part is a needle, made up of a number of horizontal parallel triangular plates rigidly attached to a metallic rod, and suspended by an iridio-platinum wire (a small coach-spring being interposed between the vanes and the needle to prevent the latter from being damaged by vibration). The fixed part of the instrument consists of two sets of cells, which attract into them the vanes of the needle. The cells are also triangular in shape, and are made by inserting brass plates in saw-cuts in a brass back block, thus ensuring parallelism and equidistance between them.

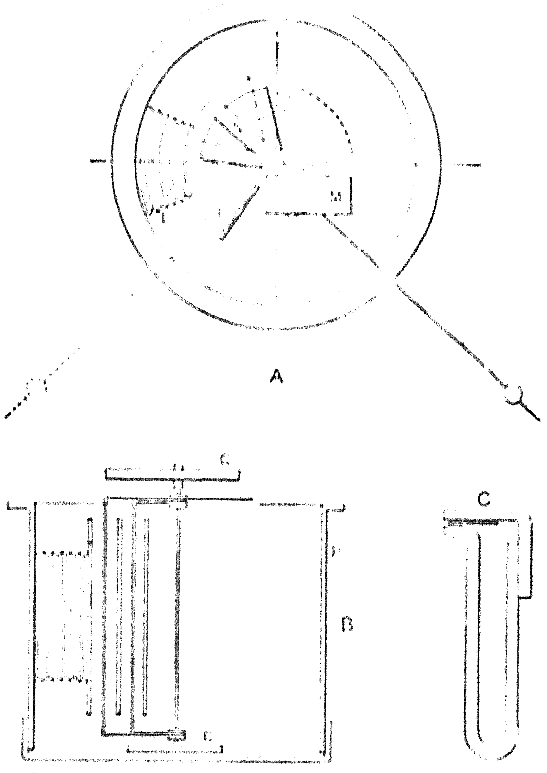

Fig. 98. A, Mechanism of Ayrton & Mather Meter. B, Electrostatic Movement. C, Damping Device, showing magnet with copper strip on needle between the poles. (I, Inductor; N, Needle; M, Magnet; E, Ebonite Block; P, Copper Pot; S, Copper Strip.)

The arrangement of the cells and needle is well shown in the figure. Attached to the bottom end of the needle is a small horizontal plate moving in a bath of heavy oil, which effectually damps the motion of the needle and prevents undue oscillation. Another form of electrostatic voltmeter is shown in fig. 98, and is designed by Messrs. Ayrton & Mather. The moving part or needle N of this instrument consists of two cylindrically-shaped vanes pivoted on centres C placed in the axis of the cylinder; these are attracted into two hollow cells or inductors I, also of cylindrical shape, the needle and the inductors being attached respectively to the two terminals of the instrument. The control is effected by gravity, and the motion of the needle N damped by attaching to it a thin copper plate moving in the gap of a permanent magnet. These instruments are only designed for measuring high potential differences.

CHAPTER VII

ALTERNATING-CURRENT POWER AND ITS MEASUREMENT

Power.—The measurement of power with continuous currents is a very simple matter. If a current of I amperes be supplied at a pressure of V volts the power = I × V watts. With alternating currents, however, this is not true, since the amount of power supplied depends on the phase relationship of the current and potential difference. The power supplied *at any particular instant* is equal to the product of the instantaneous value of the current and potential difference, so that we can plot out curves of power by taking a series of products of instantaneous values (see figs. 99, 100, and 101). If we plot a curve between the power at any instant and the time, we obtain a power curve, the area of which will represent the energy supplied. When the area is positive, *i.e.* when the current is in such a direction as that in which the applied potential difference would send it, the energy is being applied to the circuit, *i.e.* the power is positive. It will be noticed, however, in figs. 100 and 101, that at certain times the

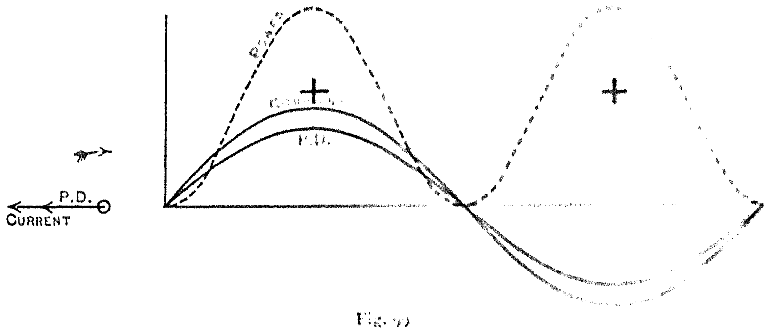

Fig. 99

current is flowing in the opposite direction to that in which the applied potential difference would send it. During this part of the period conditions are reversed, and the power, instead of being supplied by the source to the circuit, is being supplied by the circuit to the source. The cases are exactly analogous to those of a motor and dynamo. The motor corresponds to the first case, in which the current flows in the same direction as the applied potential difference would send it, and the source is supplying power to the motor. In the dynamo, on the contrary, the current through the machine is from the negative to the positive brush, *i.e.* in the opposite direction to that in which the applied potential difference would send it, and consequently in this case the power supplied is negative, *i.e.* the dynamo is supplying power instead of receiving it. Under the above conditions, therefore, we may imagine that during part of the period the circuit is behaving like a motor, and during the remainder of the period like a dynamo. The total power supplied to the circuit will be the difference between the positive and negative powers,

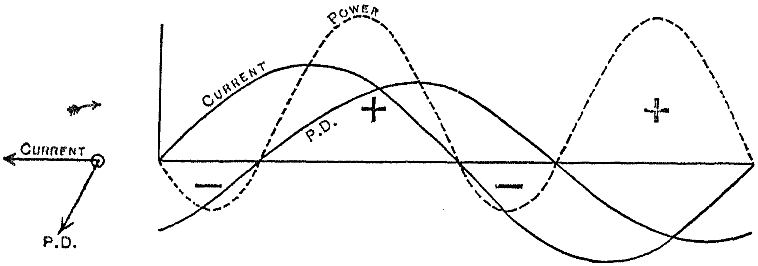

Fig. 100

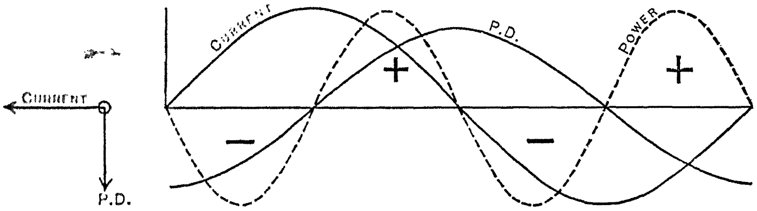

Fig. 101

and in order to determine the total energy supplied to a circuit during a period, the difference between the positive and negative areas must be measured. The average power developed will be simply the average rate of supplying energy, and is obtained by dividing the energy per period by the time occupied by the period.

The Power Factor.—It will be noticed from the curves that when the current and potential difference are 90° out of phase the positive and negative areas are equal, so that under these conditions no power is being supplied to the circuit. When, on the other hand, the two are in phase, the area is all positive. This fact may readily be used to determine the actual power supplied to a circuit for any other phase difference when the current and potential difference are sinusoidal. Using the vector notation, let us suppose that the potential difference is represented by a vector O V_0, fig. 102, uniformly rotating, and the current by another vector O I_0 lagging behind the first by an angle ϕ. The vector O V_0 can be split up into two parts, one O V'_0 parallel with O I_0 and another O V''_0 perpendicular to O I_0, and for all practical purposes these two vectors will represent the applied potential difference.

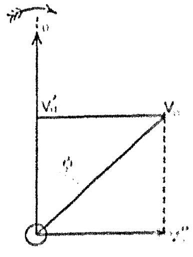

Fig 102

Now we have seen that the power corresponding to the component O V''_0 will be zero, because O V''_0 and O I_0 are 90° out of phase. The total power will therefore be that due to the component O V'_0.[1]

In order, therefore, to determine the power supplied we have to multiply the product of R.M.S. volts and current by the cosine of the angle of lag. The quantity cos ϕ is called the power factor of the circuit. A more general definition of the power factor in any circuit, which does not entail the necessity for sinusoidal waves, is the ratio

$$\frac{\text{True power}}{\text{Product of volts applied} \times \text{current}}$$

Wattmeters.—In order to obtain a measure of alternating current power it is necessary either to know the current, potential difference applied, and the power factor, or to use some instrument that will give power directly. The latter course is obviously the simplest, and such instruments, called wattmeters, are now in constant use. Some types of wattmeters are described in the section on continuous current measurements, and many of these instruments can be used equally well for alternating current circuits. The simplest form is that resembling the dynamometer, in which the moving coil carries the shunt current, the fixed coil the main current. Most instruments now, however, are deflectional, the deflection produced giving on some scale the power absorbed in the circuit. Such instruments are the Kelvin wattmeter already described and that manufactured by Messrs. Hartmann and Braun. The latter instrument, fig. 103, has a moving coil M not placed at right angles to the main coil F, but inclined to it at an angle, being pivoted on jewels. The motion of the coil is controlled by light phosphor-bronze springs, which are used also to lead the current into and out of the coil. In designing these instruments there is, however, one very important point that must not be overlooked. If the instrument is to read accurately, the time constant $\left(\frac{L}{R}\right)$ of the shunt circuit must be kept very low. Their working depends on the interaction between the main and shunt currents, and the force acting between them can only be regarded as proportional to the power if the current in the shunt is proportional to the pressure applied to it, so that if the current lags behind the potential difference in the shunt circuit the instrument will not record accurately. In order to reduce the ratio of inductance to resistance to as small a quantity as possible, the coil of the instrument is placed in series with a large non-inductive resistance; it does not do to increase the number of turns on the moving coil, as this also increases the inductance of the coil. In one of the Hartmann-Braun instruments the following figures are given:—Self induction of shunt coil, 0.0075 henry; resistance = 1000 ω for 30 volts, so that with this instrument the lag for a frequency of 50 per second would be 0.13, and, except when the power factor of the circuit is very low, the error will be inappreciable. In making measurements on losses in the dielectrics of cables where the power factor of the load is very small indeed, special wattmeters should be employed. A new form of wattmeter, in which nearly all errors are eliminated, is one devised by Messrs. Mather & Duddell. The principle of the instrument is the same as that of the ordinary "dynamometer" wattmeter. The suspension of the fine-wire coil is effected by a pair of phosphor-bronze strips, which serve to lead the current in and out, the coil being stretched between them. The shunt resistance is of special form, consisting of a manganin or constantin wire woven so as to form the woof of a piece of cloth the weft of which is silk. The instrument is made astatic by the use of two sets of coils, one vertically above the other, the currents passing through them in opposite directions.

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[1] If we denote the alternating current corresponding to O I_0 by $I_0 \sin pt$, that corresponding to O V'_0 by $V'_0 \sin pt$.

Power at any instant = $I_0 V'_0 \sin^2 pt$ and average power = $I_0 V'_0$ × average value of $\sin^2 pt$.

Now $\sin^2 pt + \cos^2 pt = 1$ and average value of $\sin^2 pt$ must be equal to average value of $\cos^2 pt$ taken for a complete period, so that average value of $\sin^2 pt = \frac{1}{2}$ and average power $= \frac{I_0 V'_0}{2}$.

Now $V'_0 = V_0 \cos \phi$, so that the average power supplied $= \frac{I_0 V_0 \cos \phi}{2}$.

If, instead of maximum values of current and potential difference we take R.M.S. values, since R.M.S. $I = \frac{I_0}{\sqrt{2}}$ and R.M.S. $V = \frac{V_0}{\sqrt{2}}$, Average power supplied = $IV \cos \phi$.

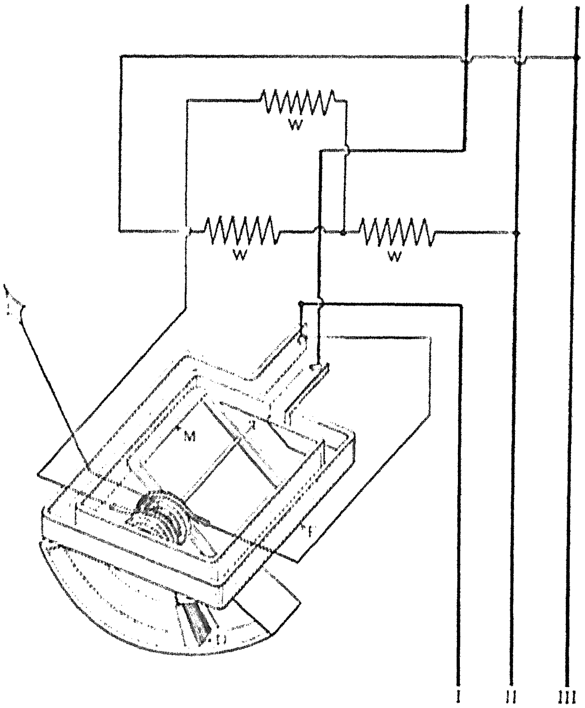


Fig. 103.—Hartmann and Braun Wattmeter

The description of some form of recording wattmeters or meters will be given later, as many of them depend for their action on the behaviour of what are known as two-phase and three-phase currents.

Power-Factor Indicators.—In many cases it is important to know the power-factor of the load on a station, as well as the actual magnitude of the power that is being generated. The power-factor as explained above can be calculated if the voltage, the current, and the power generated are known; but usually an instrument which will show directly the value of the power-factor is employed.

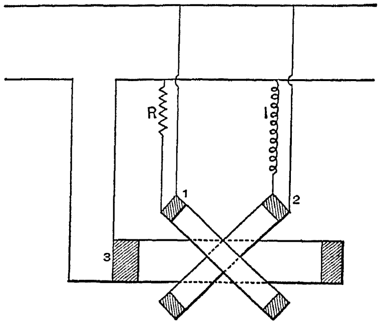


Fig. 103A

Several types of instrument are in common use. They resemble very much the ordinary wattmeter in their construction, with the difference that instead of a single moving coil, two moving coils rigidly connected together are employed. In a single-phase power-factor indicator (fig. 103A), one of the moving coils is connected through a non-inductive resistance across the mains either directly or through a pressure transformer. The current flowing in this coil is, therefore, at any instant proportional to the pressure on the circuit. The other coil is connected across the main circuit through a highly inductive resistance, so that the current flowing in this circuit lags by nearly 90° behind the P.D. on the mains.

The fixed coil of the power-factor indicator carries the main current which is being supplied to the circuit, and acts on the moving coils. When the current in the main circuit is in phase with the applied P.D., the current is in phase with the current in pressure coil (1), which is connected to the circuit through a non-inductive resistance. The force acting on the other coil is very small, since the currents in the fixed and the moving coils (2) are nearly 90° out of phase. Under these conditions, therefore, the pair of moving coils will tend to lie in such a position that coil (1) is in the same plane as the fixed coil. If the power-factor of the circuit change and the current lag behind the applied P.D., then there will be a force

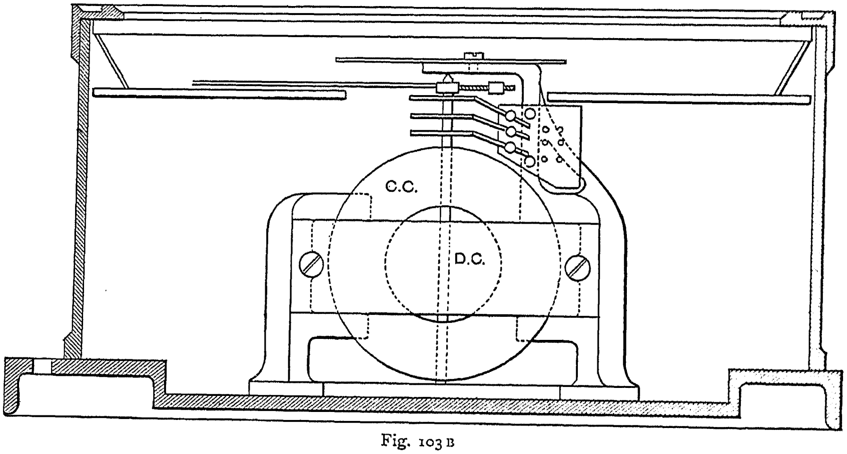


Fig. 103B

[illegible] of the combination, and the actual position of the pair of coils will depend on the ratio of the forces acting on the two coils. If the circuit is inductive, the current lagging 90° behind the pressure, the current in the main coil and in the coil (2) of the combination will be very nearly in phase, and therefore this coil will tend to lie in the same plane as the fixed coil. The position of this pair of coils with reference to the fixed current coil will serve, therefore, as an indication of the power factor of the circuit to which the instrument is connected. The arrangement of the parts in an instrument of this type, made by Messrs. Everett, Edgcumbe & Co., Ltd., is shown in fig. 103B.

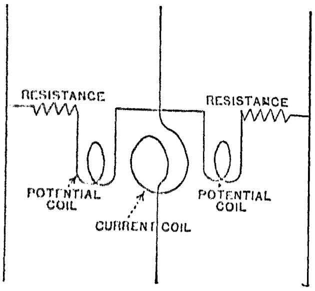


Fig. 103C

When a two phase system is used, the necessity for a highly inductive circuit to produce a current lagging nearly 90° behind the applied potential difference disappears. The second coil (2 fig. 103A) in the power-factor indicator is connected either directly or through a transformer to the other phase of the system. A power-factor indicator used in this way has the advantage over the single-phase instrument of being independent of the frequency for the accuracy of its reading. The phase difference between current and potential difference in the circuit is always 90°, whereas with the single-phase instrument the phase difference between current and potential difference varies with the frequency, and the reading of the meter on a circuit of frequency different from that for which it has been calibrated will be wrong.

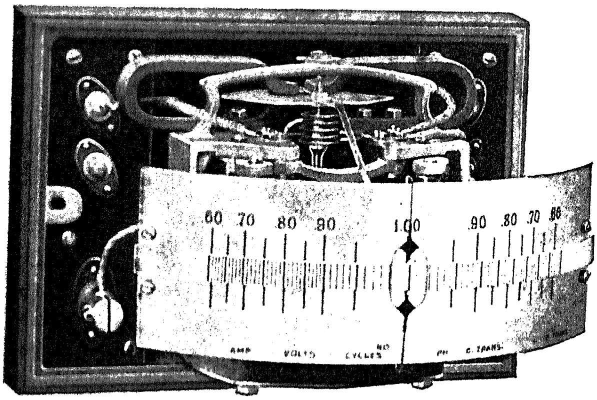


Fig. 103D.—Horizontal Edgewise Power-Factor Indicator (British Thomson-Houston Co., Ltd.)

The connections for a power factor indicator on a three phase circuit are shown in fig. 103C. When the current in the fixed coil lags behind the pressure in the phases, it comes more into phase with the current in one of the moving coils, and more nearly 90° out of phase with the current in the other, the force on one coil will be increased and on the other diminished, so that the pair (which are, of course, rigidly fixed to the same spindle) will take up a position to indicate the power-factor of the circuit. Fig. 103D shows an instrument made by the British Thomson-Houston Co. for three phase circuits. The vane seen at the top of the instrument is made of aluminium, and moves between the poles of a permanent magnet, so as to damp the motion of the revolving spindle to which it is attached.

CHAPTER VIII

TWO- AND THREE-PHASE CURRENTS

Two-Phase Currents.—Two-phase currents have lately come into notice in this country through their extensive use in America. A two-phase system is really a combination of two single-phase systems, for if we imagine two circuits in which the phases of the applied pressure (fig. 104) differ by 90°, we have a four-wire two-phase system. This method of working is sometimes employed, and all four wires are used. The most usual plan to adopt, however, is a three-wire two-phase system. Instead of having two wires for each phase a common return is employed, the current in the return wire being the sum of the currents in the leading-out wires. In determining the actual sum, the fact that the two currents are 90° out of phase has to be taken into account. Using the vector notation, we obtain the sum very easily, if vector o 1, fig. 105, represent the current in the first phase, vector o 2 the current in the second phase; the current in wire 3 is obtained by taking the diagonal of the rectangle whose two adjacent sides are o 1 and o 2. In a balanced two-phase system the currents in 1 and 2 are equal, and the current in 3 is $\sqrt{2} \times 1 = 1.41 \times$ current in outer wires. The return current, therefore, is less than it would be had the two phases remained separate, and hence the amount of copper that would be required for a transmission over a long distance is diminished. This saving, however, is comparatively insignificant, and in some cases it is found more convenient to keep the phases separated and treat them as two quite independent circuits. The great advantage which two-phase currents possess over single-phase is the ease with which a rotating field can be produced by them. A rotating field is a necessary condition for the starting of an induction motor, and therefore where much power is required two-phase current is always used in preference to single-phase, though three-phase is, for most purposes, better than either.

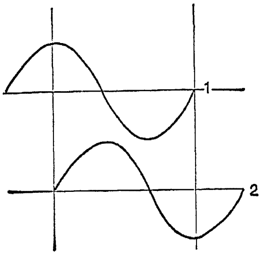


Fig. 104

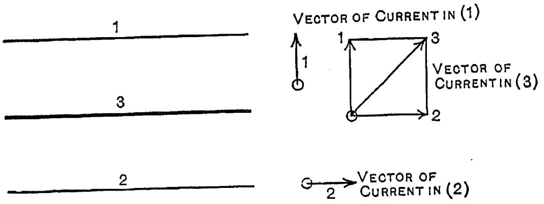


Fig. 105

The Rotating Field.—If we have two sets of coils arranged perpendicularly to one another (fig. 106), and connected respectively to the two phases of a two-phase circuit, we have all the necessary conditions for producing a rotating field. Let us suppose that at any instant the current in phase (1) is a maximum, then the current in this phase will produce a field going, let us suppose, from top to bottom of the coils. At this instant the current in (2) is zero, so that no field will be produced by it [(1), fig. 106]. Let us now consider an instant an eighth of a period later on [(2), fig. 106]. The current in phase (1) will be much less, but there will now be an equal current in phase (2), producing a field, let us suppose, from right to left. The actual field in the central space will be the resultant of these two, this resultant being obtained just in the same way as the resultant of two forces or currents; it will be a field in a direction at 45° with the original. Now consider an instant still one-eighth of a period later [(3), fig. 106]. The current in (1) will be zero, and the current in (2) will be a maximum. The resultant field will therefore be horizontal. Taking an instant still one-eighth of a period later [(4), fig. 106], the current in (2) will now be diminishing, while the current in (1) is reversed, and the resultant will be a field in the direction shown. One-eighth of a period later [(5), fig. 106] the current in (1) will be a maximum in the opposite direction, and we have a field in the direction shown.

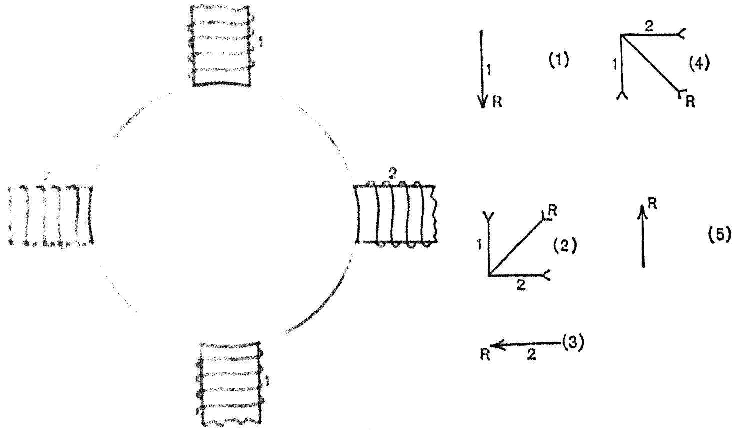


Fig. 106

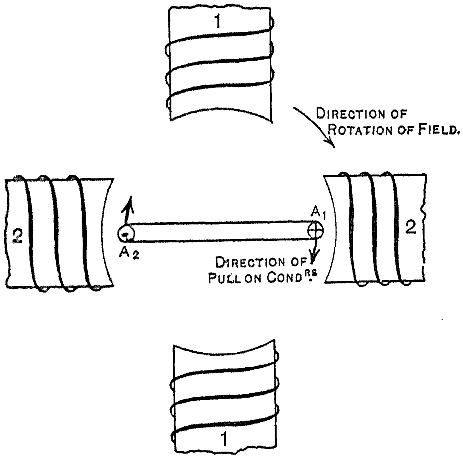


Fig. 107

During the half period considered, therefore, the effect has been (although the coils have been stationary) to rotate the resultant magnetic field produced by them through 180°, and it is a simple matter to prove that the field thus produced is constant, and will rotate at a uniform speed. As many meters are based on the effects produced by a rotating field, it will be necessary to consider briefly some of the phenomena to which it gives rise. If we imagine a rectangle (made up of copper conductors) placed in this field (fig. 107), E.M.F.s will be induced in the side conductors A_1 A_2 by the rotation of the field, due to the fact that the copper is cut by lines of force. The E.M.F. will be from front to back when the lines cut A_1, and from back to front when the lines are oppositely directed. The result will be, therefore, that an alternating current will flow through the conductors, and if the circuit have a small self-induction the current will be nearly in phase with the E.M.F. Supposing this latter condition fulfilled, we shall have a large current flowing in the coil when the field cuts it, and consequently there will be a force acting on the conductor tending to pull it round; using Fleming's rule, we find that this force will tend to drag it in the same direction as the rotating field. If the conductor system be free to revolve it will do so, and will try to increase in speed until its speed is equal to that of the rotating field. If the copper conductors be replaced by a copper cylinder or copper bell the same effects will be produced, the bell acting like a number of conductors in parallel with one another. If the coils of the system be arranged parallel to one another, giving four poles, shown in plan in fig. 108, two belonging to each phase, the same effects will be produced, and a copper disc C placed over this system will tend to revolve. Both of these arrangements are employed in meters.

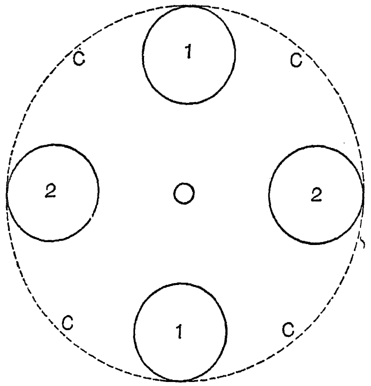


Fig. 108

Three-Phase Currents.—Another system of alternating currents, which is now very extensively used, is that known as three-phase or triphase. We may first consider such a system in which three pairs of wires are made use of, along which flow alternating currents of the same magnitude and frequency but of different phase. The phase difference between the currents in the first and the second circuits is 120°, and between the second and third also 120° (see fig. 109). In practice the three distinct sets are, however, never used, the return path being usually absent or made out of a single wire. If we take any particular instant and measure the magnitude of the currents flowing in the three wires at that instant, we shall find that the algebraic sum is zero, *i.e.* the total current in one direction is equal to the total current in the other. Take, for example,

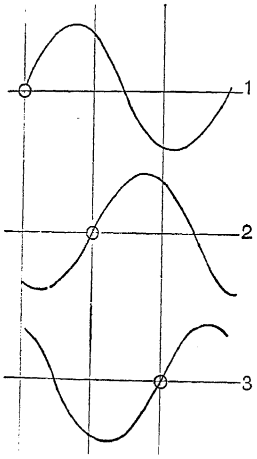


Fig. 109

in the figure the instant at which the current in 2 is zero, the + current in 1 is clearly equal to the − current in (3). We have therefore this result, that if the return path is a common one, and the system is balanced (*i.e.* the currents in the three branches are equal), the current in this common return will be zero. It will be interesting now to investigate the voltage relationship in such a system. Let us suppose that there is between each wire and the "neutral" or common return a potential difference of 100 volts. We may represent these potential differences by three vectors (fig. 110). To determine the potential difference *between* wires 2 and 3 we have to subtract the potential difference between the neutral and 3 from the potential difference between the neutral and 2; this vectorially is equivalent to adding (3) reversed to (2). Performing this operation we get a vector 01′, which is $= 2 \times 03' \cos 30^\circ = \sqrt{3} \times 03'$, representing a potential difference $\sqrt{3} \times 100$, and which leads on 2 by 30°. Thus, in the system we are considering we should have 173 volts between wires 2 and 3.

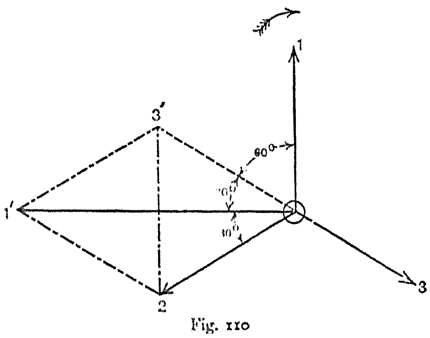


Fig. 110

It is clear that the arrangement will be symmetrical, so that between any two wires there would be 173 volts, if there were 100 volts between each wire and the neutral. This method of arrangement is known as the star or Y system, and is represented diagrammatically in fig. 111. Another arrangement sometimes used is that known as the mesh, in which the neutral wire is done away with. This is shown diagrammatically in fig. 112. In this case there is no difficulty in determining potential differences, since the potential difference in each limb of the mesh is equal to that between the lines. In order to find the relationship between the currents in the lines and in the mesh, a diagram exactly like that used for potential differences in the star system is employed, showing that the current in the line (1), for example, being equal to the difference between the currents in 2′ and 3′, is $\sqrt{3} \times$ either of those currents. To take a specific case, if there were 100 amperes in each branch of the mesh, there would be 173 amperes in each line. The star arrangement is that usually employed.

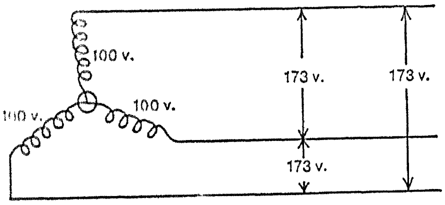


Fig. 111

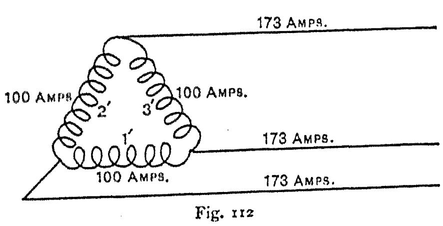
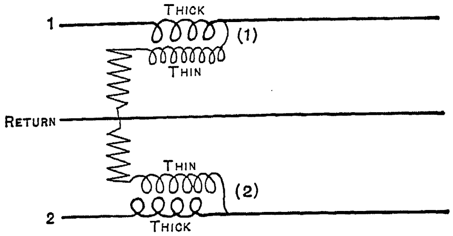
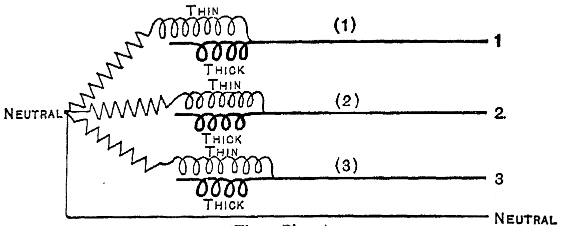


Fig. 112

Three-phase currents can be applied with great facility to the production of a rotating field by using three (or some multiple of three) coils exciting pole-pieces placed at 120° (or some submultiple of 120°) to each other. As rotating fields due to three-phase currents are not used in any meters, their production has not been described in detail. Revolving field or induction motors are, however, all worked on this principle, and are in constant and ever-increasing use on three-phase circuits.



(Two-Phase)



(Three-Phase)

Fig. 113

As has been pointed out in describing these arrangements, they are essentially modified single-phase systems, and all methods of measurement and measuring instruments described for use with simple alternating currents can be employed equally well for measuring the current or potential difference in each phase of two- and three-phase systems.

Similarly for power measurement, wattmeters can be inserted into each phase to measure the total power supplied (fig. 113). When the system is a balanced one, only one wattmeter is required, the total power being in a two-phase system twice that registered on the wattmeter, and in a three-phase system three times that registered on the instrument.

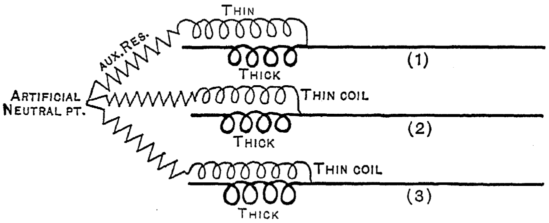


Fig. 114

When the three-phase mesh system is employed with no neutral point, an artificial neutral point must be used, and this is effected by using three wattmeters, whose shunt coils have equal resistances, the inductances being negligible, and connecting them as shown in fig. 114.

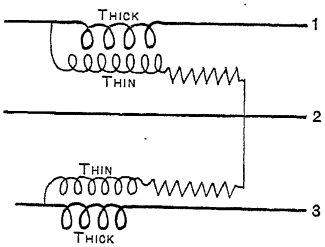


Fig. 115

When the system is balanced only one wattmeter need be used, and the artificial neutral point may be formed by the use of two auxiliary resistances connected to two of the phases, with the fine-wire coil of the wattmeter connected to the third phase (fig. 103). The resistance of the auxiliary coils must be made equal to that of the wattmeter shunt coil, together with its auxiliary resistance.

Other methods have been devised for measuring the power in three-phase unbalanced circuits which require only two wattmeters instead of three. Such an arrangement, suitable for either star or mesh connection, is shown in fig. 115. The thick coils of the wattmeter are placed in two of the mains, and the ends of their fine wire coils are connected respectively to the two mains and to the third main.[1]

CHAPTER IX

ALTERNATING-CURRENT METERS

In all that has been said so far with reference to the measurement of power in alternating-current circuits, the readings have been supposed to be taken at one particular time. It becomes necessary, however, in the great majority of cases where electric energy is supplied, not only to measure the power at a given time, but to measure the total energy supplied. An instrument for effecting this is called a recording wattmeter, or more simply a meter. Instruments designed for this purpose for alternating-current circuits may be divided into two main classes: (1) Clock Meters, the only practical example of which is the Aron meter; (2) Motor Meters, which are by far the most common; and this latter group may be split up into two divisions: (a) commutating motor meters, which are meters that can be used with equal facility on direct or alternating circuits; (b) revolving field motor meters, which depend for their action on the production of a rotating field by the alternating current.

Clock Meters. These meters depend for their action on the attracting force which is exerted between two coils carrying current in the same direction. One of these coils, which is made of fine wire and has in series with it a large non-inductive resistance, is connected across the mains (in a single-phase system), so as to carry a current proportional to the pressure applied to the distributing circuit. The other coil is made of thick wire and carries the main current. The relative position of the coils is shown in fig. 116. As in the case of the wattmeter the total attraction between the coils will depend, not only on the magnitude of the currents flowing through them, but on their phase relationship, and the average attraction between the coils is proportional to the power supplied to the circuit. The effect of this attraction between the coils will be to increase the downward pull on the bob of the pendulum, which will be equivalent to increasing the gravitational force. Thus the rate at which the pendulum will vibrate will be more rapid when the current is passing than when it is not; also, since the attraction between the coils will be *proportional* to the power supplied, the rate at which the pendulum will swing will depend on the total power transmitted.[1]

---

[1] Assuming a star connection, total power at any instant $= v_1 i_1 + v_2 i_2 + v_3 i_3$, where $v_1 v_2 v_3$ are the potentials of each branch of the star. The potential difference between (1) and (2) is $(v_1 - v_2)$.

and the potential difference between (2) and (3) $= v_3 - v_2$.

The wattmeter reading in (1) is proportional to $i_1 (v_1 - v_2)$.

The wattmeter reading in (3) is proportional to $i_3 (v_3 - v_2)$ with suitable connections.

Sum of readings $= v_1 i_1 + v_3 i_3 - v_2 (i_1 + i_3)$.

But in any three-phase system $i_1 + i_2 + i_3 = 0$,

so that $i_1 + i_3 = - i_2$.

Substituting, sum of readings $= v_1 i_1 + v_2 i_2 + v_3 i_3 =$ total power.

A similar proof may be readily obtained, assuming a mesh connection, by interchanging currents and potential differences in the above statement.

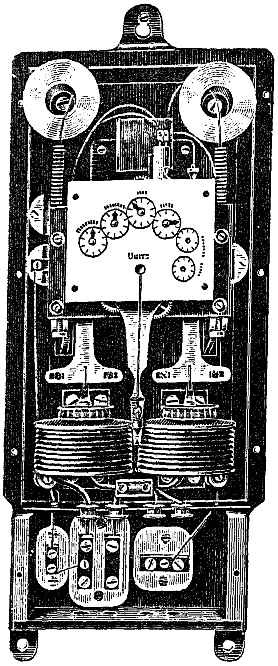


Fig. 116.—General View of Aron Meter with case removed

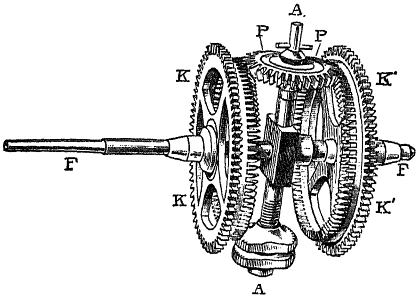


Fig. 117.—Differential Gear of Aron Meter

In the first meters of this type that were developed the clock used was first "rated" by comparison with a chronometer when no current was passing, and the total gain of the clock was then used to determine the total energy supplied to the consumer. This arrangement was, however, not at all satisfactory, and before many years elapsed two clocks were substituted, one with an attachment similar to that described above, and the other with the ordinary pendulum.

---

[1] Since for a pendulum

$n_1 = \kappa\sqrt{g}$, where $n_1 =$ number of vibrations per min.,
$\kappa =$ a constant depending on the moment of inertia and length of the pendulum,
$g =$ acceleration due to gravity;

and since the effect of the current is to increase the apparent attraction, we have, when the current is passing,

$$n_2 = \kappa\sqrt{(g + l\,\kappa w)},$$

where $l\,\kappa w$ represents the apparent increase in gravity, produced by the action of the current.

$$\text{Approximately, } n_2 = \kappa\sqrt{g}\left(1 + \frac{1}{2}\,\frac{l\,\kappa w}{\kappa g}\right),$$

$$\text{and } n_2 - n_1 = \kappa\sqrt{g}\left(\frac{1}{2}\,\frac{l\,\kappa w}{\kappa g}\right) = \kappa'\,\kappa w,$$

or the rate of gain of the clock is proportional to the power passing into the circuit.

Now energy $= \kappa w\,s$ (where s is the number of seconds during which power is supplied), and $(n_2 - n_1)\,s = \kappa'\,\kappa w\,s$, so that $(n_2 - n_1)\,s$ is proportional to the energy supplied. In other words, the total gain of the clock is proportional to the total energy supplied.

These two clocks were adjusted so as to go at exactly the same rate, and their mechanism connected by what is known as a differential gear. The two driving-wheels of this gear, shown in fig. 117, revolve in opposite directions. As long as the rate of the clocks is the same the central wheel will revolve but will not move bodily, but as soon as one wheel moves faster than the other the central wheel will move at a speed equal to the difference in rate at which the two driving-wheels are revolving. The translational movement of the central wheel therefore measures the amount of gain of one clock on the other, and the support for this wheel is geared to a counting mechanism which records its motion. The train of wheels which connects the central wheel to the counters is so arranged that the instrument records directly the number of kilowatt hours supplied through it.

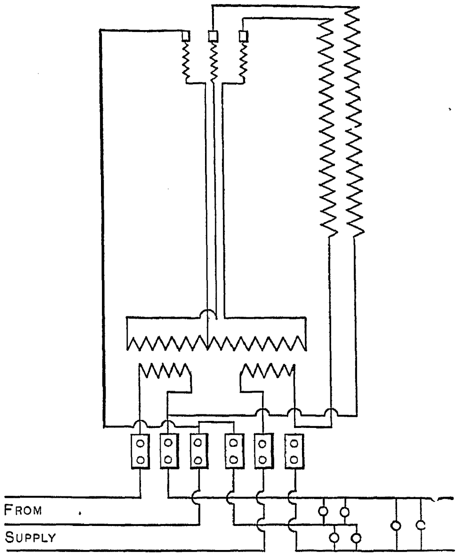


Fig. 118

The instrument has, during recent years, been greatly modified from its original form, the most considerable alterations being as follows:—Both pendulums are now acted on by the current, the connections being arranged so that an increase in the apparent force of gravity is produced on the one, and a repulsion or decrease in the apparent force of gravity on the other. The accuracy of the instrument is increased and its range is enlarged by this method of connection. The clocks are now electrically driven, and are wound by a simple arrangement every thirty seconds. An automatic cut-off is also provided, so that if for any reason either of the clocks stop, the record is not spoiled.

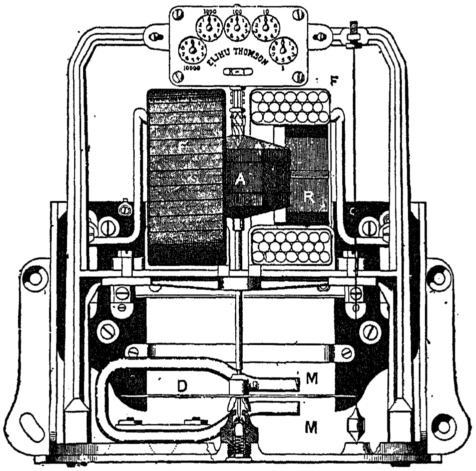


Fig. 119.—Part Section of Thomson Meter, showing principle of construction

The connection of such a meter for a three-phase system is shown in fig. 118. The meter, though it has been described as useful for measuring alternating currents, can be used with equal facility for the measurement of direct currents, and in any case is a very accurate instrument.

Motor Meters.—The second type of meter (originally suggested by Profs. Ayrton & Perry) has become very popular. The best-known example used for alternating current in this country is that devised by Prof. Elihu Thomson (fig. 119). It consists essentially of a small motor in which the field coils F carry the main current, and the armature A the shunt current. There is no iron in the field-magnet system, so that the magnetic field produced by the current is always directly proportional to it. The force f acting on the armature conductors at any instant is dependent on the strength of field H, and the strength of the shunt current c. Hence $f \propto c \times H$, and H being proportional to C, the main current, f is proportional to cC.

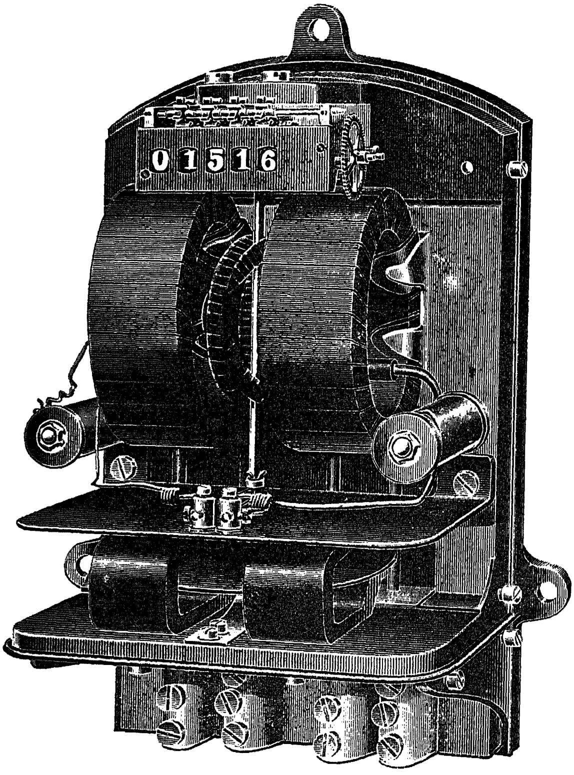


Fig. 120.—Lux Meter

Now $c = \frac{E}{R}$, if the induction of the shunt circuit is negligible, R being a constant, we have torque $\propto$ C E at any instant. If there were no resisting torque, the motor would increase in speed until the back E.M.F. it produced was equal to the E. M. F. applied. In order that the number of revolutions of the meter may register energy, the speed at which the meter is running must be proportional to the power supplied through it. This is effected if a retarding torque is applied, which is *proportional to the speed*. When such an arrangement is adopted the speed will increase until the driving and retarding torques are equal, and consequently, the speed being proportional to the retarding torque, will be proportional to the driving torque, that is, to the power supplied.

This retarding torque is produced usually, by attaching to the arbor of the armature a flat copper sheet D (fig. 119), which moves between the poles of a strong permanent magnet. Eddy currents are induced in the copper by its motion, and these eddy currents react on the magnetic field, producing a retarding torque, which is always exactly proportional to the speed at which the plate is moving. In order that the meter may record accurately, many details have to be looked to, the chief difficulty arises from the necessary friction at the bearings. To reduce this to the smallest possible amount, the bottom of the arbor, which is made of hard steel, is pivoted on a sapphire jewel, and the top supported in a cup of hard steel. The spindle is steadied at both ends by collar bearings, loosely fitting. The friction of the brushes on the commutator (which is usually of eight parts) is eliminated in the latest type by using silver strip, which is arranged to press edgewise on the commutator surface.

It is impossible to get *exact* phase equality between shunt current and applied potential difference, and in certain cases, where the meter has been used to measure the power expended in the dielectric of a cable (the current leading by nearly 90° on the applied potential difference), the meter has been found to go backwards, thus showing that the phase difference between the main current and shunt current was greater than 90°, and that therefore the instrument was no longer recording power. Except in these extreme cases the meters are, however, very reliable.

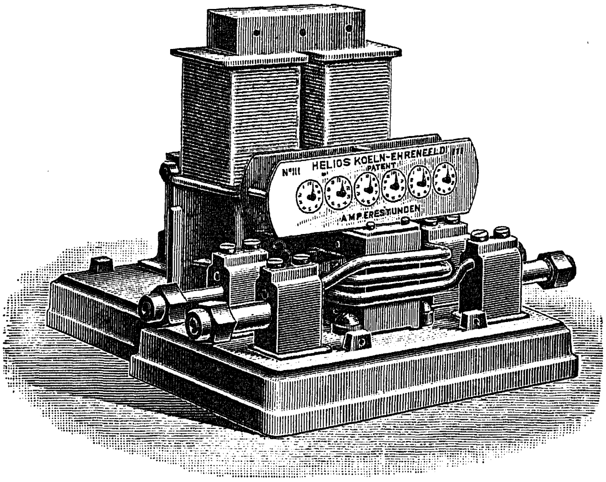


Fig. 121.—Blathy Meter

Another type of meter very similar in principle to the Thomson is that known as the Lux (fig. 120), manufactured by Luxsche Industriewerke A. G. Munich. In this the armature is made up of three coils, placed at 120° to each other. Three of the ends of the coils are connected together, and the other three ends are brought out to a three-part commutator on which press two brushes. The advantage claimed for this meter is that the current only passes through two of the coils at a time, and this only when the coils are in the most favourable position. By this means the same turning moment is produced for a smaller expenditure of shunt current. In other details the meter is essentially similar to the Thomson. For a three-phase system two meters are used, having armatures on the same arbor, the connections being the same as described for the Aron meter.

Induction Motor Meters.—The second class of motor meters is that which is essentially adapted for alternating-current circuits, since it is suitable only for such circuits. One of the earliest meters of this type is that devised by Herr Blathy, and is represented in fig. 121. One coil, wound with a few turns of thick wire, carries the main current, while another coil, wound with many turns of fine wire, carries a current proportional to the applied pressure. The winding is so designed in the latter case that the current lags by very nearly 90° behind the applied pressure. When, therefore, the current and the potential difference applied to the main are in phase with each other, we have, in the two coils, currents which are 90° out of phase, and consequently the alternating magnetization in the cores of these coils will also differ in phase by 90°.

Pivoted so as to move in the narrow gaps of the pole-pieces of these coils is a flat disc of copper. As was shown when describing the phenomena due to two-phase currents, the effect of two coils placed as shown relatively to the copper disc will be to produce a rotating field, which travels continuously from one pole to the other. This field, which is continually varying, will produce eddy currents in the disc, the interaction between which and the field will tend to move the former round. The speed at which the disc would rotate if perfectly free to move would ultimately be very nearly that corresponding to the frequency of the applied current. In order to control the speed a permanent magnet is applied, which acts on the disc and brakes it, just in the same way as in the Thomson meter. The turning force acting on the disc due to the rotating field will depend on the magnitudes of the fields in the two poles, but only that component of the field in the one will be effective in producing rotation, which is 90° out of phase with the field due to the other. In other words, only that portion of the field in the first will be effective which is in phase with the pressure applied to the second. The turning moment, therefore, will be proportional to the product of the watt-full current and the pressure applied, or to the true power supplied to the circuit. This being the case, the speed at which the motor will run when braked by a permanent magnet will be proportional to the power supplied through the meter. The number of revolutions of the disc is recorded through a worm cut on the spindle, which gears with the counting arrangement and with its attached dials.

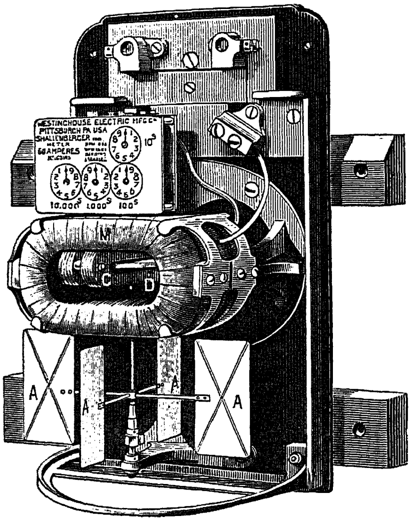


Fig. 122.—Interior of Shallenberger Meter

This meter can be made to record fairly accurately when the power factor is high, but it becomes inaccurate at low power factors, on account of the fact that the field in the second coil is not exactly 90° out of phase with the pressure applied to it.

Another very popular form, manufactured by the Westinghouse Company, is due to Shallenberger. The coils (fig. 122) in this case are placed somewhat differently, there being two shunt coils C C placed at a certain angle to the coils M M, which carry the main current. The current in the shunt coils is arranged to lag behind the applied P.D.

by nearly 90°. It is possible, by placing the two coils at the right angle with each other, to produce a [illegible] rotating field, which drives the disc. The braking is effected by air vanes, but in many of the instruments a magnetic damper is employed. The discs in these instruments are made of aluminium.

A third meter is that devised by Messrs. Hartmann & Braun, in which the copper disc is replaced by a copper bell, the coils being wound on the extremities of an **E** shaped iron frame, the two shunt coils [illegible] are wound in opposite senses on the ends of the **E**, and the main coil on the centre limb. The shunt coils are designed as in the Blathy instrument, so that the current in them lags by nearly 90° behind the applied pressure. The brake is applied through a flat copper disc attached to the same spindle running between the poles of a permanent magnet.

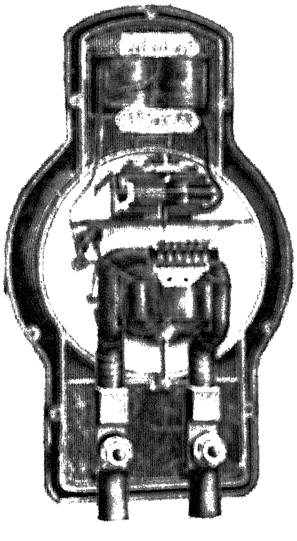


Fig. 123.—Interior of [illegible] Company's Alternating Current Meter [illegible]

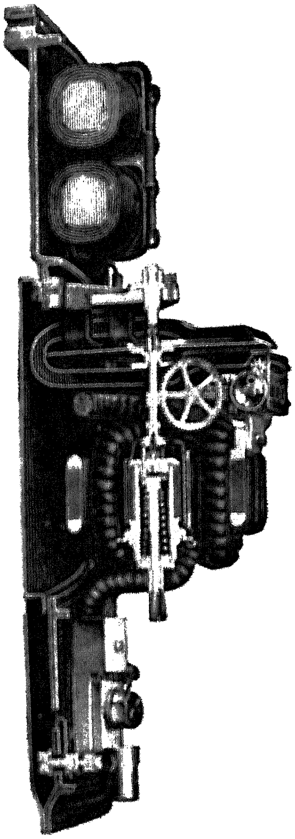


Fig. 124.—Interior of Electrical Company's Alternating Current Meter

A very similar arrangement is adopted by the Allgemeine Electricitäts Gesellschaft (figs. 123 and 124), with the modification that four pole-pieces of special shape are used, adjacent poles being excited respectively by the main and shunt currents. In these instruments, too, a special arrangement of the shunt circuit is employed to ensure exact quadrature between shunt field and current field when the potential difference and current are in phase. The shunt circuit consists of a choking coil, with a core of best transformer iron, in series with which are the shunt exciting coils of the meter, and in parallel with them a non-inductive resistance. These instruments can be made to work so that the error never exceeds

2% for loads greater than $\frac{1}{40}$th of full load. Another type of meter is manufactured by the same firm for small currents which is very similar to the Shallenberger, a single (in this case copper) disc being both driven and braked.

On the score both of simplicity of construction and cheapness of manufacture the induction motor meter offers many advantages; its accuracy, however, on loads of low power factor is never very considerable, and where such loads are frequent it is usually preferable to employ a meter such as the Aron, where greater accuracy is attempted.

3. Continuous-Current Generators

CHAPTER I

GENERAL REMARKS

In the following article on continuous-current generators the authors have considered only typical modern traction and power generators, such as are in use in traction stations, collieries, engineering works, &c. They have not gone into ancient history, nor have they attempted to write an article on the theory of the dynamo electric machine. For such the reader is referred to the well-known standard works of Mr. Gisbert Kapp, Professor Silvanus Thompson, and Messrs. Parshall and Hobart. The authors have simply attempted to show how such theory is applied to the design of commercial generators in an engineering works in order to produce a dynamo of high efficiency over a great range of load, sparkless in operation under all loads and such overloads as are likely to be met with in practice, which will run within the ordinary limits of temperature rise generally specified, and the cost of materials of which will enable the makers to compete with their machines in the open market.

The methods and formulæ employed are to a great extent worked out in the authors' own way, and are the results of practical experience. Of course free use has been made of tables and curves published in many standard works, including those named above, but all of those given have been actually employed by the authors in practical work.

Some of the methods may appear somewhat unorthodox, but, as employed, have been found to give very satisfactory and reliable results.

Although the development of alternating-current generators, particularly for multiphase work, has been startling and rapid in the last decade, the development of the direct-current generator in the same time has been no less so.

Ten years ago most of the dynamos built in this country were designed for electric-light work. For this class of work the two-pole, smooth-core, bar-drum machine, then in general use, was a very good machine in its way. Although heavy and expensive as regards material, its behaviour on the comparatively steady loads usual in lighting work was very good. Units larger than 100 to 150 kilowatts were the exception.

The growth of power-transmission work in collieries, workshops, &c.,

but more especially for electric traction, creating a demand for large generators capable of standing rapidly fluctuating loads, with sudden and heavy overloads, without the necessity for any adjustment of the brushes to prevent sparking, hastened the development of multipolar generators, and at the present time few dynamo makers build bipolar machines in any but very small sizes. Even when the output is so small—under, say, 15 to 20 kilowatts—that there is not much to choose in operation or efficiency between bipolar and multipolar designs, there is generally still a saving of material effected by adopting a multipolar type. This type also lends itself to a better mechanical arrangement of parts.

Questions of economy of material and sparkless operation at all loads with fixed-brush position have caused the adoption and almost universal use of the slotted type of drum-wound armature.

This type of armature, with the magnetic induction at the root of the teeth run up to a density of, say, 130,000 magnetic lines per square inch at full load, and properly proportioned generally, can be made quite sparkless with fixed-brush position from no load to 25 per cent overload, and yet the watts lost in field excitation at load need not exceed 1 per cent of the rated output. With the introduction of multipolar types of machine the tendency has been in the last few years towards simplicity of design. There have been a considerable number of inventions and suggestions which involve a radical departure from the simple type, such as reversing lugs, commutating poles, cord windings, &c., but none of these are in extensive use at present. The tendency seems to be towards the development of the simple type of machine and to the elimination of all these ingenious, but, as it seems to the authors, unnecessary devices. It seems to them that the natural conservative tendency of manufacturers to retain old types as long as possible is responsible to a great extent for the appearance of these devices.

New conditions arise and new requirements have to be met. The manufacturer, with a natural horror of altering patterns, rather than revise and improve his designs, attempts to work in old and unsuitable types and to fulfil the conditions imposed by the use of one or another of the devices mentioned. The result may be satisfactory for the moment, but the use of such devices will generally be found to be very limited, and when the matter is taken up carefully and from a scientific standpoint, it will in most cases be seen that there is so much room for improvement along straightforward and simple lines that there is no necessity to depart from them, at any rate in any but very special cases.

The same tendency towards simplicity is seen in the armature windings of modern traction generators. Except in small sizes of machines under, say, 75 kilowatts capacity, the winding almost exclusively employed is plain multiple circuit-lap winding. In the smaller sizes mentioned two-circuit wave-winding, or series-winding as it is sometimes called, is used with advantage, as the difficulty of commutation found when using these windings on large machines, especially with a greater number of poles than four, is not so great.

The difference between Lap and Wave types of windings will be understood by a reference to figs. 129 and 131, Chapter II. In the case of lap windings as many sets of brushes are necessary as there are poles in the machine. In the case of wave or series windings, two sets of brushes only may be used, whatever the number of poles, although the number of sets of brushes may be made equal to the number of poles if necessary.

There is a considerable saving of labour effected by the use of series-wound armatures, owing to the reduced number of armature conductors, as the number of conductors for such an armature is $\frac{1}{p}$ times the number necessary to produce the same voltage if the armature had been lap-wound, where p is the number of pairs of poles.

With the development of multipolar designs armatures of machines have become large in diameter and narrow in length. Continental makers were inclined to go to an extreme in this direction, while English makers have generally clung to a longer, smaller-diameter type of armature. In larger and more modern machines, however, English, American, and Continental practice has coincided to a greater degree, and there is now a very great similarity in the general proportions of machines. The authors favour proportions in which the pole face is approximately square with pole-cores and bobbins of circular section. Very little is to be gained by narrowing the length of machines below these proportions, and machines with greater length and less diameter are heavier in proportion, and bobbins and pole-pieces become of a less convenient and less economical shape for winding.

Using these proportions, and with the poles covering .7 of the armature circumference, a six-pole dynamo, having an armature of 36 inches diameter, would have an armature length of 12 inches; in a ten-pole machine, having an armature diameter of 60 inches, the length would again be 12 inches; also in a machine of twelve poles, with 72 inch armature diameter, the length would be 12 inches. The proportion of length to diameter is thus $\frac{1}{p}$, p being, as before, the number of pairs of poles—a very convenient proportion. If, as is generally the case, the yoke is cast as a plain ring separate from the poles, the poles being bolted to the inside face of the ring, the same pole pattern could be used in the case of the three machines mentioned above, and also the same bobbin-formers. If the armature strength, *i.e.* the ampere turns of the armature as given by formula 4, and magnetic flux per pole were kept the same, the bobbin windings would also be the same, and one could be as certain of good operation in the case of the bigger machines as in the smaller.

Of course considerations of speed limit the use of proportions such as mentioned above, and it is not possible to make a series of machines all of the same armature length and size of pole-piece (but of varying diameters and numbers of poles) which would work in for all the standard ratings necessary in practice. It will, however, be found that not more than five or six different armature lengths and standard pole pieces will be required for a very complete series of machines from 20 kilowatts up to 1000 kilowatts.

As regards the number of poles for any given output the considerations to be taken into account are the maximum armature reaction per pole permissible, and the maximum voltage per commutator bar to be allowed.

The latter may be determined from the maximum permissible reactance which can be allowed per armature coil. This point is further treated in Chapter II.

It can, however, be taken that good results will be obtained if machines are built with the number of poles given in table below.

No. of Poles.	Kilowatt outputs.	
	220 to 250 volts.	500 to 550 volts.
2	10	10
4	80	80
6	200	250
8	300	400
10	350	500
12	400	600
14	500	700
16	550	800
18	700	900
20	750	1000
22	800	1100
24	900	1200

For power and traction work the use of compound wound generators is general. The compound winding is generally made to raise the voltage on full load. For tramway traction work this overcompounding is usually as much as 10 per cent, *i.e.* a machine giving 500 volts when unloaded would give 550 volts when fully loaded at constant speed. In ordinary power transmission in steel-works, mills, &c., 5 to 8 per cent is generally sufficient, but in colliery work, in the case of generators driving underground haulage motors, the authors have sometimes over-compounded the machines as much as 15 to 20 per cent in order to keep the voltage constant at the bottom of deep mines. The over-compounding of these generators has also a good deal to do with their excellent operation as regards sparklessness and fixed-brush position. The strengthening of the magnetic flux at full load is a condition most favourable to sparkless running.

CHAPTER II

DESIGN OF ARMATURES

In commencing to design a continuous-current armature for any given output the simplest and most direct way nowadays, when so much data is available to settle the general proportions of diameter and length, is to refer to data of other machines and pick out preliminary proportions which can be altered and adjusted as the design proceeds. Kapp and others have given formulæ which are necessarily only of an approximate character. According to Kapp:

$$\text{Output in kilowatts} = \frac{K_1 D^2 L \text{ revolutions per minute}}{10^6}, \qquad (1)$$

where K_1 = a constant,
D = diameter armature in inches,
L = length " "

Values for the constant K_1, taken from actual machines, are given on the curve shown in fig. 125 for different outputs.

The curve is plotted in values of K_1 and "specific output". The specific output is simply the output of the machine in watts divided by the speed in revolutions per minute, and is a quantity which is constant for a given machine, no matter for what output and corresponding speed it is used. It is useful as a comparative figure. Using these values in the above formula the value D^2L can be obtained. If the number of poles to be used in the design has been settled, and the proportions of length of armature to diameter recommended in Chapter I have been adopted, an approximate diameter can be obtained.

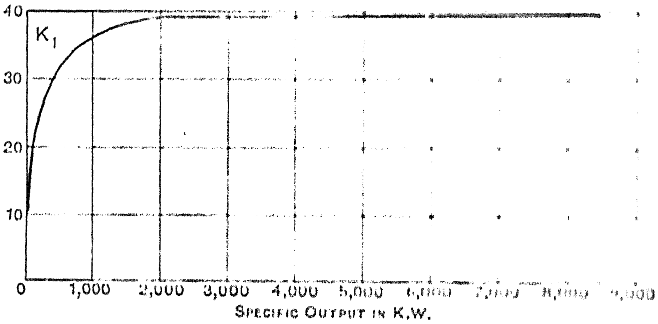


Fig. 125

For example, assume that—

Required output in kilowatts	=	250
Speed in revolutions per minute	=	400
No. of poles settled on	=	6

$$\text{Specific output} = \frac{250{,}000}{400} = 625$$

The value of K_1 from curve fig. 125 for this specific output = 31.5.

We then have, from formula 1:

$$D^2 L = \frac{250 \times 10^6}{31.5 \times 400} = 19{,}800.$$

The number of pairs of poles or $p = 3$.

If we adopt the proportion of length to diameter described in Chapter I, viz.:

$$L = \frac{D}{3},$$

$$\text{then } D^2 L = \frac{D^3}{3} = 19{,}800,$$

$$\text{and } D = \sqrt[3]{19{,}800 \times 3} = \text{(say) } 39 \text{ inches.}$$

Our armature would, therefore, be 39 inches diameter, 13 inches long.

Of course regard will be had to the limit of peripheral velocity; this should not exceed, on generators from 100 to 150 kilowatts capacity and upwards, from 4000 to 4500 feet per minute. In exceptional cases of very high speed the design will have to be modified and a longer length and smaller diameter taken.

At the ordinary speeds met with in practice, however, the above formula will be found applicable.

It would probably have been found better, however, to consult a table of sizes of existing machines.

Having settled the general proportions of the armature, the next step would be to determine the magnetic flux per pole entering the armature from the field-magnets, and to do this we should first settle upon the magnetic density to which we are going to saturate the air-gap. In modern generators this density is generally pretty high, usually from 50,000 to 55,000 lines per square inch. Multiplying this density by the area of the air-gap in square inches gives the number of magnetic lines of force or magnetic flux per pole entering the armature. It is convenient to reckon this quantity in millions of magnetic lines, or megalines, as this simplifies calculation.

Calling this quantity ϕ we can obtain the total number of inductors which we have to arrange on the surface of the armature from the fundamental formula.

$$N = \frac{E \times 100}{\phi \times n} \times \frac{p_1}{p} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Where N = total number of inductors,
E = total internal voltage to be generated,
ϕ = magnetic flux per pole in megalines,
n = speed in revolutions per second,
p_1 = number of bifurcations of the current through the armature,
p = number of pairs of poles.

The quantity p_1 depends on the type of winding used. If plain multiple-circuit lap-winding is used, p_1 is equal to p, being 1 in the case of two-pole armatures, 2 in the case of four-pole armatures, 3 in the case of six-pole, and so on. In the case of two-circuit or series windings p_1 is always 1, no matter what the number of poles may be. If duplex, triplex, or quadruplex windings are used, as is sometimes necessary in the case of special heavy-current machines, the values of p_1 given above would be doubled in the case of duplex windings, trebled in the case of triplex, and four times in the case of quadruplex coupling. It is seldom, however, that one has to resort to the use of these windings, especially for series-wound armatures.

The quantity E is equal to—

$$V + e + e_1, \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

where V = volts required at terminals and e = internal drop of voltage due to armature resistance. e_1 = voltage drop due to resistance of brush contacts and leads. The last named will be generally in the neighbourhood of 1 per cent of the terminal voltage, while e will usually be taken at 2½ to 3 per cent for preliminary calculation. Of course the exact results may be calculated when the design is finally completed.

It will be well to check the figures obtained of the total number of inductors. These should not exceed such a number that the ampere turns of the armature will be greater than 50,000 ampere turns per inch length of the air-gap, using magnetic densities in the air-gap of 50,000 to 55,000 lines per square inch.

The ampere turns of the armature will be given by the formula:

$$\text{Ampere turns} = \frac{N\,C}{4\,p\,p_1}, \quad \dots \quad \dots \quad \dots \quad (4)$$

C being total current, the other symbols as before.

The above rule, which the authors have found to be a good general limit for practical work, can be stated simply as—

$$\frac{N\,C}{4\,p\,p_1\,\delta} < 50{,}000, \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

δ being the length of air-gap from iron to iron in inches. The number of armature inductors being satisfactorily settled within the above limit, the number of slots will next be determined and their dimensions fixed. The depth of slot can generally be made from 5 to 5½ times the length of the air-gap in machines of over, say, 50 kilowatts size, and under this size from 5½ to 6½ times the air-gap length. The width of slot will be settled from the permissible magnetic density at the root of the teeth at full load. This should not exceed 130,000 to 135,000 lines per square inch, nor should it be much under this, as working with the teeth pressed to this density has the great advantage that it causes the magnetic reluctance in the path of the armature cross-magnetizing field to increase greatly as the load increases, thus tending to reduce distortion of the field-flux in the air-gap and poles, and greatly assisting in producing a machine having fixed-brush position whatever the load.

A number of conditions have to be taken into account in determining the best number of slots to be used. If these are taken too few the width of the teeth will come out too great, and eddy currents will be generated in the pole faces if solid, unless one gives up the advantages of solid pole cores and uses a laminated type. On the other hand, if too many teeth are taken the core becomes expensive to build, and too much space is lost in the insulation in the slots. However, by an appropriate choice of the number of conductors per slot, the number of slots can generally be made such that there is something between 1.1 and 1·5 slot per inch periphery of the armature, a number which will be found to be a good working compromise between the two conditions mentioned above.

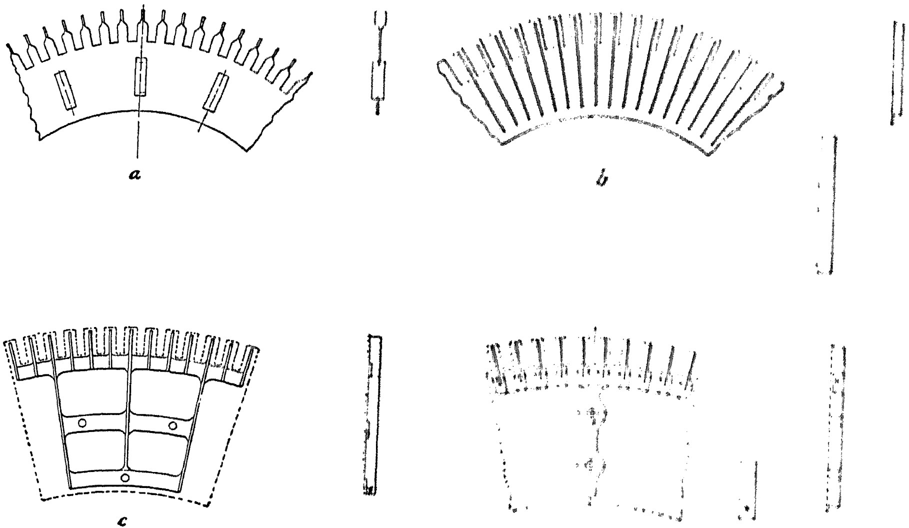


Fig. 126.—Types of Ventilators

The radial depth of armature iron under the teeth will be proportional with regard to the permissible hysteresis loss in the armature. Generally it will be found that a density of 60,000 to 65,000 lines per square inch will give good results.

Ventilating ducts should be provided in modern armatures at least every 4 inches, and these should be generally from $\frac{3}{8}$ inch to $\frac{1}{2}$ inch wide. Various types of ventilators are employed, and fig. 126 shows drawings of some of these. The authors favour the types shown in *a* and *b*. These are very simple and effective, being made from $\frac{1}{16}$-inch-thick plates, stamped with teeth of the same dimensions and number as the rest of the discs. In the case of fig. 126, *a* the tops of these teeth are turned round at right angles to the plate. Pieces of bar-iron $\frac{1}{2}$ inch square are fitted in slots punched in the plates below the teeth as shown, in order to keep the body of the armature discs apart. In fig. 126, *b* the teeth are not turned, but strips of $\frac{1}{16}$-inch-thick iron are fitted to the ventilator plate at each tooth as shown.

In reckoning up the net effective length of armature iron, the space occupied by these ventilators must be deducted from the total length of armature over the discs, and 10 per cent of the remainder must again be deducted for space lost owing to the insulation between the discs.

The necessary cross section of the armature conductors will be determined from the maximum permissible current density in amperes per square inch we can employ in them for a given temperature rise of the armature surface above that of the atmosphere. This permissible current density will vary for a given rise, as the watts lost due to hysteresis and eddy currents in the core are greater or smaller, and in the case of an average type of armature, such as that illustrated in fig. 142, may be arrived at in the following way. Taking the peripheral surface of the armature over core and conductors in square inches, it will be found that in ordinary well-ventilated types of armature there will be an increase in temperature above that of the atmosphere of 20° F. to 27° F. per watt per square inch dissipated.

The total watts lost in the armature will be made up of—

1. Watts lost due to hysteresis in core.
2. „ „ eddy currents in core and inductors.
3. „ „ resistance of armature inductors

The peripheral surface and temperature rise being fixed, the total watt loss in the armature from all these causes will therefore be also fixed. As the dimensions of the iron core of the armature have been determined, the hysteresis and eddy current loss can be calculated. Deducting this from the total permissible loss in watts, we obtain the permissible watt loss in the armature conductors. The necessary current density can then be found from the formula:

$$\text{Current density} = \frac{81750\ W p_1}{m\ N\ C}, \quad \dots \quad \dots \quad 6$$

where

W = permissible watts in inductors as given above.
p_1 = number of bifurcations of armature current.
m = mean length of one inductor in yards.
N = total number of inductors.
C = total current.

The above formula is arrived at in the following manner:—

The permissible watt loss $W = C e$ where e is the voltage drop due to armature resistance.

$e = C R$ where R = resistance of armature between brushes, and $R = \frac{r m N}{4 p_1^2}$, where r = resistance per yard of one inductor, the other symbols as described above; therefore watts lost in armature due to copper resistance $= W = \frac{C^2 r m N}{4 p_1^2}$; and as $r = \frac{.00002405}{A}$, where A = area of cross section of inductor in square inches, $W = \frac{C^2 .00002405 m N}{A 4 p_1^2}$.

Current density in amperes per square inch is given by $\frac{C}{2 p_1 A}$, and from above $\frac{C}{2 p_1 A} = \frac{2 p_1 W}{00002405 m N C}$, *i.e.* current density $= \frac{81750 W p_1}{m N C}$.

The hysteresis and eddy current losses, which are generally lumped together, and the total termed core loss, would usually be calculated by designers from standard curves of core loss actually measured from similar types of machines. They can be estimated, however, from curves of hysteresis loss in the discs at varying densities. Fig. 127 shows a curve of magnetic density and hysteresis loss in watts per pound at 1 alternation per second in good average armature iron.

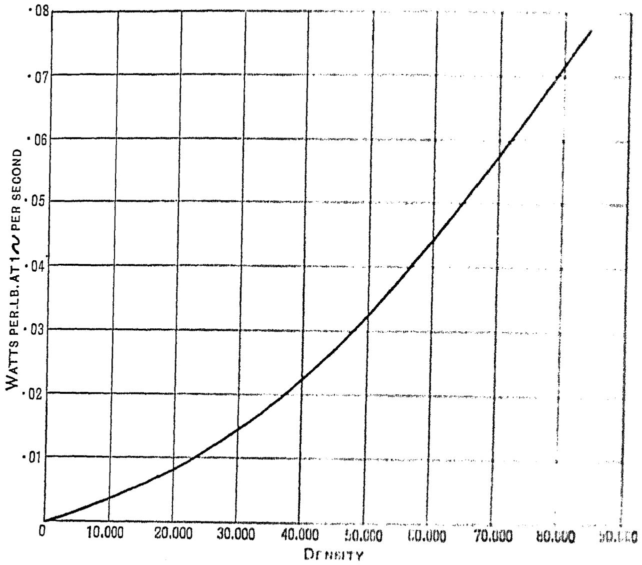


Fig. 127

The eddy currents losses are difficult to calculate, and in practice it is generally sufficient to add 50 to 70 per cent to the values obtained from the above curve to allow for eddy currents. If the core has been built carefully with a minimum of filing or machining of the discs, 50 per cent should be ample allowance; but if the slots have been filed or milled to any great extent 70 per cent up to even 100 per cent will have to be added to arrive at a correct value of the core loss.

The losses in the iron of the armature below the teeth and those in the teeth can be figured separately and added together to arrive at the total core loss. Of course, in using the above curve it must be remembered that the results are given at one alternation per second, and will have to be multiplied by the number of alternations the core is undergoing per second, which is $= p n$.

In calculating the losses in the teeth, and also (as described farther on in Chapter III) in arriving at the number of ampere turns of excitation on the magnets necessary to drive the magnetic flux through the teeth, it is not correct to take the apparent density at the root of the teeth as the actual density in the iron of the teeth. A considerable portion of the magnetic flux traverses the slots and ventilating ducts, so reducing the density in the teeth considerably below the apparent value arrived at if it is assumed that all the flux flows through the iron of the teeth. In fig. 128 is shown a curve of corrected and apparent densities.

This curve is slightly below that given by Messrs. Parshall and Hobart in their book on *Electric Generators*, which the authors have found to give values rather higher than generally met with in practice. The above curve is plotted for teeth having an average width equal to .8 width of the slot.

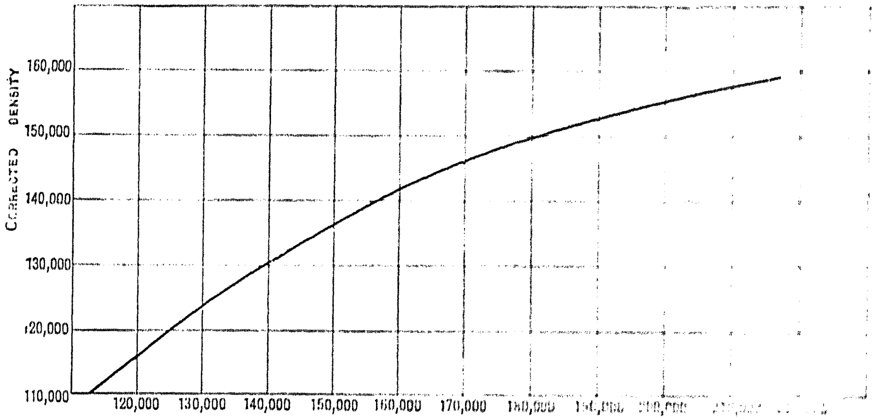


Fig. 128

The following example may be of interest as illustrating the method of calculating armature losses given above. An armature having the dimensions given below, and of the slotted drum type, lap wound for six poles will be taken.

Diameter over teeth, 36 inches.
Length over laminations, 12 inches.
Total length over inductors, 25 inches.
Mean length of one inductor, 1.03 yard.
Total number of inductors, 888.
Full load current, 420 amperes.
Density in laminations below teeth, 66,000 lines per square inch.
Apparent density in teeth at root, 134,000 lines per square inch.
Corrected density in teeth from curve, 125,000 lines per square inch.
Weight of core below teeth, 1476 lbs.
Weight of teeth, 222 lbs.
Speed, 450 revolutions per minute.
Temperature limit, 60° F. rise above the atmosphere.
Alternations per second $= p \times n = 3 \times 7.5 = 22.5$.
Watts per lb. lost in hysteresis from curve $= .053$.
Watts per lb. at 22·5 alternations $= .053 \times 22.5 = 1.2$.

Add 50% for eddy currents = 1.2 + .6 = 1.8 watt per lb.
Watts core loss below teeth = 1476 × 1.8 = 2650 watts.
Hysteresis loss in teeth = 3.36 watts per lb. at 22.5 alternations.
Add 50% for eddy currents = 5.04 watts per lb.
Loss in teeth = 5.04 × 222 = say 1120 watts.
∴ Core loss total = 2650 + 1120 = 3770.

Peripheral surface of armature = 36 × 3.1415 × 25 = 2820 square inches, taking average ventilation, say 27° F. rise per watt per square inch = 2.22 watts per square inch permissible = 6200 watts total loss allowable.

6200 − 3770 = 2430 watts allowable for copper loss for 60° F. rise in temperature; using formula 6 and filling in this copper loss.

Permissible current density $= \dfrac{81750 \times 2430 \times 3}{1.03 \times 888 \times 420} = 1540$ amperes per square inch.

We should, accordingly, from the above figures, limit the current density in the armature under consideration to between 1500 and 1600 amperes per square inch.

The above figures are worked out by means of an ordinary 10 inch slide-rule, which is quite accurate enough for all ordinary dynamo design.

From the density thus obtained the necessary area of the armature inductors can be settled. Formula 6 could, of course, be so arranged as to give this direct without incidentally bringing in the density, but it will be found simpler in the end to proceed by steps as shown, as one is able to mentally check the results obtained more readily and to detect errors or discrepancies before proceeding very far on wrong lines.

Making proper allowances for insulation in the slots, there should be just room for inductors of the above size. If there is more than enough, the armature is large for the output required and can be rated higher than intended, or a smaller diameter would have sufficed. If the room in the slots is insufficient then the armature is too small for the output required at the stated speed, and the design must be altered accordingly.

As a general rule, the more the armature winding is split up, *i.e.* the greater the number of commutator segments employed, the better will its performance be from the point of view of sparking. In machines designed for 500 volts to 600 volts the limit will generally be set by the smallest permissible width of commutator segment at the top, the diameter of the commutator, of course, having to be kept within reasonable limits. With the ordinary types of carbon brushes which are employed, using peripheral speeds of commutator not exceeding, say, 3500 feet per minute, and employing general armature and pole-piece proportions and densities already described, it will generally be found sufficient for good performance to limit the average volts per commutator bar to 8 to 10 volts in 500-volt machines and 7 to 9 volts per bar in 250-volt machines, the average voltage per bar being given by the formula:

$$\text{Volts per bar} = \frac{2pV}{N_1}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

where N_1 = number of commutator bars,
p = number of pairs of poles,
V = terminal voltage.

Mr. H. M. Hobart gives, in a paper read in Section IX of the Glasgow Engineering Congress, 1901, a method of calculating the reactance voltage per segment which will be found to give good results. Of course these results are strictly relative, but they give a value which is exceedingly useful as a guide in new designs and in adapting standard designs to special requirements.

The reactance voltage per segment, which is the voltage induced in one section of the armature due to the section being threaded by a varying magnetic field—that is, the magnetic field induced by the varying current flowing in all the sections which are under one line of brushes while commutation is taking place—depends on the inductance of the coil and the frequency of reversal of the current commuted. The inductance, measured in henrys, of any coil is the total linkage (or the product of the number of turns and the magnetic flux set up in the coil when 1 ampere passes through it) divided by 10^8. This is proportional to the square of the number of turns. It will therefore be seen how necessary it is to keep down the number of turns per segment if the reactance voltage is to be kept low. It will generally in large generators be found difficult to obtain good values for the reactance voltage unless the number of turns per segment is not above 1, *i.e.* 2 inductors per segment of the commutator.

In the paper referred to, the slot-enclosed portion of the coil is called the embedded length, and the part of the coil forming the end connections the free length. Values are given for the field per ampere turn per centimetre length of coil which, reduced to C.G.S. lines per ampere turn per inch length of coil, are as follows:—

10 C.G.S. lines per ampere turn per inch of embedded length.
2 C.G.S. lines per ampere turn per inch of free length.

Using these values, and remembering that the embedded portion of a coil lies over, or close to, the corresponding portion of the coil short-circuited under the next line of brushes, and that the magnetomotive force of this part is consequently twice that of the free portion, we may calculate the total linkage.

An example will make this clear. If we take a case where we have two inductors, *i.e.* 1 turn per segment and 4 turns short-circuited under each line of brushes with—

Embedded length per turn	24 inches.
Free length per turn	50 „
Revolutions per minute of armature	300
Diameter of commutator	28 inches.
Thickness of brush	·875 inch.

Linkage for embedded length $= 8 \times 10 \times 24 = 1920$
Linkage for free length $= 4 \times 2 \times 50 = 400$

Total linkage 2320

Then if L is the inductance in henrys, $L = \frac{2320}{10^5} = .0000232.$

The periodicity of reversal is given by

$$\sim \; = \frac{n \pi d_1}{2 t} = \frac{6 \times 3.1415 \times 28}{2 \times .875} = 300, \text{ say}, \quad \ldots \quad \ldots \quad (8)$$

where $\sim$ = periodicity in alternations per second,
n = revolutions per second,
d_1 = diameter of commutator in inches,
t = thickness of brush in inches.

Then if the current to be commuted by one line of brushes is, say, 50 amperes, the reactance voltage which is given by—

$$\text{reactance voltage} = 2 \pi \sim \text{L C} \quad \ldots \quad \ldots \quad (9)$$

would work out to = 2 × 3.1415 × 300 × .0000232 × 50 = 2.17 volts.

According to Mr. Hobart this value should not exceed 3 volts, in order that good results may be obtained and heavy overloading be possible without sparking troubles arising. With the proportions of polar arc to pitch, gap-densities, and armature ampere turns per pole, recommended by the authors, no trouble has been found as regards sparking with machines having a reactance voltage, estimated as above, of 4 to 4½ volts per segment in sizes of 250 to 300 kilowatts, although, where possible, this value in the authors' machines is kept below 4 volts, and 3 volts may be taken as a good safe limit.

In calculating the reactance voltage per segment of series-coupled armatures one may proceed as described above, remembering that the current to be commuted in this case is half the total current, as the value of p_1, *i.e.* the number of bifurcations of the armature current for series coupling, is always 1, and is independent of the number of poles.

If the proportion of length to diameter of armature mentioned in Chapter I is adopted, viz.—

$$\text{Length} = \frac{\text{diameter}}{p}$$

the calculation of reactance voltage on the above lines may be simplified by using the formula:

$$\text{Reactance voltage} = \frac{48\,\text{S}_2\,\text{L}\,\text{Z}\,\pi \sim \text{C}}{10^8 p},$$

where L = armature length in inches
Z = number of turns short-circuited under one line of brushes,
C = total current,
$\sim$ = periodicity of reversal,
S_2 = turns per segment.

As regards methods of winding and coupling up the inductors of an armature, only those methods in general use on modern generators will be considered in these pages. For information about the numerous other types of windings, such as double and triple lap-wound or series-wound armatures, interpolated segments, &c., none of which types are in anything like general use, the reader is referred to the well-known German work by Professor Arnold and to that by Messrs. Parshall and Hobart on armature windings.

Drum windings in different forms are in almost universal use for modern traction generators. Ring windings are only used for very special cases. Drum-wound armatures of modern generators by various makers differ mainly in methods of bending and forming the end connections. The barrel type of winding, in which the end connections are laid out flat on cast-iron barrel ends, is the method most favoured in modern practice. In this type the whole winding rotates at the same peripheral speed, giving excellent cooling effect, a point of very great importance in modern generators where the thermal limit of output is the great point to be considered, as the efficiency of these machines will, even when they run with temperature rises above those permissible in practice, be quite high enough for ordinary requirements.

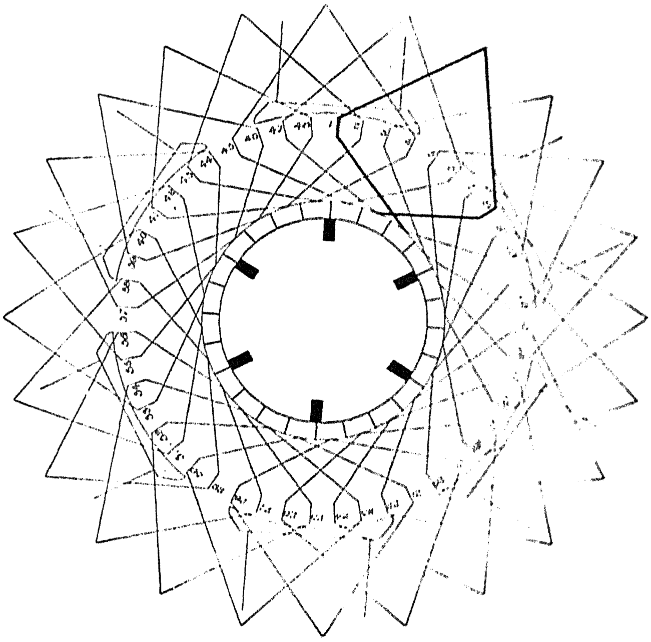


Fig. 129.—Diagram of Lap Winding

It will generally be found that on all machines for 250 to 600 volts, from, say, 70 kilowatts upwards, simple lap-winding, or multiple circuit single-winding as it is called, will be the best type to employ. It will only be on very special types of machines that any more complicated winding is necessary. Fig. 129 shows a diagram of an ordinary lap winding. The radial lines represent inductors in the slots, the connecting lines inside represent the end connections to commutator, and those outside represent end connections at the back of the armature. The thick black line traces out one coil or section of the armature between two commutator segments. The diagram shows 48 inductors and 24 commutator bars. An inductor at the front end is connected to the 9th ahead of it and at the back to the 7th behind it. The pitches are thus 9 at the front end and 7 at the back. The conductors will generally be arranged two or more in a slot in two layers as shown in fig. 130. It will be found convenient to number them as shown, *i.e.* odd numbers at the top and even numbers at the bottom.

The following are the rules governing ordinary multiple-circuit single windings:—

1. The number of conductors must be even.
2. The front and back pitches must be odd, and must differ by two.
3. The average pitch, which will be an even number, must not be very different from $\frac{N}{2p}$

where N = total number of inductors,
p = number of pairs of poles.

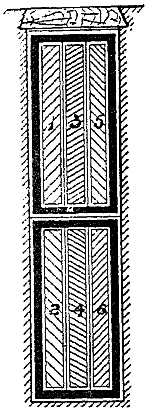


Fig. 130.—Diagram of Slot

With machines of any size, with lap-wound armatures, it is practically impossible to avoid small pressure differences between the circuits of the armature due to unequal strength of the magnetic circuits. These inequalities may be due to eccentric boring of poles, unequal quality of iron in each pole, brackets and projections on yoke, or to various other causes, and may cause serious sparking trouble at the commutator due to the unequal loads on individual lines of brushes.

In order to counteract this, it will be found necessary, with lap winding, to use equalizing rings or cross connectors. These will be used to couple up points in the armature circuits which should be at the same potential, whereby any internal currents generated by the differences of pressure mentioned may be prevented from flowing through the brushes and causing sparking. It will generally be found sufficient if 4 or 5 points in the circuits of an armature between the brushes are coupled up thus.

If 4 points are used there will be 8 rings necessary, and if 5 points 10 rings necessary, and so on. If p be the number of pairs of poles in the machine, each ring will be coupled to the armature at p equidistant points.

In a paper by Professor Arnold in the Electrotechnische Zeitschrift, March, 1902, a full description of the methods of coupling these rings is given. The connections can be carried out at the back of the commutator, at the back of the armature, or on the armature circumference.

In machines built by some German makers, the binding bands on the armature circumference are used as equalizing rings.

Fig. 131 shows a diagram of a wave-connected winding or series winding. The thick black line, as before, shows a section or coil between two commutator segments. A reference to the figure will show that the pitch at each end is always forward in this type of winding, and not alternately forward and backward. Also there are only two paths through the winding from a positive to a negative brush, no matter what the number of poles may be. The quantity p_1, or the number of bifurcations of the armature current, is, therefore, always one with series windings and is independent of the number of poles.

Two lines of brushes only are necessary to collect the current, a great advantage in smaller types of generators and motors. Where, in order to keep down the length of the commutator, as many lines of brushes as there are poles are employed, the advantages of this type of winding are overbalanced by the increased tendency to spark due to unequal division of the current between different sets of brushes. This type of winding is therefore seldom employed for machines above 50 to 60 kilowatts capacity, although below this size excellent results and considerable advantages can be obtained by its use.

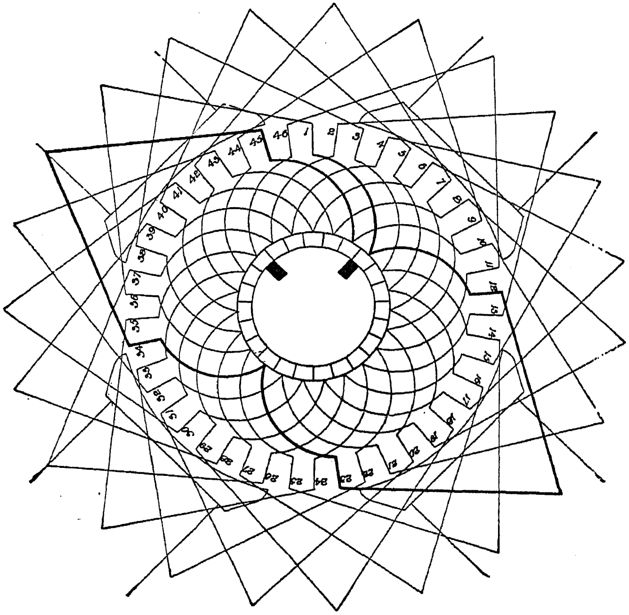


Fig. 131.—Diagram of Wave Winding

The disadvantages increase the greater the number of poles employed, and series-wound armatures are not to be recommended for machines having more than four poles.

The armature drop due to resistance, and also the effect of reaction, is the same for this type of winding as for similar parallel-wound armatures.

The principal conditions governing the use of this type of winding are:—

1. The number of commutator bars must be odd if p, the number of pairs of poles, is even, and must be even if p is odd.

2. If y is the average pitch

$$y = \frac{N \pm 2}{2p}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

where N = total number of conductors,
p = number of pairs of poles.

3. Both back and front pitches must be an odd number.

They may be equal or may differ by 2 in four-pole machines for a given number of inductors.

In using wave windings on the smaller types of machines, and on motors, it will often be found convenient to use a number of slots which is not an even multiple of the number of commutator bars, as, for example, in an armature wound with 198 inductors.

99 commutator bars.
50 slots.

In this case a dead section or 2 dead inductors may be used to fill up. Thus in the above armature there would be four inductors per slot or 200 total, *i.e.* 198 + 2 dead inductors.

CHAPTER III

FIELD-MAGNET DESIGN

The number of different types of field-magnet which have been employed or proposed for dynamos is very great, but in modern electrical generators the types in actual use have become much more uniform.

Figs. 132 A, B, C, D show the main features of practically all the types which are at present employed.

The bipolar type, fig. 132 A, is very little used except for small machines. In multipolar types the general practice for large machines is to employ cast-iron yoke rings with steel poles bolted to them. These are either cast solid or made up of laminated steel plates. In machines with an armature diameter not exceeding, say, 40 inches, the authors believe that better results are obtained by making the yoke ring as well as poles of cast steel, the cast-steel poles being bolted on inside the yoke. Above this size cast-iron yoke rings offer decided advantages, being stiffer and stronger, and not so liable to alter shape after machining as those of steel. For small sizes of generators the authors prefer solid cast-steel poles with bobbins of round section. The saving of wire due to the smaller length of mean convolution with round bobbins in large machines is considerable, and although laminated poles may, and undoubtedly do, tend to equalize the magnetic strength of each pole and to check eddy currents in pole faces, machines can easily be built with solid poles having efficiencies of 93 to 94 per cent, which seems to render the use of laminated poles with their extra expense and inferior mechanical qualities unnecessary.

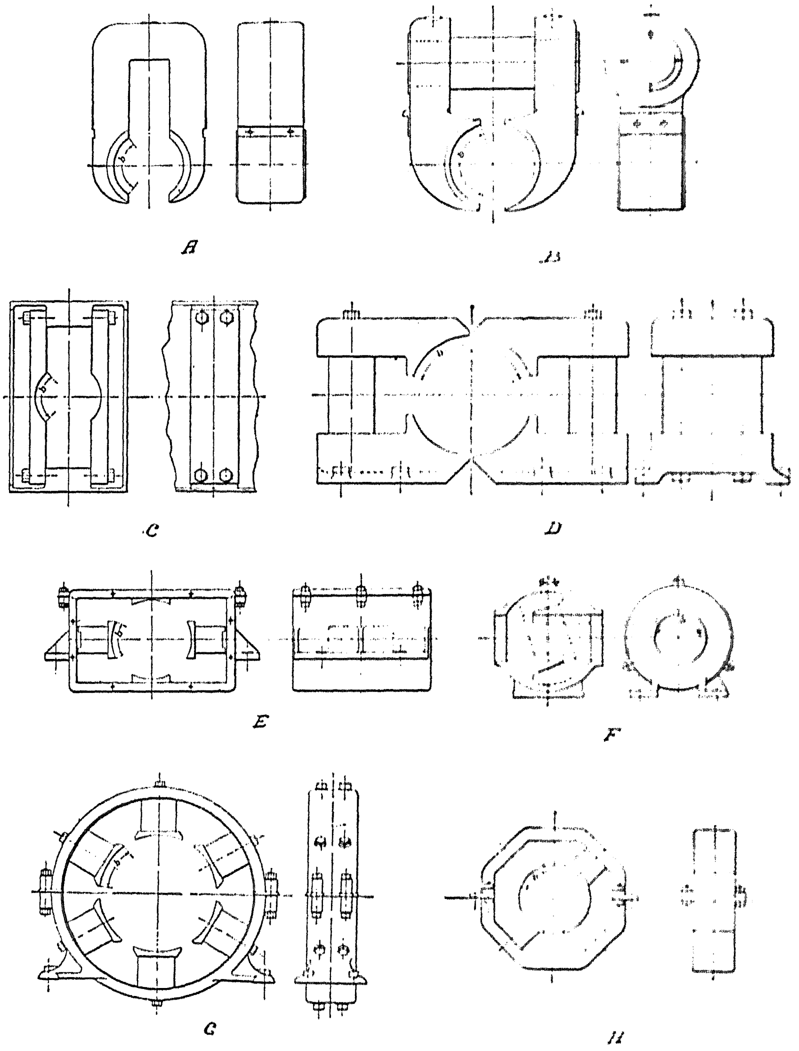


Fig. 132.—Types of Field-Magnets

The pole span (see B in fig. 132) or length of polar arc (on curve) is usually about .65 to .72 of the pitch of the poles. .7 is a good average proportion. The edges of the pole-piece should either be chamfered or rounded off, or it should be made of an elliptical shape, in order to prevent too sudden variations of flux in the teeth when entering under and leaving the pole face. As described in Chapter II, the polar span and length of armature being settled, the area of pole face (*i.e.* the area of the air-gap) is fixed, and, assuming a density in the air-gap of 50,000 to 55,000 lines per square inch, the flux entering the armature from the pole face, or useful flux, is determined. The flux through the magnets will, of course, be greater than this, as magnetic leakage will occur all round the magnetic circuit. The flux is a maximum through the yoke at A (fig. 134), becoming gradually less and less until at the armature surface it is reduced to the useful flux cutting the conductors. It is convenient, however, to assume that the useful flux passes through the armature core, teeth, and air-gap, the leakage all occurring at the pole face, and that a certain increased flux, which we may call the magnet flux, passes through the pole-pieces and the yoke, a coefficient of leakage being taken by which the useful armature flux will be multiplied to get the magnet flux. This assumption simplifies magnet calculations very considerably, and gives quite accurate results for ordinary work.

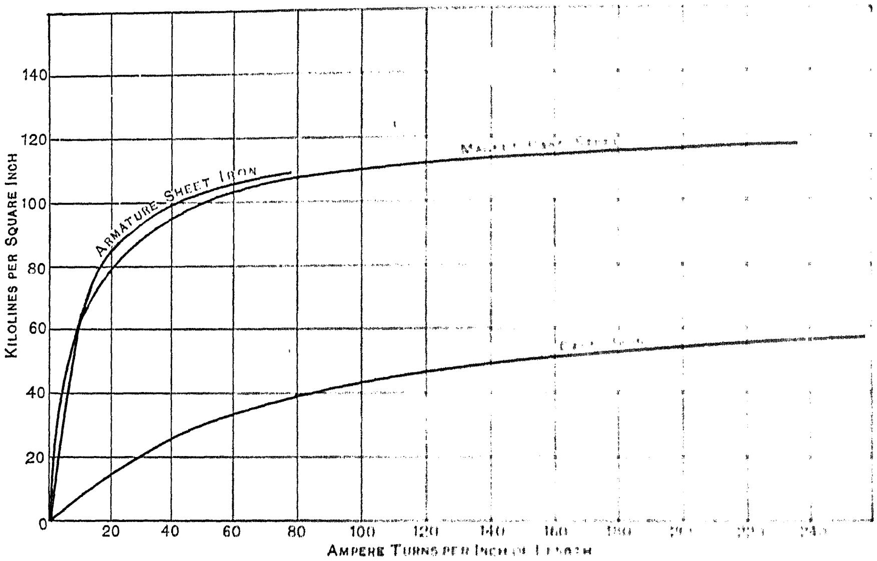


Fig. 133

On types of field-magnets similar to fig. 132, which are those in general use, the leakage coefficient may be taken as varying between 1.16 in machines from 100 kilowatts and upwards to 1.2 in machines of 10 to 15 kilowatts, provided slotted armatures are used. In old-fashioned smooth-core machines this figure varied between 1.3 and 2.

However, on all modern generators of usual type, and of sizes above 100 kilowatts, this value may be taken as 1.16 with good results.

The magnetic densities at full load in lines per square inch usually employed in modern machines, are as given below.

Armature core, sheet-steel ...	...	65,000	lines per square inch
Armature teeth at root	...	135,000	" "
Air-gap	...	55,000	" "
Pole cores, cast-steel ...	..	102,000	" "
Yoke, if cast-steel ...	...	90,000	" "
Yoke, if cast-iron ...	...	45,000	" "

Using the above densities, the area of the different portions of the magnet-frame can be settled. The length of the pole cores left for bobbin space will be settled by assuming a length in comparison with similar machines, and then calculating out, as will be described further on in this chapter, whether sufficient bobbin space has been allowed to accommodate the necessary amount of field winding.

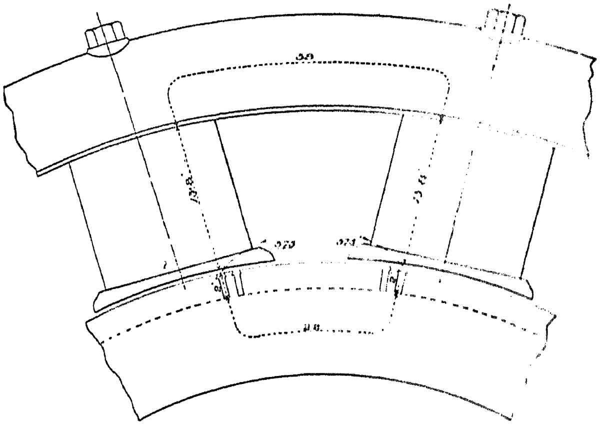


Fig. 134.—Diagram of Magnetic Circuit

Fig. 133 shows curves of ampere turns per inch of length and magnetic density in lines per square inch for different materials used in magnets and armature cores of modern generators. The curves given are taken from good average samples. By means of these curves the ampere turns necessary to drive the magnetism through the different parts of the magnetic circuit can be calculated if the mean lengths of path of the magnetic lines and the densities in each part are known. The sheet-steel curves will be used when considering the armature and teeth parts of the magnetic circuit, and the cast-steel or cast-iron curves for the poles and yoke according to the material of which these parts are made. For the air-gap no curve is necessary, as the magnetic flux in air is directly proportional to the magnetomotive force, which is given in ampere turns per inch of length of path in air by the formula:

$$\text{Ampere turns per inch} = .3133\ B''$$

where B'' = density in lines per square inch.

An example will illustrate the method in the simplest manner. We may take a machine of the dimensions shown in fig. 134.

Assume the volts at light load 230, at full load 240, amperes 1220, speed 90 revolutions per minute, number of inductors 1584, the yoke of cast-iron, twelve poles of cast-steel, and armature plates of mild-steel segments.

The drawing shows the mean path of a magnetic line dotted in. The lengths are as follows:—

Length of path in yoke ring	38	
„ „ poles	13.8 + 13.8	= 27.6″
„ „ air-gap	$\frac{3}{8} + \frac{3}{8}$	= .75″
„ „ teeth	2″ + 2″	= 4″
„ „ armature body ...	... ...	22″

The length of the air-gap is taken from iron to iron, *i.e.* from the top of the teeth to the face of the pole. Twice this amount is taken as the magnetic length.

It can be seen from the figure that the air-gap enters twice into the total path, as do the lengths of pole-pieces and teeth.

As mentioned in Chapter II, the apparent density at the root of the teeth, as figured on the assumption that the whole of the armature flux goes through the roots of the teeth under the pole-piece, must be corrected by the curve (fig. 128) in order to get at the actual tooth density to be taken in the following calculations:—

Assuming a magnetic flux in the above machine of, say, 10 megalines per pole through the armature, and taking a leakage coefficient of 1.16, the flux through the magnets, *i.e.* yoke and pole-pieces, will be 11.6 megalines.

We may then proceed to calculate the ampere turns per magnetic circuit, and to do so it will be convenient to make out a table as follows:—

	Area in square inches.	Density in lines per square inch.	A.T. per inch length.	Magnetic lengths	Total ampere turns.
Magnet ring ...	280	41,250	88	38	3350
Magnet poles...	114	101,000	56	27.6	1550
Armature ...	154	65,000	10	22	220
Teeth... ...	77	{ 130,000 123,700	510	4	2040
Gap	189	52,800	16,600	·75	12,480
					19,640

i.e. for a flux of 10 megalines per pole we require 19,640 ampere turns per magnetic circuit, neglecting for the moment the ampere turns necessary to compensate for reaction of armature.

In working out a new design it will generally be found convenient to plot an excitation curve of the machine, as the ampere turns per circuit can then be read off for any flux.

Fig. 135 shows such a curve for the machine given above. It is only necessary to assume, say, 5 or 6 values of magnetic flux, and using the figures for magnetic lengths and areas in the above table to calculate (using the curves of fig. 133) ampere turns per circuit for each value of flux assumed. Plotting flux and ampere turns we get a curve which is exceedingly useful in predicting the probable behaviour of the machine, as well as in calculating the field windings. Of course, after the machine is built, this curve can be obtained experimentally by running the machine without load at a constant speed, varying the field excitation by means of resistance or otherwise, and taking readings of volts at terminals and excitation ampere turns. The excitation curve can then either be plotted in volts and ampere turns, or volts can be reduced to flux per pole by using formula 2 in another form, as follows:—

$$\phi \text{ per pole} = \frac{V \times 100}{n \times N} \times \frac{p_1}{p}, \quad \ldots \quad \ldots \quad (11)$$

symbols having same meanings as in 2.

The advantage of plotting the curve in flux and ampere turns is that the curve is then a standard curve for that magnet frame and armature, and can be used for that frame, no matter what voltage the armature is wound for.

If the proportions and magnetic densities in magnets, air-gap, and armature recommended in this paper are employed, it will be found

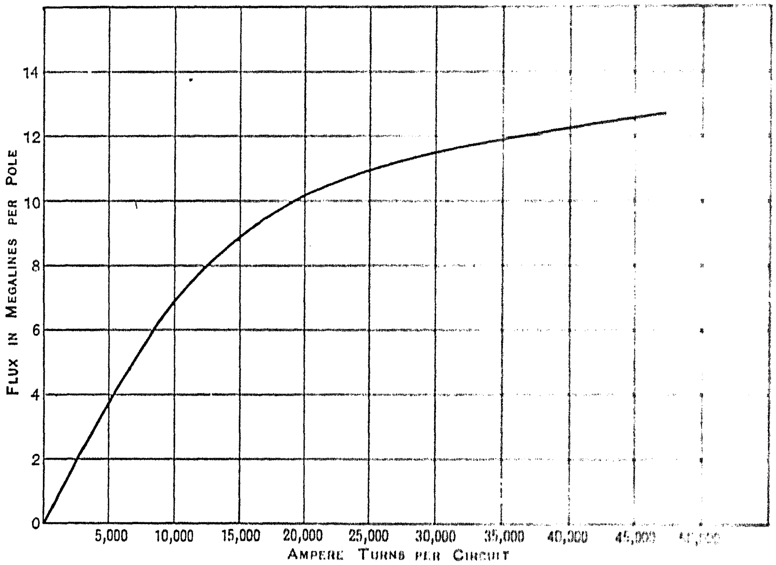


Fig. 135

that the load-flux point will be well up round the bend of the excitation curve. Should this point come below the bend or low down on the bend, the machine will be unstable, and have bad regulating qualities. If compound wound, there will be a considerable hump in the external characteristic curve, the terminal voltage rising very much about half-load and falling away again as full-load is reached.

Using the curve in fig. 135, assuming our generator is to be compound wound, we can determine the size and number of turns of shunt and series windings as follows:—

Our shunt winding will be required to give 230 volts at the terminals on open circuit. The total internal voltage generated by the armature will be equal to the terminal voltage, as there is practically no internal voltage drop due to the armature resistance, because no current flows through the armature except the very small shunt current. The magnetic flux necessary in megalines per pole can then be calculated by formula 11:

$$\text{Light-load megalines} = \phi = \frac{230 \times 100}{1.5 \times 1584} \times \frac{6}{6} = 9.68$$

From the curve (fig. 135) we find that for a flux of 9.68 megalines per pole 18,250 ampere turns are needed for the magnetic circuit. If we decide to allow a temperature rise at the bobbin surface above the ordinary temperature of the atmosphere of, say, 50° F., we must wind on a wire which will give about 11 per cent more ampere turns when cold, *i.e.* at ordinary atmospheric temperature, say 60° F. Therefore 18,250 + 2000 = 20,250 ampere turns to be provided by the shunt at 60° F.

The diameter of wire necessary can be calculated from the ampere turns direct, but, as a square root comes into this calculation, a quicker way to arrive at the diameter is to divide the terminal voltage at the coils by the ampere turns required, and by the mean length of one turn in yards, which gives us the resistance per yard of the necessary wire. From an ordinary wire table we can then find a wire of standard gauge having the nearest resistance per yard to that required. Of course if we are using, as above, ampere turns per magnetic circuit, we must remember that the machine has as many magnetic circuits as there are pairs of poles, *i.e.* p magnetic circuits. In the design of field shown, which is typical of most modern generators, there are two bobbins to each magnetic circuit, therefore the terminal voltage of the dynamo must be divided by p in order to get the voltage across the two coils, which is to be used in calculating the size of shunt wire. In the generator we are considering this works out as under:—

Volts at dynamo terminals = 230

Volts across two coils = $\frac{230}{6}$ = 38.3 volts

Mean length of one turn in yards = 1.3

Resistance per yard = $\frac{38{\cdot}3}{20250 \times 1.3}$ = .00145 ohms

The nearest standard wire to this is .144 inch diameter S.W.G., which has a resistance per yard of .00145 ohm. To allow a little margin, however, we should probably wind on .148 inch B.W.G.

We have next to determine the number of turns of series required on the bobbins in order that the terminal voltage at full load shall be 240 volts.

The total internal voltage generated at full load will have to be equal to the required terminal voltage plus the volts lost due to the resistance of armature, brushes and brush leads, series coils and connections. The volts lost due to armature resistance will be given by the formula—

$$e = \frac{r\,m\,\mathrm{N}\,\mathrm{C}}{4\,p_1^{\,2}}, \qquad \ldots \qquad \ldots \qquad \ldots \qquad (12)$$

where r = resistance per yard of armature inductors,
m = mean length one inductor in yards,
N = total number of inductors,
C = full-load current of dynamo,
p_1 = number of bifurcations of armature current,
e = volts lost due to armature resistance.

The volts lost due to resistance of brush leads and brushes, &c., should not exceed, in generators of over 100 kilowatts output, 1 per cent of the terminal voltage. In most modern machines it is rather under this. In the case of the generator we are considering we have that—

$$r = .000334 \text{ ohm}$$
$$m = 1.08 \text{ yard}$$
$$N = 1584$$
$$C = 1220$$
$$p_1 = 6$$

$$\text{therefore } e = \frac{.000334 \times 1.08 \times 1584 \times 1220}{4 \times 36}$$

$$e = 4.85 \text{ volts}$$

Drop in brush leads, &c., 1% = 2.4 volts
Total drop = 4.85 + 2.4 = 7.25 volts

And the armature must generate a total internal voltage of 240 + 7.25 = 247.25 volts in order that there may be 240 volts at the terminals at full load of 1220 amperes.

We can now calculate the full-load magnetic flux in megalines per pole by again using formula 11.

$$\text{Full load } \phi = \frac{247.25 \times 100}{1584 \times 1.5} = 10.4 \text{ megalines}$$

On again referring to our curve (fig. 135), we find that for a flux per pole of 10.4 megalines we need 21,700 ampere turns per circuit. The shunt and series winding, however, has to provide more than this amount. It has to provide 21,700 ampere turns plus an amount to compensate for the demagnetizing effect of the armature current on the field, and also for the decrease of flux due to the magnetic distortion in the pole-pieces caused by the cross-magnetizing action of the armature. A detailed explanation of these effects is scarcely necessary in this article, but it can be found in most modern books on dynamo design, notably those of Professor S. P. Thompson, Mr. Gisbert Kapp, and Messrs. Parshall and Hobart.

The authors think that it is scarcely necessary to separate these effects in practical work. The constants of the ordinary reaction formula can be increased slightly to allow for the effect of distortion, and this will give quite good practical results.

The ampere turns per magnetic circuit necessary to compensate for armature reaction are given by the formula:

$$\text{Reaction ampere turns} = \frac{K \times a \times N \times C}{360 \times p_1} \quad \dots \quad \dots \quad (13)$$

where K = leakage coefficient,
a = angle of lead of brushes (from neutral position),
N = total number of armature inductors,
C = total current,
p_1 = number of bifurcations of armature current.

The angle α, or angle of brush lead on most modern multipolar generators, can generally be taken as about .3 to .33 of the angle between two adjacent pole tips. On types of two-pole smooth-core machines this angle was generally equal to .5 of the interpolar angle, *i.e.* the brushes were just under the horn of the leading pole-piece. As a matter of fact, in most modern machines, the actual brush lead angle is slightly less than .3 of the interpolar angle as given above; but the above figures may be taken to allow an extra amount for the effect of distortion.

To return to our example, the reaction ampere turns will be equal to—

$$\frac{1.16 \times 2.5 \times 1584 \times 1220}{360 \times 6} = 2600 \text{ ampere turns}$$

These 2600 ampere turns have to be added to the 21,700 ampere turns, which we found from the excitation curve were necessary to drive the load flux of 10.4 megalines through the magnetic circuit, giving 21,700 + 2600 = 24,300 as the total ampere turns per circuit which we must wind on the bobbins to maintain 240 volts at the terminals of the generator when a current of 1220 amperes is being taken from the armature. Of this total our shunt winding already provides 18,250 ampere turns with 230 volts at the terminals. The terminal voltage at load, however, is 240 volts, and at this voltage our shunt winding will provide

$$\frac{240 \times 18250}{230} \text{ ampere turns} = 19000, \text{ say}$$

We assume that the shunt regulator has not been touched, as would be the case in practice. The ampere turns remaining, therefore, to be provided by the series coils are

$$24300 - 19000 = 5300$$

With a current of 1220 amperes the actual number of turns will be

$$\frac{5300}{1220} = (\text{say})\ 5$$

or 2½ turns per bobbin.

We have now to determine if the space we have provided on the bobbins is sufficient to enable enough shunt wire to be wound on (after space has been allowed for the compounding coils) to keep the bobbin surface temperature below the specified limit. To do this we take into account the cooling surface of the bobbin in square inches and the total watts lost in the shunt and series coils. The temperature rise of the bobbins above that of the atmosphere can be taken, within the limits of ordinary working, as proportional to the watt loss per square inch of external surface. On a modern multipolar generator of the type of our example there would be a temperature rise above that of the atmosphere at the bobbin surface of 60° F. per watt per square inch. In smaller machines below, say, 100 kilowatts with less natural ventilation the temperature rise per watt per square inch would be taken as 70° F. to 75° F., and in old types of two-pole generators with very indifferent ventilation and large bobbins the rise per watt per square inch would be from 90° F. to 100° F. The difference in these figures for multipolar and two-pole types is probably accounted for by the fact that in the multipolar types of generator the pole cores and yoke ring, owing to their shape, act as much more efficient radiators to carry off the heat from inside the bobbins than does the iron of the two-pole magnet frame.

A current density in the series coils of 750 amperes per square inch is generally accepted as good practice in modern machines. From this we can fix the section of the conductor of our series coils and, having done so, can calculate the C^2R loss in them in watts. Assume this in our present case to be 106 watts per bobbin. The surface of one bobbin in square inches we will take to be 420 square inches; for a 60° F. rise in temperature above that of the atmosphere we must limit the watts per bobbin lost in both shunt and series to 420 watts. This leaves 420 − 106 = 314 watts as the permissible loss in the shunt wire. The voltage across each bobbin being 20 volts, the necessary resistance of shunt wire to be wound on per bobbin is therefore—

$$\frac{20 \times 20}{314} = 1.27 \text{ ohm per bobbin required}$$

It is then a simple matter to determine if the space left, in our preliminary design, for shunt wire is sufficient.

Had the machine been arranged as a simple shunt-wound generator instead of compound-wound as assumed in the preceding example, the above calculation would have been somewhat simpler. The shunt winding would then have to supply the total ampere turns per magnetic circuit necessary to maintain 240 volts at the terminals at full load. This amount we found to be 24,300 ampere turns. We must, therefore, in calculating the resistance per yard of the shunt wire, use this value instead of the light-load value of 20,250 ampere turns which we used before. Adding 11 per cent as before to allow for increase of resistance due to temperature rise of the bobbin, we get 24,300 + 2680 = 26,980 ampere turns. The resistance per yard of the shunt wire is, therefore—

$$\frac{40}{26980 \times 1.3} = .00114 \text{ ohm per yard}$$

The nearest standard wire to this is .165 diameter, but we should probably use .176 diameter wire in order to give a margin for regulation. If we assume the same temperature rise as before, viz. 60° F., we have, with 420 square inches of bobbin surface, 420 watts as the permissible loss per bobbin. The whole of this is now taken as shunt loss. The necessary resistance per bobbin is, therefore, with 20 volts across each bobbin as before—

$$\frac{20 \times 20}{420} = .95 \text{ ohm per bobbin}$$

If we arrange to wind with .176 wire, which gives 31,100 ampere turns, we need not wind as much as .95 ohm per bobbin of shunt wire, as we shall have a certain amount of the shunt regulator resistance left to reduce the 31,100 ampere turns to the amount we require, viz. 26,980, and this will reduce the voltage across the bobbins to

$$\frac{26980}{31100} \times 20 = 17.4 \text{ volts}$$

The permissible resistance per shunt bobbin will, therefore, be—

$$\frac{17.4 \times 17.4}{420} = .72 \text{ ohm per bobbin of .176 wire}$$

In order to find the amount of resistance we require in the shunt regulator to enable us to reduce the voltage of the generator to, say, 230 volts at no load, we should take from our excitation curve the ampere turns necessary for this voltage—in the present case, 18,250. And as the shunt with no regulator resistance in circuit would give 31,100 ampere turns, and has a resistance of .72 × 12 = 8.64 ohms, the total resistance of the shunt circuit plus regulator in order to reduce the 31,100 ampere turns to 18,250 will be

$$\frac{31100 \times 8.64}{18250} = 14.7 \text{ ohms}$$

The regulator resistance must, therefore, be—

$$14.7 - 8.64 \text{ ohms} = 6.06 \text{ ohms}$$

On most modern compound-wound machines the series and shunt coils are wound separately and arranged with space in between them for ventilation instead of, as used to be the practice, the series coils being wound on the top of the shunt wire. This can be noticed in the illustrations given farther on of machines by Messrs. Crompton & Co., the British Thomson Houston Co. the British Westinghouse Co., and the machines designed by the authors.

It will be seen from the foregoing bobbin calculations that considerable advantage is gained by keeping the mean convolution or mean length of one turn of wire on the bobbins as small as possible. Round bobbins are, therefore, from this point of view, desirable. They are also easier and cheaper to wind than the oblong or square-shaped type. One point must be noticed with regard to the bobbins of modern generators: their dimensions hardly alter on machines of very different outputs and sizes. In the case of old two-pole designs one of the difficulties inherent in the type, when large sizes were reached, was the unwieldy size of the magnet bobbins. On modern generators the number of poles increases as the size of machine increases, and their dimensions and weights of bobbins and pole-pieces alter very little.

CHAPTER IV

COMMUTATOR AND BRUSH GEAR, &c.

The successful operation of modern multipolar generators depends to a large extent on the correct design of commutator and brush gear. The essential points with regard to the commutator to consider are as follows:—

1. The bars must be held in place absolutely rigidly. It should not be possible to drive down a bar by striking the commutator surface with a mallet or hammer.

2. The bars must not be liable to be loosened by unequal expansion of the commutator due to frequent heating and cooling.

3. Sufficient radiating surface must be provided to dissipate the watts lost, due to brush contact resistance and friction, without excessive rise of temperature.

4. It must be possible to reduce the diameter of the commutator by a reasonable amount in the course of successive turnings and grindings without in any way weakening or loosening the bars.

5. The copper bars and the mica insulation between them must wear evenly, to avoid the formation of ridges of mica between bars, which will cause chattering of the brushes and sparking.

As regards the first point, this depends on the goodness of the end clamping arrangements for holding bars.

Fig. 136 shows different types of end-clamps in general use.

Fig. 136, *a* shows an arrangement used by the authors which has given every satisfaction. The top part of the clamping ring is slightly coned in order to give a good true seating for the bars, whilst the clamping down is done by the underside of the ring. This section of clamp-ring was adopted after very careful experiments and with excellent results. The commutator sleeve is of cast-iron, while the end ring is of cast-steel.

Fig. 136, *b* is a design by Mr. Parshall used on a 1600-kilowatt generator, and is arranged so that a segment of the clamp can be removed and repairs effected on a portion of the commutator without disturbing the rest.

The diameter of modern commutators is generally large in comparison with the armature diameter. From a study of the method of computing reactance voltage per segment, given in Chapter II, it will be seen that in order to keep the value of this within the prescribed limits it is necessary to have a large number of commutator segments. In fact, in most modern machines above, say, 200 kilowatts, and wound for voltages of 220 or 550 volts, it will be found that it is impossible to keep the value of reactance voltage per segment within limits unless the maximum number of commutator bars is employed; that is, unless the number of commutator-bars is equal to half the number of armature inductors. This at ordinary speeds, and with the proportion of magnetic surface generally employed, will necessitate a commutator diameter equal to something like .7 to .8 of the armature diameter, as the thickness of the commutator bars at the top must, for mechanical reasons, be not less than .2 of an inch, and should be more if possible. The thickness of mica generally employed between the segments is about

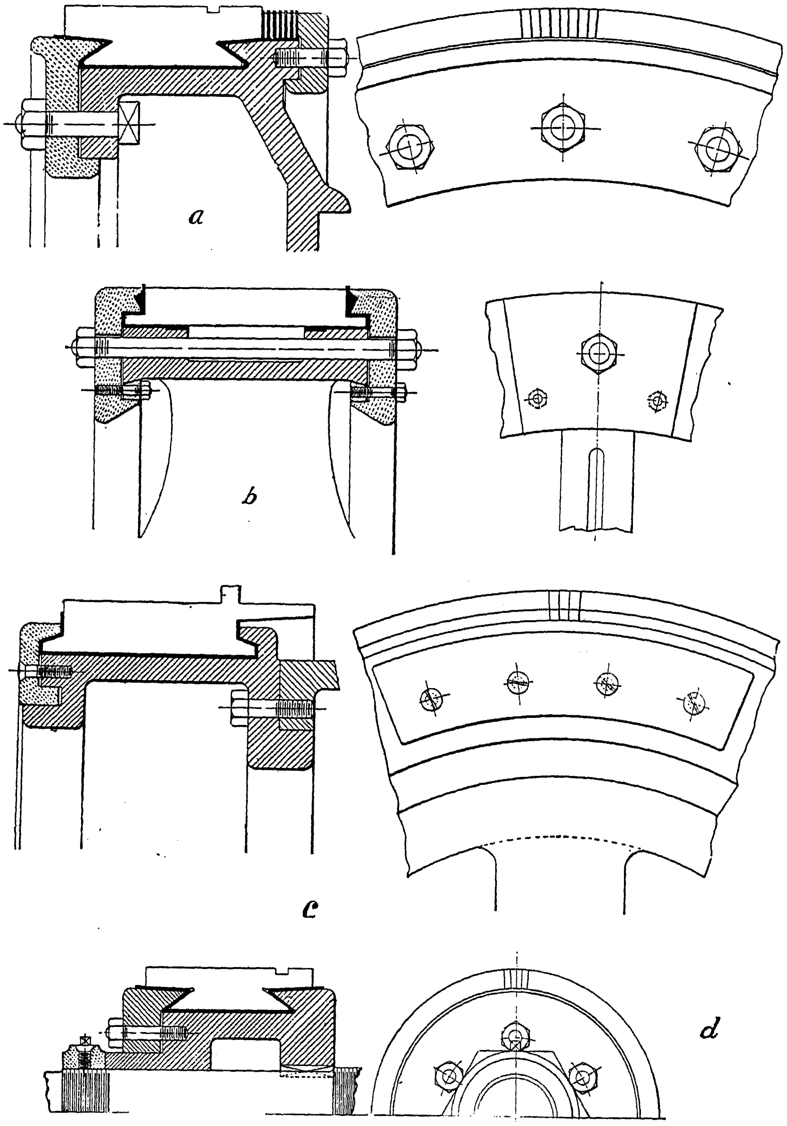


Fig. 136.—Types of Commutator Clamps

.03 of an inch. As a rule pure mica will be found too hard for this purpose, except for small commutators. If it is employed it will generally be found, after some months working, that ridges of mica have formed between the segments, which, if not trued down with an emery wheel, will rapidly get worse and cause sparking to appear at the brushes. Several insulation-makers manufacture a special quality of mica plate for use between commutator segments. Care must be exercised in selecting these materials, however, as, although a few types give excellent results, others are quite worthless.

The insulating end-rings for insulating the commutator end-clamps from the bars are built up of mica strips stuck together by an adhesive material, such as shellac. They are then moulded under hydraulic pressure, the moulds being heated to drive off the shellac used as binding material.

It is a great mistake to make these end-rings too thick, a thickness of one-tenth to one-eighth of an inch being ample for generators wound for 500 to 600 volts. If made much thicker than this the rings are liable to have soft places and ridges on the surface, which will cause trouble due to dropped bars, as the commutator expands and contracts under the changes of temperature it will meet with in practice.

As regards the temperature rise at the commutator surface on most modern commutators the temperature rise per watt per square inch of surface may be taken as from 20° to 25° F. For an average temperature rise above the atmosphere of 60° to 70° F., therefore, we may allow 3 watts loss per square inch of surface of commutator. The losses at the commutator surface are made up of the loss due to friction of the brushes pressing on the commutator and the loss due to the electrical resistance of the brush surface. Carbon brushes press on modern commutators with a pressure of generally about 1½ lb. per square inch of contact surface. Taking a coefficient of friction of about ·3, as given in Messrs. Parshall and Hobart's book, and which accords very well with the results of practical experience, the friction loss in watts per square inch of brush surface works out as .01 time the peripheral velocity of the commutator in feet per minute, a very simple rule.

The C^2R loss at the contact surface of brushes may be taken—with a pressure of 1.5 lb. per square inch and a current density of 30 amperes per square inch, which is the usual density—as 27 watts per square inch.

An example will illustrate the application of the constants given above to determine the probable temperature rise of a given commutator. Of course the results must necessarily be very approximate, as the condition of the commutator and the adjustment of the brushes may, if bad, increase the losses by two or three times the amount figured from the constants given. By the use of these constants, however, we can design a commutator which will, when running under proper conditions, give satisfactory results as regards temperature rise.

Assume data as under:—

Diameter of commutator	60 inches
Rubbing length	10 inches
Peripheral speed	1800 ft. per min.
Area of brush surface per line of brushes ...	8 square inches
Number of lines of brushes	12
Total current of dynamo	1440 amperes

Peripheral surface of commutator = 60 × 3.14 × 10 = 1884 square inches.

Total contact surface, both of positive and negative brushes, = 8 × 12 = 96 square inches.

Watts lost due to friction = .01 × 1800 × 96 = 1730 watts.

Watts lost due to contact resistance of brushes = 27 × 96 = 2590 watts.

Total = 2590 + 1730 = 4320 watts.

Watts per square inch of commutator surface $= \frac{4320}{1884} = 2.28$

Probable temperature rise = 2.28 × 25 = 57° F.

The current density per square inch of brush surface will be

$$\frac{1440}{96} \times 2 = 30 \text{ amperes per square inch.}$$

It will be noticed that while, in computing the watts lost due to friction and contact resistance, the total area of all the brushes is taken, half this only is taken in computing the current density. A little reflection will show why this is so.

As regards the wearing depth of modern commutators this is generally specified as a radial depth of 1 inch, *i.e.* 2 inches may be turned off the diameter before the commutator requires renewing. This is really ample as, when properly operated with carbon brushes, the wear is exceedingly small, and a commutator with the above allowance should last a very long time before renewal is required.

The use of carbon brushes is now general. In fact, it would be impossible to run modern direct-current generators on a traction load with any other type of brush without excessive cost of upkeep of both commutator and brushes. There are many types of brushes made of various materials and designed to give a contact resistance varying between copper and carbon, but none of these are in general use. Some of these varieties may be of service in very special cases, such as machines for plating or welding intended to give large currents at low voltages, and have been so used by the authors, notably on welding machines, but on practically all generators designed for traction and power work carbon brushes will be found used. Extended experience with this type of machine leads the authors to endorse the opinion expressed by Messrs. Parshall and Hobart, that carbon brushes of a hard structure run much better than brushes of a softer, closer quality. The harder type more readily acquires on the contact face a bright steely smooth surface, which enables the machine to run for very long periods with remarkably little wear on either carbons or commutator. In some cases, where the peripheral speeds of commutators have been well above the average, say, between 3000 and 3500 feet per minute, and in consequence brush friction losses have been large, the authors have reduced these losses by treating the carbons. These are heated to a dull red and plunged into machine-oil. The effect of this is to give the brush contact the necessary amount of lubrication without causing smearing of the commutator, and it also has another advantage—that it stops all trace of the grinding noise

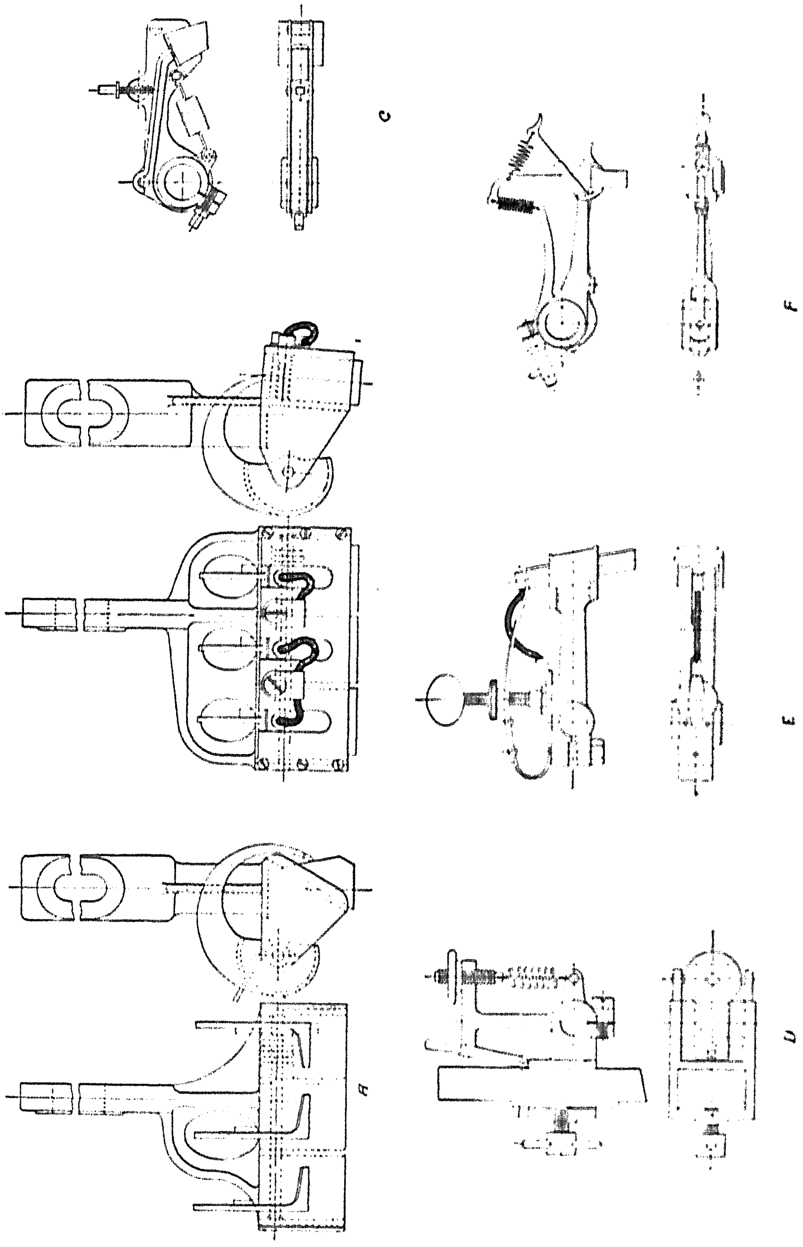


Fig. [illegible].—Types of Brush Holders.

so common with carbon brushes, thus causing the commutator to run very sweetly.

Other makers boil their carbons in paraffin-wax, which has much the same effect as the above treatment, but has the disadvantage that it cannot be so readily and easily carried out by the ordinary dynamo attendant as can the first-mentioned process.

In looking at the modern generator one point which becomes apparent is the great attention given nowadays by makers to the design of the brush-holders and gear. The tendency here, as in other points of design, is towards simplicity and at the same time solidity.

Fig. 137 shows various designs of brush-holders used on modern machines. Fig. 137, A and B illustrate types used by the authors on their machines. The reaction type, as shown in fig. 137, A, gives very good results when used on a commutator running quite true and having no movement due to vibration. But in large generators, coupled direct to the crank-shaft of slow-speed engines, however true the commutator may be turned, there is always a slight eccentric movement due to the working of the crank-shaft in the bearings causing the brushes to pump up and down slightly. In the reaction type brush this constant movement has in time a bad effect on the contact face of the holder, and causes heating trouble. In the type shown in fig. 137, B, designed by the authors, this defect is remedied by the addition of a flexible connection between the carbon block and the holder.

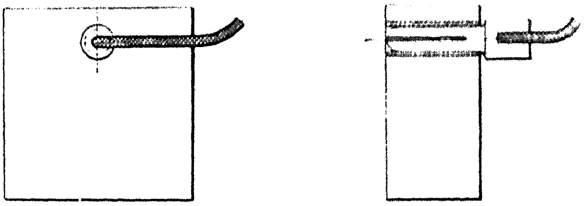


Fig. 138.—Sketch of Carbon Blocks

The carbon blocks are of the shape shown in fig. 138. A small brass tube is let into the block; into this a split pin slides; and from this split pin a flexible connection is carried away to the holder. When a new carbon block has to be inserted the split pin is pulled out of the old block, which is then removed from the holder, a new block is slipped in place and the split pin reinserted into the brass tube with which the block is fitted.

This type of holder has the great advantage that the moving parts are as light as possible, thus lessening the chance of sparking occurring, due to vibration or to roughness or unevenness of the commutator surface. Also the whole length of the commutator is utilized, giving increased efficiency and avoiding the formation of ridges on the commutator periphery between the brushes.

The type of brush most favoured on the Continent is that known as the girder or finger type, as shown in fig. 137, C. Very good results can be obtained from this type of brush, but it has several disadvantages. The distance between the carbon block and the brush spindle brings the end of the brush spindle opposite and close to commutator lugs, which are at a considerably different potential from it. If care is not taken in arranging the brush gear there is a liability to flashing over between these when the machine is subjected to the heavy and sudden overloads to which traction and power generators are liable.

Elaborate arrangements, such as used to be invariably seen on generators, for raising the brushes from the commutator and holding them clear are nowadays hardly ever fitted, and in fact are quite an unnecessary complication. Modern commutators, if properly looked after at the starting of the machine, require very little attention beyond

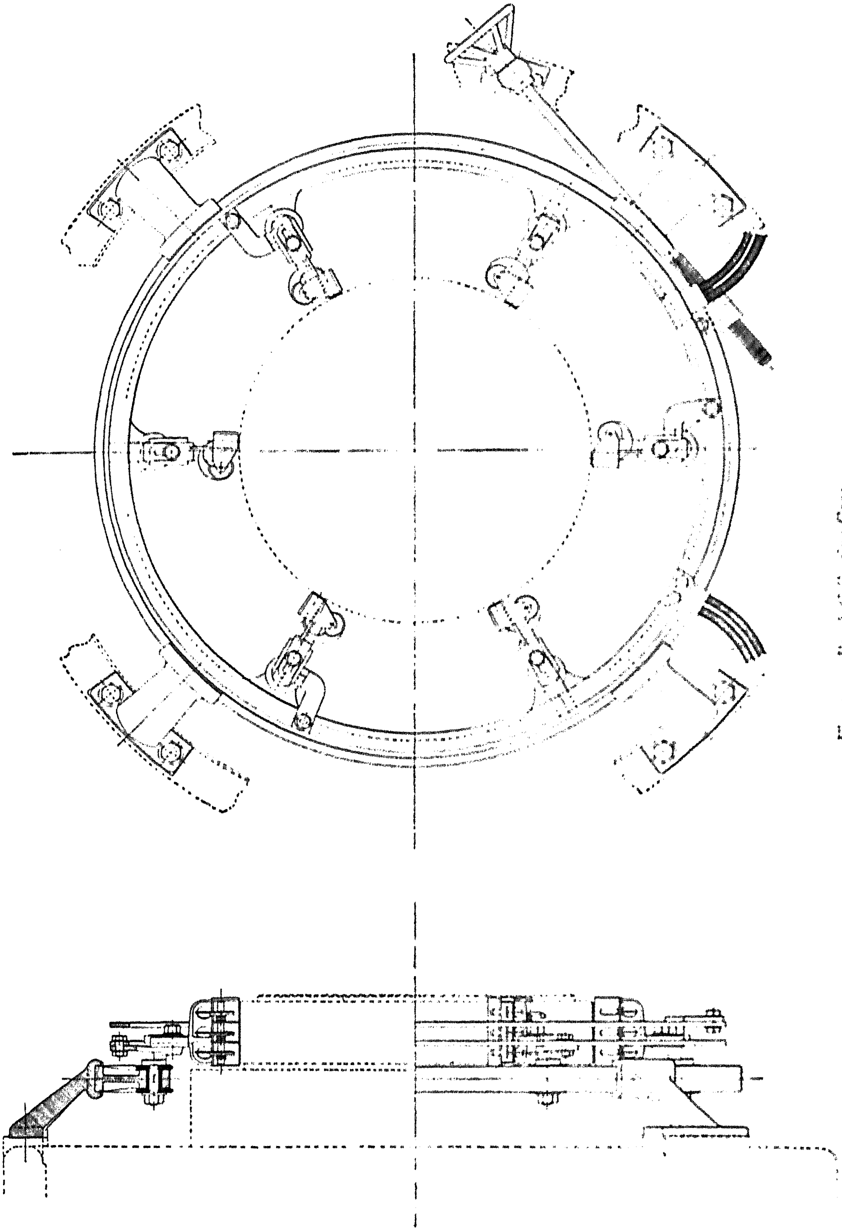


Fig. [illegible].—Brush Collecting Gear

occasional cleaning with a dry rag during running. At long intervals they may require grinding up in position with an emery grinder, but the necessity for frequent raising of the brushes and rubbing the commutator with emery cloth has quite disappeared, and with it the need for complicated hold off catches. There are nevertheless to be found consulting engineers who still specify that these are to be fitted.

The general practice in modern machines is to support the brush-holder ring carrying the collecting gear from the magnet yoke by means of brackets bolted to facings on the yoke. Many makers, notably those on the Continent, support the collectors by a spider carried from the bearing.

The authors think the former plan, as shown in fig. 139, to be much the best. The machine is then self-contained, and the same machine can be used as a direct-driven generator coupled to high-speed engine, as fig. 147, or arranged on the crank-shaft of a slow-speed type of engine, as in fig. 143, or arranged for rope driving, fitted with three bearings. The method of supporting the brush gear from the magnets leaves the commutator free and open for inspection, a point which all central-station engineers, or anyone responsible for the running of large generators, will readily appreciate.

CHAPTER V

EXAMPLES OF MODERN GENERATORS

Fig. 140 shows a direct-current engine-type generator designed by the authors for direct coupling to high-speed engine. This machine has an output of 650 amperes at 230 volts, or 150 kilowatts at 380 revolutions, and the principal dimensions are as follows:—

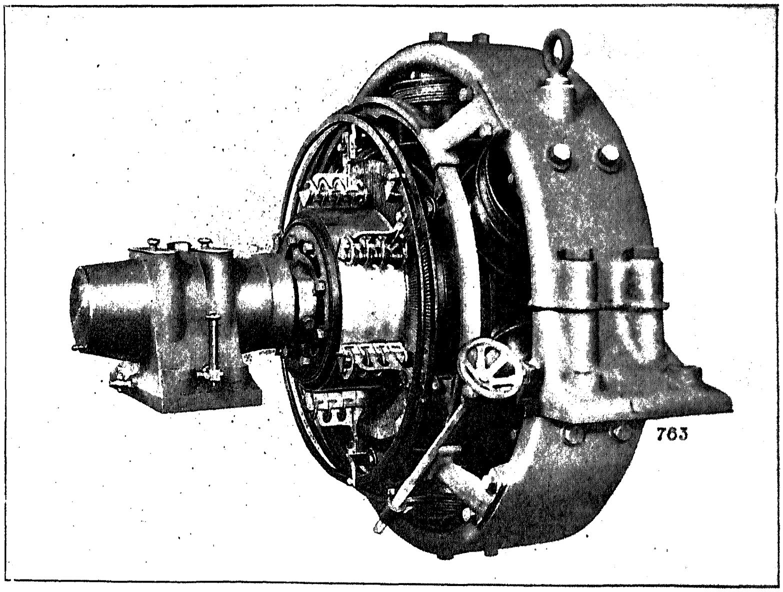
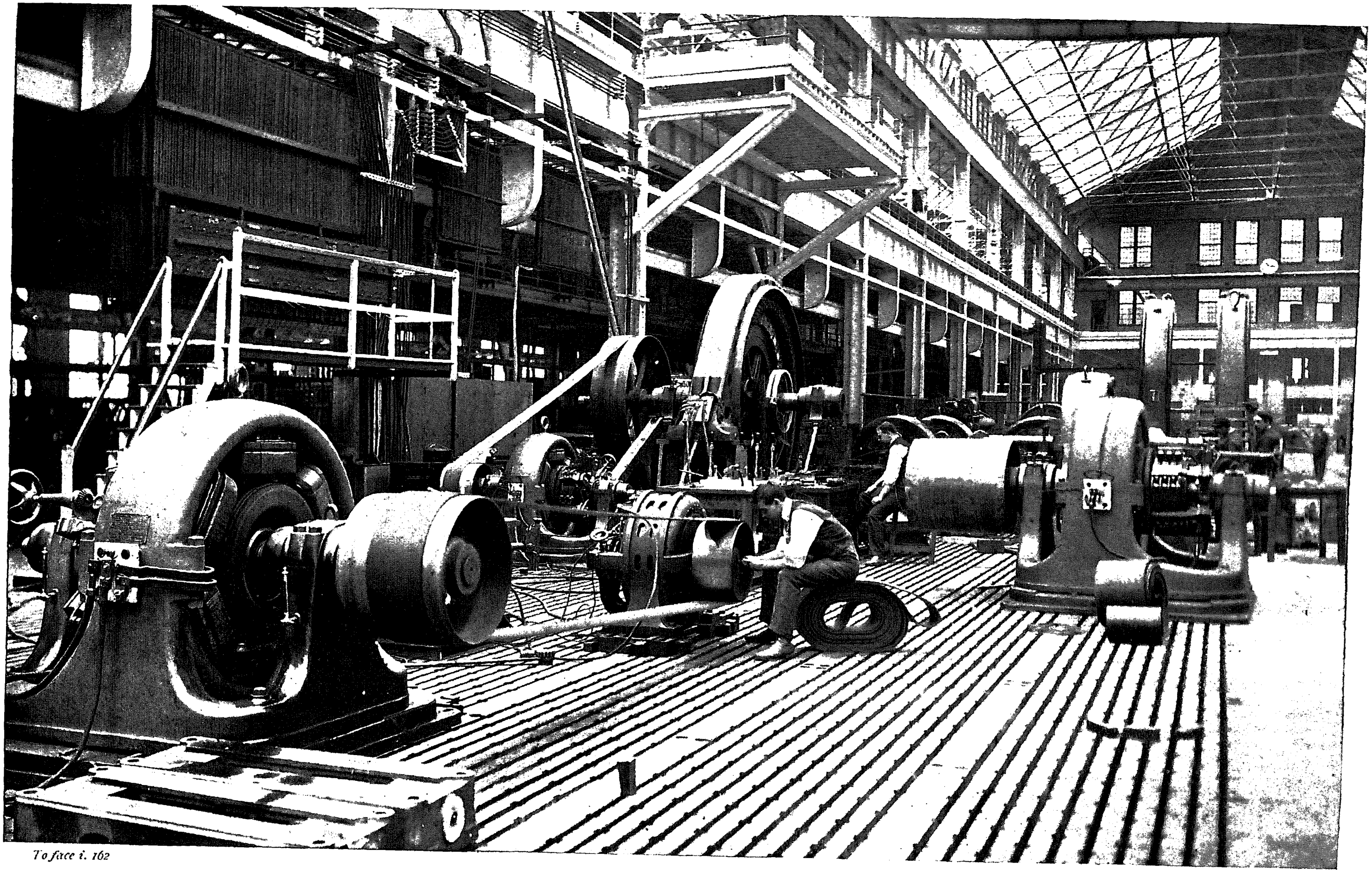


Fig. 140.—Direct-Current Engine-Type Generator



To face i. 162

DYNAMO TEST PLATE (Westinghouse)

Number of poles	6
Armature, diameter over all	36 inches
„ length over conductors	24·75 „
„ diameter at bottom of slots	32.8 „
„ internal diameter	21.25 „
„ length over iron	12 „
„ effective length, magnetic iron	9·9 „
„ depth of slot	1.6 inch
„ width of slot	·45 „
„ number of slots	124
„ width of tooth at root	.384 inch
„ depth of conductor	·7 „
„ width of conductor	.125 „
„ space factor	.486 „
„ number of ventilating ducts	2
„ width of each duct	.5 inch
„ core	mild steel
„ spider	cast-iron
„ yoke	cast-steel
Magnet poles	„
Brushes	carbon

Magnetic Core—

Length of pole face	12 inches
Length of pole arc	13·3 „
Pole arc ÷ pitch	·7
Radial length	11.75 inches
Diameter	10.5 „
Bore of field	36.625 „
Depth of air-gap	.3125 inch
Outside diameter of yoke	66.5 inches

Commutator—

Diameter	28 inches
Rubbing length	10 „
Number of segments	248
Width of segment at top	.3250 inch
Thickness of mica insulation	.03 „

Brushes—

Number of sets	6
Number in one set	5
Contact size of each brush	2 square inches
Number of inductors	496
Inductors per slot	4
Style of winding	lap or six-circuit single
Mean length of inductor	1 yard
No load voltage	230
Resistance between brushes	.00402 ohm
Current density in inductors	1118 amps. per sq. inch
Average voltage between commutator segments	5·6

Armature ampere turns	5
Ampere turns to overcome armature reaction	1710
Reactance voltage per segment	3.28
Armature flux (full load)	8.1 megalines
Leakage coefficient = K	1.15
Flux in magnet cores	9.3 megalines
Section of armature	114 square inches
Magnetic length through armature	14.75 inches
Area of teeth under one pole	63 square inches
Magnetic length of teeth	3.2 inches
Area of gap	173 square inches
Length of gap	.625 inch
Section of magnet core	88.6 square inches
Magnetic length of core	23.5 inches
Area of yoke	67 square inches
Magnetic length through yoke	33 inches

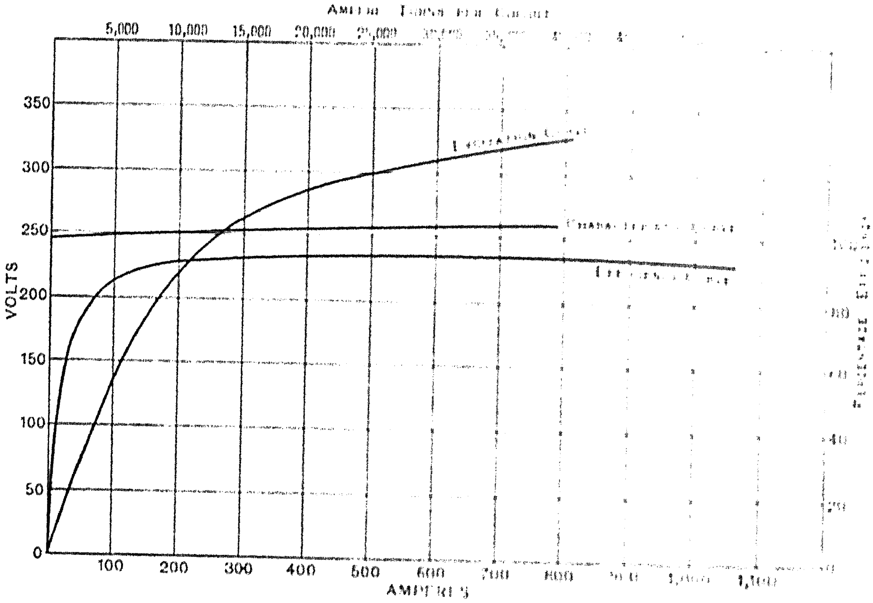


Fig. 141

Bobbin Windings—

Ohms per bobbin (shunt)	10
Size of wire	.080
Number of shunt turns per bobbin	1030
Cross section of series wire	.8 square inch
Number of series turns per bobbin	$3\frac{1}{2}$
Ampere turns (shunt) per circuit	12533
Ampere turns (series)	4550
Total ampere turns at full load	17083

This machine may be taken as a typical modern generator of moderate size for power work. The machine was designed for use in a steel works in which were several electric cranes. Two of these dynamos were arranged to run in parallel. There was also a heavy hoist at the blast-furnace fitted with a motor which frequently demanded as much as 500 to 550 amperes suddenly from the circuit; the variation of the engine speed from no load to full load is 5 per cent. The dynamos were arranged to overcompound 5 per cent, *i.e.* the voltage at the bus-bars rose from 230 volts at light load to 240 volts full load.

Curves of performance of this machine are given in fig. 141. It will be noticed from the efficiency curve that the commercial efficiency of the machine is 94 per cent at full load, and is very nearly constant down to below half-load, a property which is of great value in the case of machines working on fluctuating loads, such as are met with in traction power-stations.

The probable temperature rises at the end of six hours' run, as calculated on the lines given in the preceding chapters, are as follows:—

Armature—

Watts lost due to armature copper resistance	1680
Watts lost due to iron core and teeth	3800
Total	5480

Armature surface = 2800 square inches

$$\text{Watts per square inch} = \frac{5480}{2800} = 1·96$$

taking 27° F. rise per watt per square inch, probable rise at armature surface

$$= 1·96 \times 27 = 53° \text{ F.}$$

Field Coils—

Watts per bobbin to be dissipated = 241
Surface per bobbin in square inches = 270
= ·89 watt per square inch

Probable temperature rise

$$= ·89 \times 60 = 53.5° \text{ F.}$$

Commutator—

Watts lost due to brush resistance and friction = 3300
Commutator surface = 880 sq. inches
= 3.75 watts per square inch
= 3.75 × 20 = 75° F. rise

The actual value obtained on test were as given below:—
Armature, 49° F. rise after six hours' run.
Field bobbins, 43° F. after six hours' run.
Commutator, 62° F. after six hours' run.

The greatest discrepancy between calculated and observed results arises in the case of the commutator temperature, but this is hardly to be wondered at, for, as mentioned in Chapter IV, the condition of the commutator and adjustment of the brushes may increase or diminish the losses considerably, and so affect the temperature rise.

Fig. 142 gives a section through the centre of the machine, showing the details of construction of the armature and commutator. The spider,

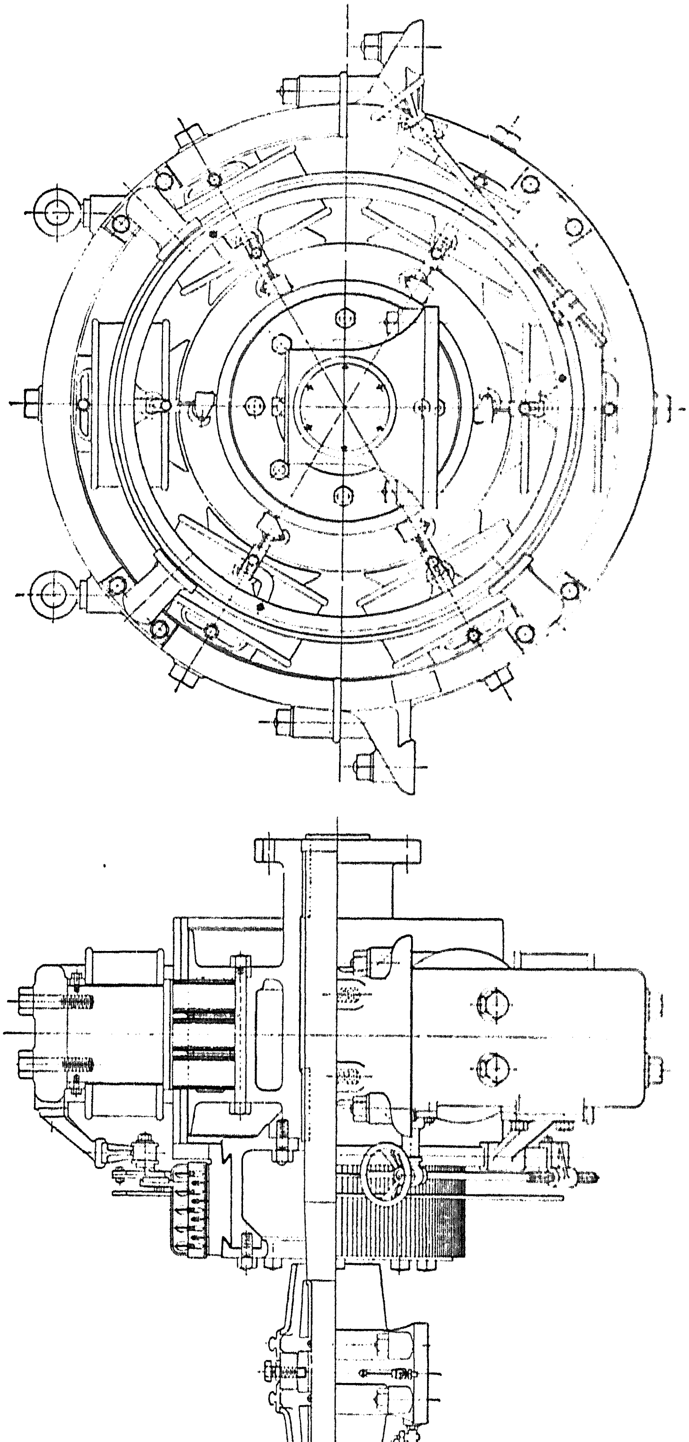


Fig. 142.—Arrangement of 150 K.W. Generator

which is of cast-iron, is of very solid construction, keyed to the shaft by two keys driven in from both ends. One of the armature end-plates clamping the core discs, and also supporting the windings, is cast solid with the spider; the other is drawn by turned steel bolts, which also form keys for driving the core discs. The half-coupling for bolting to the engine fly-wheel is cast solid with the spider, thus the torsional stresses between the fly-wheel and armature are taken directly by the spider and not by the shaft. The commutator sleeve is of cast-iron with a heavy cast-steel end clamping ring. The commutator centre is bolted to a flange cast on the spider, making a very solid and lasting construction.

This last point, more especially on large motors driving pumps or haulage by means of spur-gearing, is of great importance, as the connection between the commutator centre and the armature cannot be too solid, otherwise trouble will be experienced with the coupling lugs connecting the commutator bars and inductors.

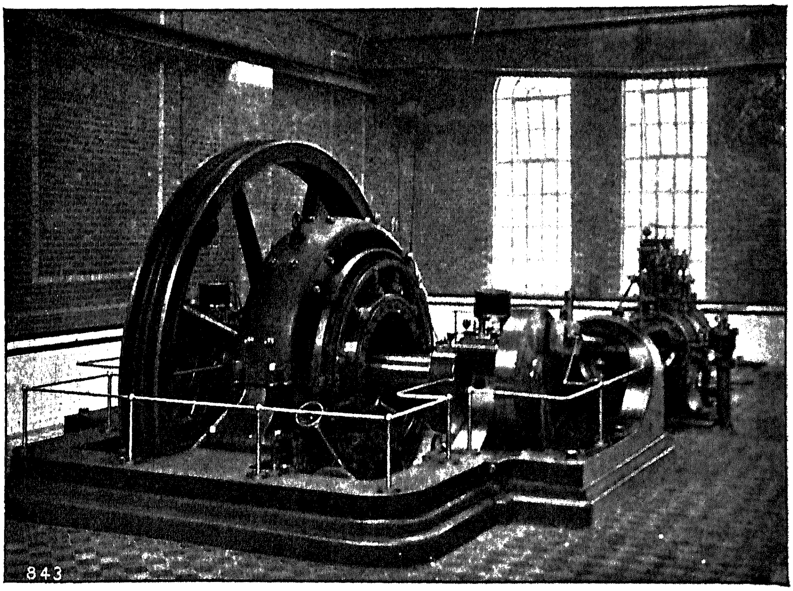
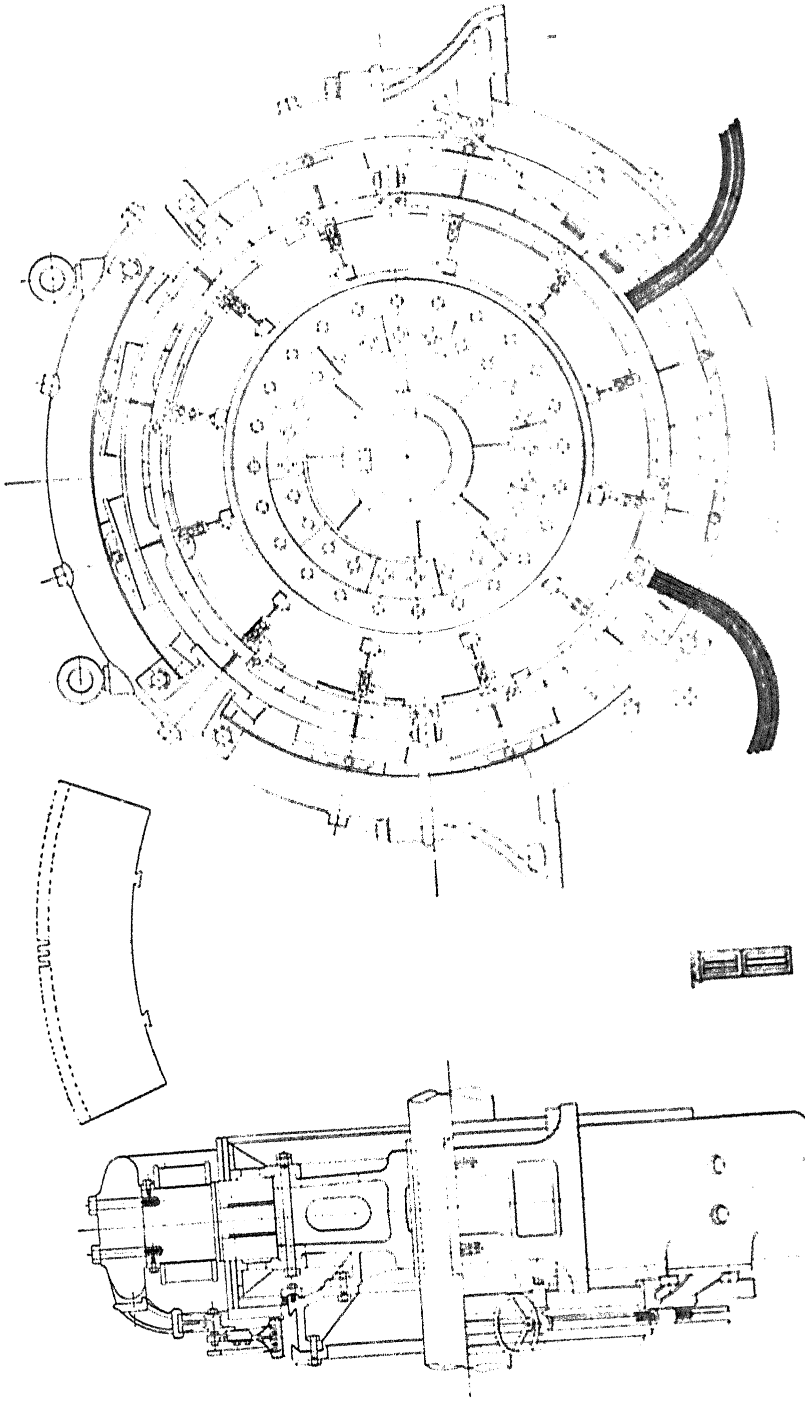


Fig. 143.—300 K.W. Crank-Shaft Generator

The brush gear is supported from the yoke by cast-iron brackets. The rocker ring is adjustable by means of a hand-wheel and screw. The brushes, however, can be set so that they do not require adjustment from no load to 25 per cent overload. The hand-wheel and screw, therefore, could very well be dispensed with, but are fitted simply in accordance with the usual practice.

The outer bearing is of the self-aligning type, with white-metal bearing surfaces. Continuous lubrication is provided by means of oiling-rings. This type of bearing is typical of modern standard practice, and is of the type almost universally adopted on this class of machine.

Fig. 143 is a photograph of a slower-speed type of generator designed



by the authors for coupling on to the crank-shaft of a large horizontal engine. This is a type which is very much in use in modern tramway power-stations both on the Continent and in this country. On the Continent the engines are generally of the horizontal type, as shown in the photograph; but in this country and in the United States the majority of the generators for this class of work are coupled to slow-speed engines of the vertical type.

The general features of this machine are much the same as those of the machine just described, but on a larger scale. For instance, the brush gear is supported from the yoke, the commutator centre is bolted direct to the armature spider, &c.; but it will be noticed that the yoke ring is of cast-iron in this case instead of cast-steel. As already mentioned in a former chapter, it is becoming the practice in the case of machines of this size to use cast-iron in the construction of the yoke ring, as a cast-steel yoke ring, when designed with proper regard to the magnetic proportions of the rest of the machine, becomes weak from a mechanical point of view and apt to alter its shape after being erected.

The magnetic density in lines per square inch in the cast-iron is usually taken as from 40,000 to 45,000, or about half that employed in a cast-steel yoke.

A reference to the sectional drawings shown in fig. 144 will show that the armature core is built up of segmental plates. Each segment forms one-twelfth of a complete circle, and has two dove-tail projections left on the inside edge, which slip into dove-tail grooves machined out on the periphery of the cast-iron spider, so keying the plates to it. Where the edges of the segments butt together, $\frac{1}{50}$th of an inch is left for working clearance. With good stampings it should never be necessary to file or tool the slots. Nothing more than drifting up by means of folding drifts driven into the slots should be necessary in order to make their sides dress up smooth. If tooling or filing has to be resorted to, trouble will almost certainly be experienced from heating in the core.

It will be seen from fig. 144 that the tops of the slots are recessed, and wood wedges driven in to keep the inductors in place.

This method of holding the inductors in large machines is now general. Some makers, indeed, use it even on very small machines of less than 20 kilowatt capacity. It has the great advantage that binding bands can be dispensed with and repairs can be easily effected, as the removal of the wood wedges is not a great matter, whereas the rewinding of several bands on large armatures is a troublesome and lengthy job. The authors have used both hornbeam and beech for the wedges in the slots; both give very good results, but on the whole they are inclined to favour the use of beech, which is also more easily obtained in this country.

The dimensions and technical data of the machine described in the foregoing paragraphs are as follows:—

Number of poles	12
Kilowatts	300
Amperes	1250
Terminal volts	240
Revolutions per minute	90
Armature core	mild steel
Armature spider	cast-iron

Yoke … … … … … … …	cast-iron
Magnet poles … … … … … …	cast-steel
Brushes … … … … … … …	carbon
Armature, diameter over all … … … … …	78 inches
" length over conductors … … … …	27½ inches
" diameter at bottom of slots … … …	74 "
" internal diameter of core … … …	60 "
" length over discs … … … …	13 "
" net length, magnetic iron … … …	10.8 "
" depth of slot … … … …	2 "
" width of slot … … … …	·45 inch
" number of slots … … … …	264
" width of tooth at root … … …	·43 inch
" size of conductor … … … …	·8 × ·09 inch
" space factor … … … …	·48
" number of ventilating ducts … … …	2
" width of each duct … … …	·5 inch
Magnet core—	
Length of pole face … … … … …	13 inches
" pole arc … … … … …	14.5 "
Pole arc ÷ pitch … … … … …	·7
Radial length of magnet pole… … … …	12.25 inches
Diameter of magnet core … … … …	12 "
Bore of field … … … … … …	78.75 "
Depth of air-gap … … … … …	·375 inch
Outside diameter of yoke … … … …	118 inches
Commutator—	
Diameter … … … … … …	60 inches
Rubbing length … … … … …	6 "
Number of segments … … … … …	792
Width of segment at top … … … …	·2072 inch
Thickness of mica insulation … … … …	·03 "
ushes—	
Number of sets … … … … …	12
Number in one set … … … … …	3
Contact size of each brush … … … …	2 × 1 inches

General Particulars

umber of inductors … … … …	1584
ductors per slot … … … …	6
pe of winding … … … …	12-circuit lap winding
n length of inductor … … …	1.08 yard
ad voltage … … … …	230
ance between brushes … … …	.0039 ohm
density in inductor … … …	1447 amps. per square inch
voltage between segments … …	3.6 volts
ampere turns … … …	13750
ead … … … …	3 deg

Ampere turns to overcome armature reaction	3166
Reactance voltage per commutator segment	3.2 volts
Armature flux, full load	10 megalines
Leakage coefficient = K	1.15
Flux in magnet coils	11.5 megalines
Section of armature	154 square inches
Magnetic length through armature	18.7 inches
Area of teeth under one pole	77 square inches
Magnetic length of teeth	4 inches
Area of gap	189 square inches
Length of gap	.75 inch
Section of magnet core	114 square inches
Magnetic length of core	24.5 inches
Area of yoke	280 square inches
Magnetic length through yoke	30 inches

Bobbin windings—

Ohms per bobbin (shunt)	1.25
Size of wire	.148 diameter
Number of turns of shunt per bobbin	660
Cross section of series wire	1.35 square inch
Number of turns per bobbin (series)	3½
Ampere turns (shunt) per circuit	20,333
Ampere turns (series) per circuit	8533
Total ampere turns per circuit	28,866

This machine is arranged to stand 25 per cent overload for a period of three hours, and is therefore in output lower than it would otherwise be. The calculated probable temperature rises at the end of a six hours' run were as follows:—

Watts lost due to armature copper resistance = 6000
" " iron core and teeth = 4000
Total 10,000

Armature surface = 6400 square inches = 1·56 watt per square inch
Probable rise at armature surface = 1.56 × 27 = 42° F.

Field coils—

Watts per bobbin to be dissipated = 442
Surface bobbin, square inches = 460 = .96 watt per square inch
Probable temperature rise = .96 × 60 = 57.5° F.

Commutator—

Watts lost due to brush resistance and friction = 2950
Commutator surface = 1130 square inches
= 2.6 watts per square inch = 52° F.

The actual values obtained were—

Armature	48 F.
Field	53° F.
Commutator	62° F.

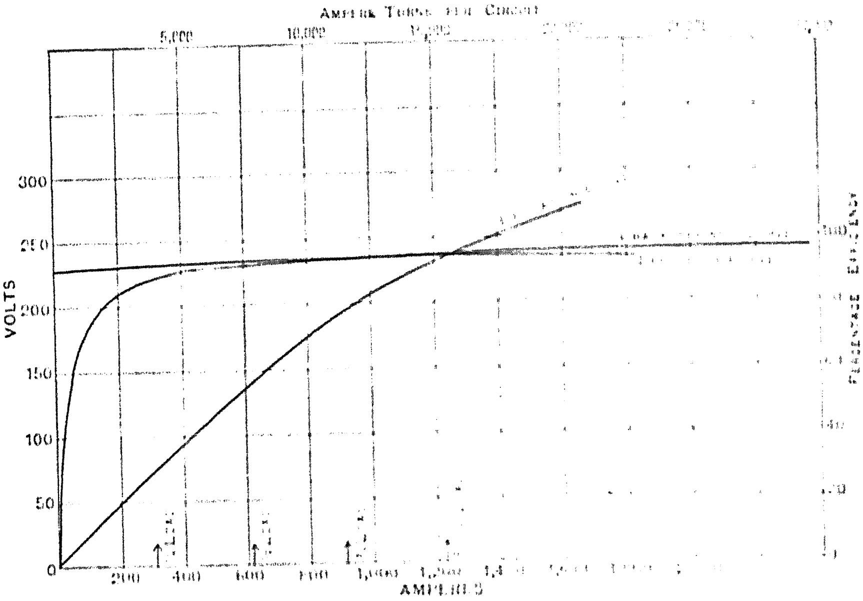


Fig. 145

Fig. 145 shows the curves of performance of the machine. It will be noted that the external characteristic curve of volts at terminals and current output in all these machines is slightly convex or "hog-backed", instead of being quite straight from no load to full load. This is due to two causes. The portion of the excitation curve over which the machines are worked is not quite straight, and also the field ampere turns necessary to compensate for armature reaction are not quite proportional to the load at heavy loads. The curve in fig. 146 was taken from the actual machine illustrated in fig. 147, and gives a rough idea of how the reaction ampere turns necessary on the field vary with the load. The curve was taken with constant speed, constant terminal voltage, varying load and field excitation, and fixed-brush position.

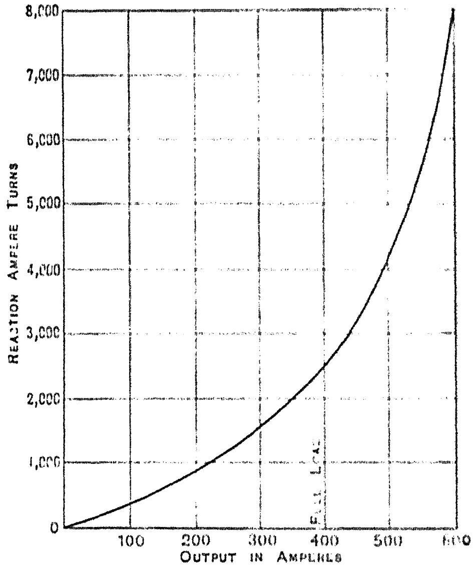


Fig. 146

The effect of the "hog-backed" shape of the characteristic curve is not very important, unless the convexity gets very great. If this is so, it will probably be found that the machine is worked too low down on the excitation curve, *i.e.* that the magnetic densities in the yoke, poles, &c., are too low.

To give an example of the working of the method described in Chapter I of proportioning the length and diameter of multipolar machines, the photograph in fig. 147 illustrates a machine designed by the authors a few years ago, and generally similar in type to that shown in fig. 140, although differing somewhat in a few details. It is shown coupled to a Scott & Mountain enclosed high-speed engine, with forced lubrication, running at 325 revolutions per minute.

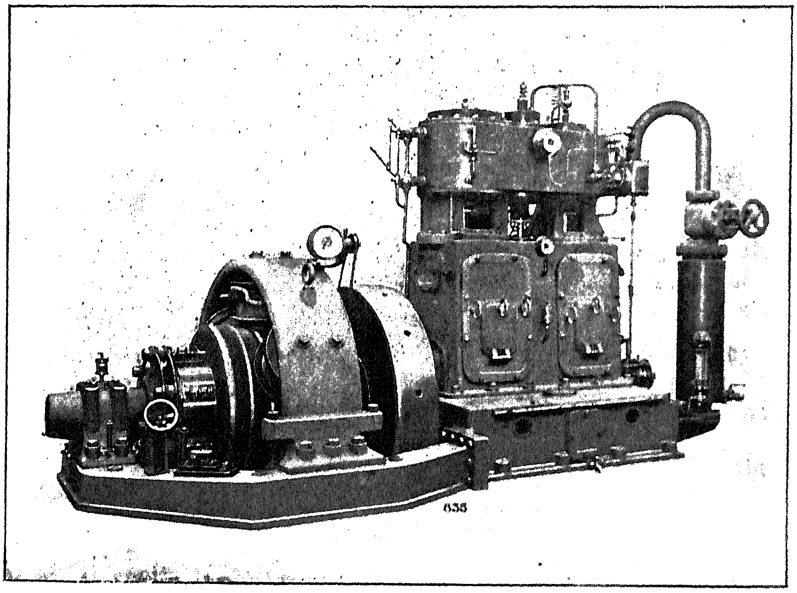


Fig. 147.—200 K.W. Combined Plant

A few particulars of this machine are given below, and it will be seen on reference to these that the length of the armature over core discs, gap-area, magnetic flux per pole, radial depth of armature, are exactly the same as those of the crank-shaft generator just described.

Number of poles	6
Kilowatts	200
Revolutions per minute	325
Terminal volts	500 to 600
Amperes	335
Armature—	
Diameter over all	39 inches
Internal diameter	22 ,,
Length over iron	13 ,,
Effective length of magnetic iron	10.8 ,,
Number of ventilating ducts...	2
Width of each duct	.5 inch
Magnet core (cast-steel)—	
Length of pole face	13 inches
Length of pole arc	14.5 ,,

Magnet core (cast-steel) (continued)—

Pole arc ÷ pitch	·7
Radial length	12.25 inches
Bore of field	39.625 "
Depth of air-gap	.3125 inch

Yoke (cast-steel)—

Outside diameter	72 inches

Commutator—

Diameter	28 inches
Rubbing length	6 "
Number of segments	480

Brushes—

Number of sets	6
Number in one set	3
Contact size of each brush	1.65 square inch

Armature no-load voltage	500
Number of inductors	960
Inductors per slot	6
Style of winding	6-circuit single or lap
Mean length of one inductor	1.08 yard
Resistance between brushes	.00925 ohm
Average voltage between commutator segments	7·5 volts
Armature ampere turns	8940
Ampere turns to overcome armature reaction	1370
Reactance voltage (full load)	2.8

Bobbin windings—

Ohms per bobbin	34
Size of wire	.058 inch
Number of shunt turns per bobbin	3154
Section of series wire	.33 square inch
Number of series turns per bobbin	11½
Ampere turns per circuit (shunt)	18,500
Ampere turns per circuit (series)	7716
Total ampere turns per circuit (full load)	26,216

The number of poles is 6; the diameter of the armature, 39 inches; and length, 13 inches. In the case of the large machine the length of the armature is the same, but the diameter is 78 inches, or 39 × 2 inches, and there are twelve poles of the same size as in the smaller machine. As mentioned in Chapter I, if tests and figures are available relating to the performance of the 39-inch machine, the probable behaviour of the 78-inch machine can be predicted with considerable accuracy. In the same way the behaviour of a machine having, say, 20 poles and an armature 130 inches diameter and 13 inches long could be predicted from the tests of the 39-inch machine.

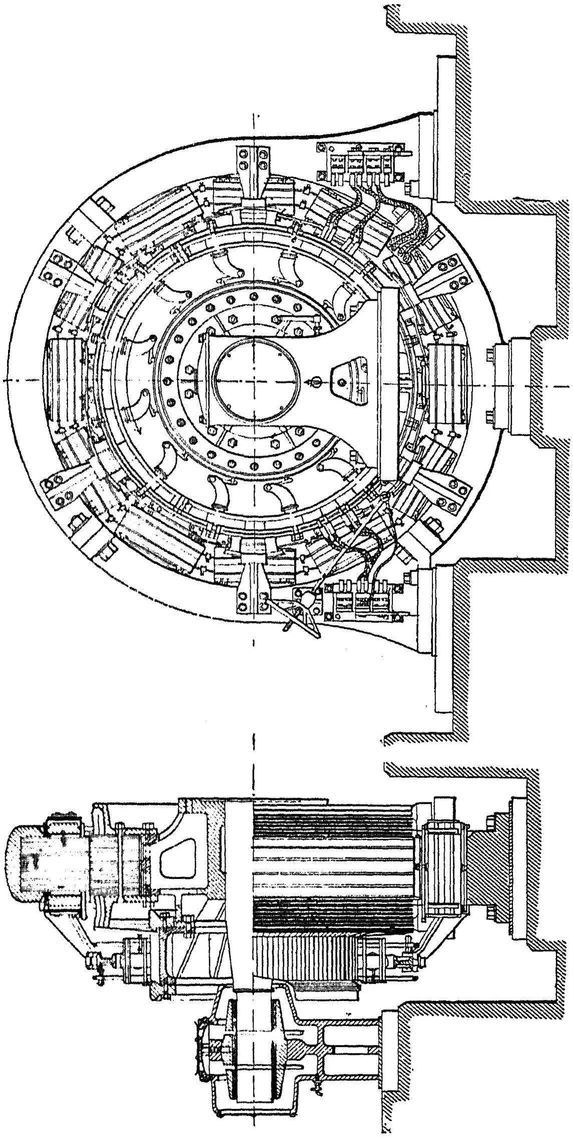


Fig. 148.—840 Kilowatt Crank-Shaft Generator

Fig. 148 is a drawing of a machine built by Messrs. Crompton & Co. of Chelmsford, for power work. The design is exceedingly massive and substantial from a mechanical point of view. Messrs. Crompton & Co. have kindly furnished the following particulars:—

Output	840 kilowatts
Volts	600
Amperes	1400
Revolutions per minute	230
Number of poles	12
Overload capacity	1900 amperes for two hours

Armature—

Diameter	90 inches
Length over core discs	18 ,,
Net effective length of iron	15 ,,
Number of slots	250
Inductors per slot	6
Length of air gap (iron to iron)	[illegible] inch
Total number of inductors	1500
Number of commutator segments	750
Diameter of commutator	58 inches

The armature is parallel-coupled and fitted with equalizing rings. The flux density at the root of the teeth is 100,000 C.G.S. lines per square inch, and in the iron of the armature body about 32,300 lines per square inch. These values are properly kept low in order to reduce the core loss of the machine, as the speed, 230 revolutions per minute, is high for a machine of the above diameter, and gives a frequency of alternation of 23 per second.

The magnets are made up of laminated pole cores cast into the cast-iron yoke, and fitted with cast-steel pole shoes. The winding of the bobbins is compound, and it will be noticed that the series coils are wound outside, but quite independent of the shunt coils, with a space left for ventilation between.

The densities in the various portions of the magnetic circuit are as follows:—

Flux density in the air-gap	45,000 lines per sq. inch
,, ,, wrought-iron magnet cores	107,000 ,,
,, ,, cast-iron yoke	37,000 ,,
,, ,, teeth	100,000 ,,
,, ,, armature body	32,300 ,,

The electrical efficiency of the above machine is as high as 98.5 per cent, and the guaranteed full-load commercial efficiency is 96.5 per cent. The guaranteed temperature rise above that of the atmosphere is 70° F. after thirty-six hours' run at full load. The above machine is, of course, fitted with a slotted drum-wound armature and carbon brushes.

The British Thomson Houston Company have designed and erected some very large direct-current generators. Fig. 149 is a photograph of the field frame of a generator designed for an output of 2700 kilowatts at 75 revolutions. The following are some particulars of this machine, kindly supplied to the authors by the British Thomson Houston Company (Limited) of Rugby.

The generator is an M.P. 36-pole machine, giving an output of 2700 kilowatts at 575 volts, full load, at a speed of 75 revolutions per minute. The diameter of the armature is 21 feet 8 inches, and of the commutator 15 feet 7 inches. The outside diameter of the frame is 31 feet, the total weight of the armature is 160,000 lbs., and the total weight of the entire machine is 310,000 lbs.

Magnetic frame and poles.—The magnet frame is of cast-iron, and divided horizontally, so that the upper half can be lifted to permit inspection of or repairs to either armatures or fields. On account of the large size of the frame, the lower and upper halves were each subdivided into three sections to facilitate transportation.

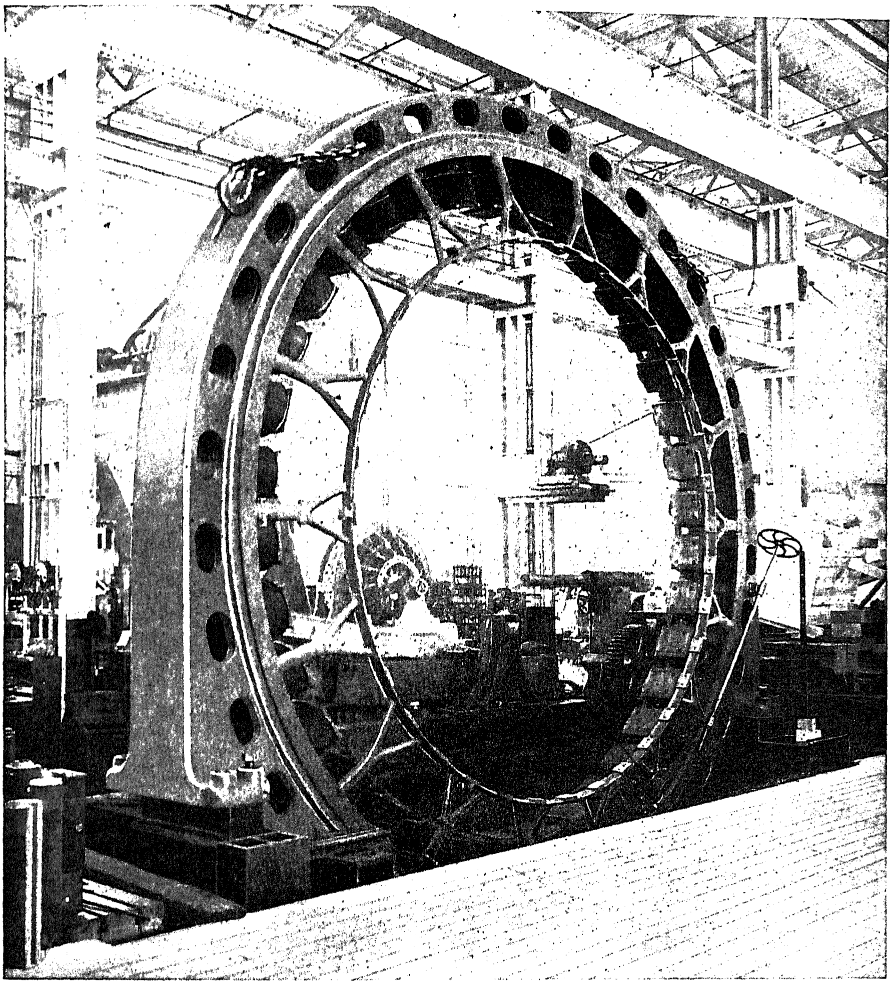


Fig. 149.—Field Frame of 2700 Kilowatt Generator

The magnet cores are 36 in number, and of cast steel, the pole-pieces being laminated and securely fastened to the same; they are fastened to the magnet yoke by bolts which can easily be removed, so that inspection or repair of any particular pole-piece with its field winding can be readily made. The pole faces are composed of laminations of two lengths, arranged alternately, so as to produce the graduated field effect given by a chamfered pole-piece. In the old generator this effect was produced by chamfering the solid pole face.

The field coils are wound on spools, having a double sheet-iron body and malleable-iron flanges, the shunt winding being next the magnet frame and the series winding below it.

Armature.—The armature spider is of cast-iron, and so designed that shrinkage strains are avoided. The armature core is built up of laminations, japanned before being assembled to prevent eddy currents. These are secured to the spider by dove-tail projections, and are held in place by end flanges of cast-iron, which also serve to support the end windings.

Space bolts are inserted at equal intervals between laminations to provide ventilation ducts.

None of the bolts entering into the construction of the armature pass through the laminations.

The armature spider is so arranged that it produces a fan effect, forcing the air through the ventilating space between the laminations and into the windings, thus reducing heating of the generator.

The armature is multiple-circuit drum-wound, and provided with equalizer rings, connected to points of equal potential at the back of the winding. The rings are secured with insulation pieces attached to the arms of the spider on the back of the armature, and are readily accessible for inspection.

They tend to compensate for differences in the magnetic strength of the poles, due either to difference in material or length of air-gap, which effect tends to improve the commutation of the generator.

Armature coils are made in halves, back connection being made by soldering two ends together. This form of winding has the advantage that, in case it is necessary to remove one-half of the coil for repairs or inspection, the other half may be left in place without disturbing the adjacent coils.

Commutator.—The commutator is supported on an extension of the armature spider. The commutator bars are of hard-drawn copper, and made with a dove-tail on the lower edge to secure them in place.

The side mica used in the construction of the commutator is of soft quality and as thin as the requirements for insulation will permit, so that even wear of mica and copper is secured.

Brush-holders and Brushes.—The brush-holders are so constructed that the tension of any brush can be readily adjusted, or the brush removed, while the machine is in operation. The brush is inclined slightly forward of a radial position, the angle in reference to the commutator being such as to largely eliminate the friction of the brushes in the guides, causing it to better follow any inequalities due to the movements of the commutator.

Insulation.—The insulation between the field coils and magnet frame, and also between armature windings and the armature core, is tested at 4000 volts alternating current for 10 seconds, or 2000 volts for a period of 60 seconds.

The temperature of the generator is guaranteed not to rise more than 63° F. after a run of 24 hours at full rated load. The rise will not be more than 99° F. with an overload of 50 per cent for two hours, following a run of 24 hours at rated load.

The generator is further guaranteed to carry an overload of 50 per cent at rated voltage for two hours, and 100 per cent momentarily, without movement of the brushes, without injurious sparking.

The generator is compounded for 525 volts no load and 575 volts full load, full load being 4695 amperes.

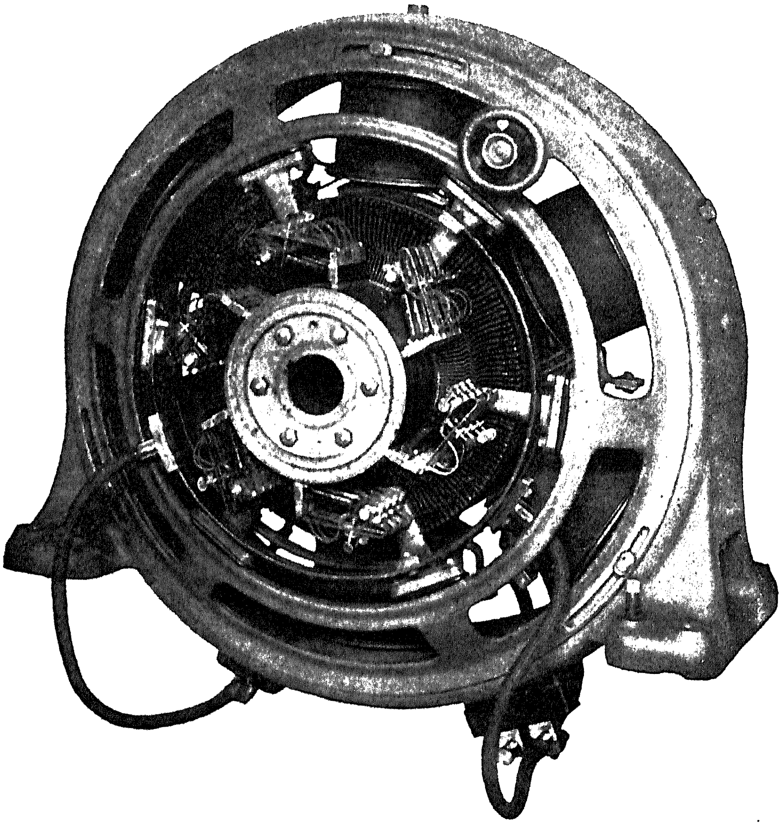


Fig. 150.—Generator for Direct Coupling to Engine

Fig. 150 shows another form of generator, manufactured by the above Company for direct coupling to engines. These machines have iron-clad armatures of the slotted drum-wound type. The field frame is cast from specially-selected soft iron of the highest permeability, while the pole-pieces are of soft steel and are solid. The bobbins are of the round pattern. The armature windings, which are separately insulated, consist of straight copper bars requiring but two joints for each turn, and repairs can easily be made. The conductors are held in the slots by wood wedges, as illustrated in fig. 144, rendering the use of binding bands on the core unnecessary. The commutator sleeve, which is of cast-iron and is pressed on the armature spider, is independent of the shaft.

The following is the standard basis of rating of the above generators. The temperature of no part of the armature or field-coils—as measured by the thermometer after a run of 24 hours at full-rated load with normal conditions of ventilation will rise more than 60° F. above that of the surrounding air. The guaranteed rise of the commutator is 72° F. They, however, introduce a proviso that the temperature of the air must not exceed 77° F., and in the event of this temperature exceeding 77° F. the above rise in temperature is to be corrected .28 per cent for each degree

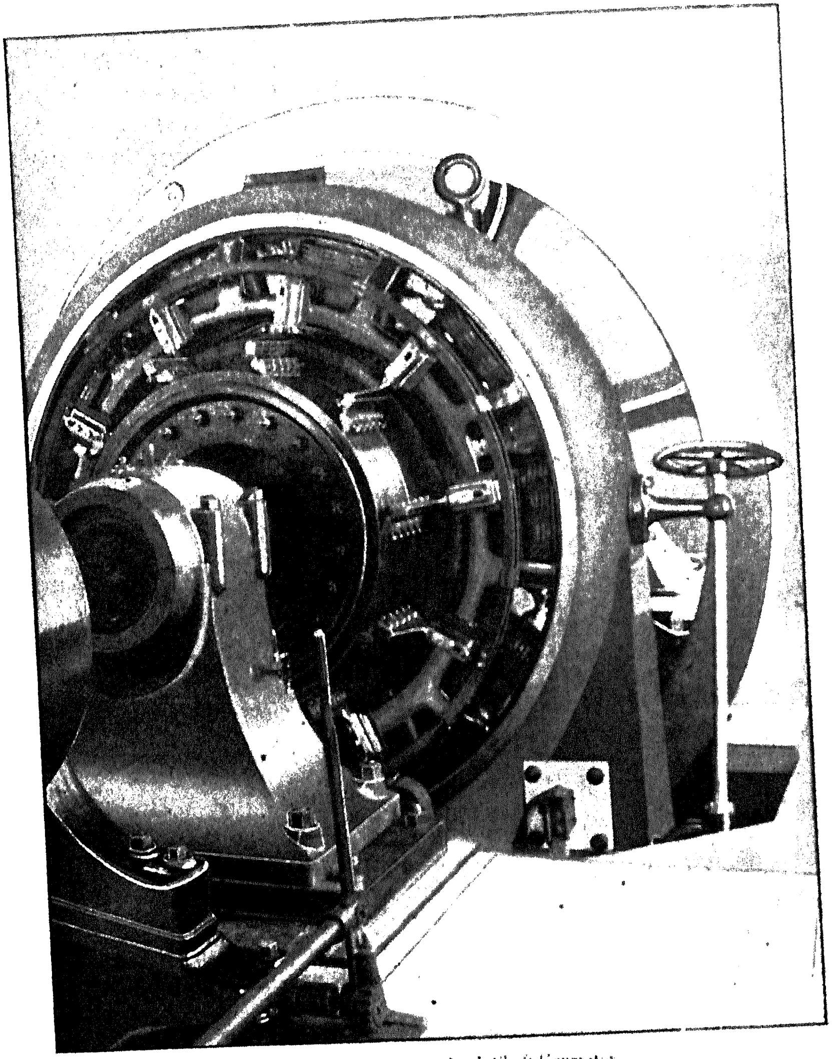


Fig. 151.—400 Kilowatt Crank-Shaft Generator

Fahrenheit that the temperature of the surrounding air differs from 77° F. With the load then increased 50 per cent above the rated amperes and at the rated volts for two hours, the temperature of no part of the generator will be more than 99° F. above the atmosphere. These generators will carry an overload in current of 100 per cent momentarily without injurious sparking.

Fig. 151 is a photograph of a 400-kilowatt crank-shaft generator, showing the general type of machine built by The British Westinghouse Company. Particulars and data of the machine illustrated are not available, as the Company is averse to publishing such data. However, the general design of the generators built by this Company consists of a circular yoke of cast-iron carrying inwardly-projecting poles of laminated soft steel. The field castings are divided vertically and set upon a guide-plate. This vertical division of the field affords excellent facility for inspection or removal of the armature or field coils without the necessity of removing the armature from its bearings. These machines are generally compound wound, and are over-compounded to raise the pressure at full load in accordance with the best standard practice. The shunt and series coils are removable at will. Another feature of these machines is that the armature core is built up upon a cast-iron spider which also has the commutator sleeve cast solid with it. Larger machines have retaining-wedges of hard fibre driven into the slots as in the case of some of the other machines described in this paper. Carbon brushes are, of course, used with all these machines. The brush gear is carried in the approved method, *i.e.* from brackets projecting from a ring concentric with and supported by the field yoke.

The foregoing machines are typical of modern direct-current generators. Most machines built within the last few years for traction and power work follow pretty closely on the lines of those described.

Of the machines described in the foregoing chapter, those described in the fullest detail are of the author's own design. This is simply due to the fact that full data of these were easily available, whereas the authors were unable to obtain such complete information regarding other makers' machines.

CHAPTER VI

COMMUTATING-POLE MACHINES

In the following consideration of the design of motors having commutating poles, the same rule will be followed as in the previous chapters on continuous-current generators. No attempt will be made to go deeply into the theory of commutation, but attention will be brought to bear more particularly on conditions affecting the design and manufacture of this type of motor.

The design of motor machinery falls into three broad classes:—

Small motors used for general purposes, and manufactured in large quantities.

Small motors similar to the above, but for special purposes, such as hoist and crane work, &c.

Larger motors working up to considerable powers.

Under the designation "small motor", machines up to 60 horse-power are included, and by many manufacturers they are stocked in quantities. The powers range from about 5 horse-power at 1200 revolutions per minute to about 700 revolutions per minute in the larger size mentioned above.

The chief consideration in the manufacture of these motors is cheapness, and any device which will tend to lower their cost is of the greatest value. The field system is so simple that no alteration to the accepted design is likely to cheapen its construction. In the case of the armature—which is of complicated structure—great ingenuity has been exercised to reduce the labour in manufacture; anything which will lead to a simplified type of armature would, therefore, reduce the labour bill, and would be of the greatest use to designers.

It is usually found that in these machines the output is limited by the amount of permissible sparking rather than by the temperature rise, and in many cases where heating has been the limit, slight alterations in structure have allowed an increase in output far beyond the point where sparking became destructive. Of course this does not apply to totally enclosed machines.

Until the advent of commutating poles, designers had usually relied upon the following characteristics for good sparkless running, all tending to make a costly armature:—

Few armature turns,
Few turns per coil,
Large commutators,
Large number of sections, and
Strong magnetic field.

In other words, they depended upon natural commutation as much as possible. The many attempts to design devices to force commutation had all ended in failure until Mr. Hobart published his researches into the conditions governing commutation.

From that not far distant time, designers have perfected their knowledge of the phenomena underlying the commutation of current, and are able to calculate the effect with certainty.

Before going further, it would be better to arrange our knowledge of the actions and reactions present in D.C. armatures during load, and to see what additions and alterations must be made to known theory in the design of commutating poles.

First, we will recapitulate the theory as it applies to dynamo machinery without interpoles. The flux passing from the field magnets through the moving armature coils generates in them electric currents. These currents magnetize the armature, creating a magnetic force in opposition to the impressed force, *i.e.* it reacts on the main field. This will be made more apparent by the study of the diagram shown in fig. 151 A. A two-pole machine has been chosen for the sake of simplicity.

Assuming for the moment that the brushes are in an upright position—that is, midway between the poles—all the wires lying to the right of a line drawn through the brushes would be carrying currents flowing to the observer, and all wires on the left would be carrying currents flowing away from him. It is now seen that this would make the upper portion of the armature a "S" pole, and the lower portion a "N" pole—that is, there are two poles of like sign at right angles; the resultant field would therefore lie at an angle to the axis of the poles. In a machine unprovided with a commutating device it would be found impossible to collect current sparklessly in this position, as the inductance of the coils under short circuit at this moment would be so great that a large current would flow in them, setting up destructive sparking. *It is imperative, therefore, that a field must be found of sufficient strength* to overcome the action of inductance, and this can be done by bringing the short-circuited coil under the influence of opposite polarity to the one which it has just left; in effect the brushes are tilted round the commutator in the direction of rotation in a dynamo, and the reverse direction in a motor; this action now brings into play a series of reactions on the main field.

It has been seen that the action of the armature carrying current with the brushes in the neutral position was to distort the main field, therefore it will be distorted to a greater extent still if the brushes are brought farther round the commutator and, of course, the magnetic pole which lies immediately above that position on the armature.

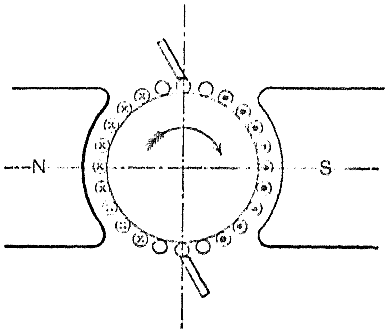


Fig. 151 A

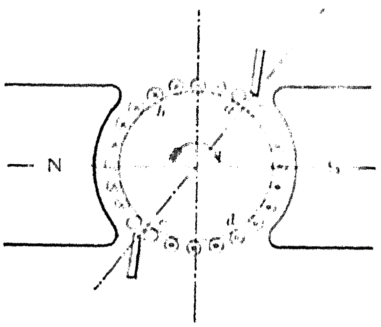


Fig. 151 B.

It is now assumed that a suitable field has been found, and that sparking has ceased. If we now turn to fig. 151 B, where the above conditions are diagrammatically shown, it will be seen that a belt of current lying between the lines *a* and *b*, *c* and *d* is in such a position as to oppose the impressed magnetic field, shutting out, as it were, some of the main field.

So that we now find that in our search for a suitable reversing field we have considerably distorted our main field, and brought into play a large demagnetizing force.

If it were possible to keep the brushes in the neutral position, *i.e.* midway between the poles, armature demagnetization would be obliterated and distortion would be at a minimum.

Bearing the above dissertation in mind, it would be a much more rational idea to take a piece of the main pole of suitable strength, as it were, and rotate it back over the armature to the neutral position, as in fig. 151 C.

It is now only a step to the realization of the modern commutating pole.

We are now in a position to consider the theory of the commutating pole at length.

In a closed-coil armature each section or coil lying between two commutator segments must during one revolution be transferred from the influence of an "N" pole to that of an "S", and the current which was flowing in one direction in the armature will, after passing the brush, be flowing in the reverse direction. It follows that the current flowing in this coil during the time it is under the brush must die away and then increase in the reverse direction, so that the moment it leaves the brush, the current is flowing in the same direction as that portion of the armature it has now joined. Should it not be so, destructive sparking will be set up; but if the period of commutating has been long enough to allow the above condition to be brought about, commutation will be perfect.

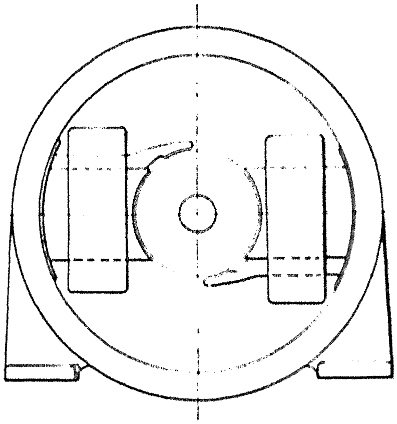


Fig. 151C

In the previous portion of the book some of the conditions existing in the short-circuited coil were considered, so they will only be briefly analysed here.

To make the foregoing more clear, we consider the graphic diagram, fig. 151D, in which the line A B represents time, and ordinates the value of the current flowing in the positive direction till the moment it comes to the point *a*, when commutation begins; if it is to be sparkless, it must be completed before it passes from under *b*. The current is here shown as decreasing at a uniform rate, and if such was the case there would be constant current over the whole brush contact, and it must be here remembered that this could only happen with no self-induction in the short-circuited coil; self-induction being always present, current will not decrease at uniform rate, and the line cannot therefore be straight, but will probably be as shown in diagram, fig. 151E.

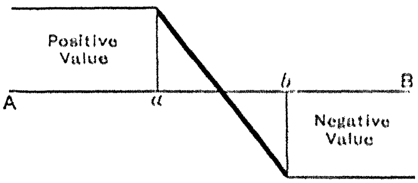


Fig. 151D

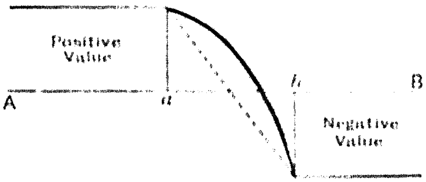


Fig. 151E

Many forms have been suggested for this curve, but owing to the difficulty experienced in an experimental determination of its exact limits, it is idle to conjecture further.

The self-induction of the coil short-circuited, however, is not the whole question of commutation, and by many writers on the subject the coil alone is considered, and is taken, as it were, away from the armature and laid on a dissecting-table. This coil is still a part of the winding, and even if the original current in the coil had ceased to flow, sparking might result. No doubt the magnetic momentum of the short-circuited coil plays the greatest part in producing sparking, but the voltage generated in the coil by cutting lines of force produced by those turns not undergoing commutation, and by the voltage-drop in the short-circuited coil at the instant of leaving the brush, also play their part.

We see, then, that to produce ideal or straight-line commutation we must obliterate the effect of self-induction, also the effect of the armature field, and this can be done by introducing into the coil a field equal and opposite to that produced by the linkage of lines and the armature field, and here we have the tool ready to our hands in the interpole.

Again recapitulating, we have defined ideal commutation and the method which must be adopted to obtain it.

We know that the condition during commutation consists of the following three effects:—

1st, and most important, the magnetic momentum of the coil under short circuit.

2nd. The short-circuited coil cutting lines of force, which lines are set up by the armature turns not undergoing commutation.

3rd. The voltage drop due to the cessation of current in an inductive circuit.

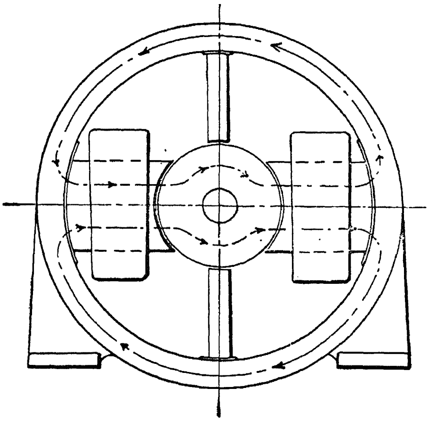


Fig. 151 F

The interpole must have an effect the antithesis of the natural condition.

The magnetomotive force of the poles must be greater than, and opposite to, that produced by the armature.

The ampere turns, or M.M.F. of the armature, is known with certainty, and the magnetic inertia can also be calculated sufficiently near for practical purposes.

In the following pages we will discuss the distribution of the magnetic fluxes present in the field and linked with the armature turns.

In the diagram, fig. 151 F, the main flux is shown threading the magnetic circuit as in an ordinary machine without commutating poles. These poles, though shown, are left without windings; this is merely to indicate that they are in the position of zero magnetomotive force, and in their very nature could not become part of the magnetic conducting circuit.

Fig. 151 G shows the reverse condition. The interpoles are now provided with a winding, and the main poles are left blank. On examination, the magnetic conditions are exactly similar in effect to those produced by winding the main poles.

From this it might be assumed that these two fluxes are entirely distinct and each could exist without the other, and such would be the case if one of the windings were to be disconnected in any way; but with both windings excited and in normal operation it is quite impossible that the commutating fields could take the path shown in fig. 151 G.

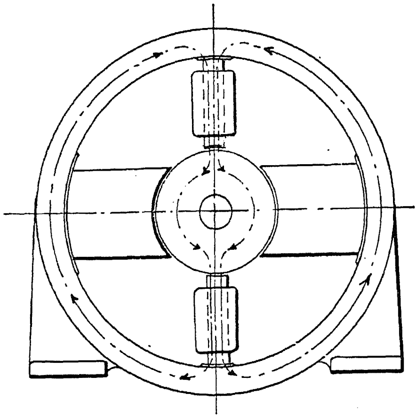


Fig. 151 G

Now we saw previously that the commutating field was in effect part of the main magnetic circuit rotated back over the armature in the place of the brushes, and with it the short-circuited coil being removed under the influence of the reversing field, therefore it is perfectly logical to expect that the commutating field will take the path of the main field, and being part of the main field diverted, it cannot in any way react on the main field to distort it.[1]

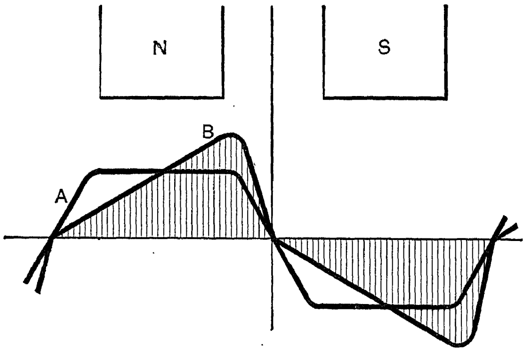


Fig. 151 H

So far we have only considered the unloaded armature, and we must now observe the conditions existing when the machine is carrying current.

The effect of the armature turns is precisely the same in this type of machine as in a machine without commutating poles, considerably distorting the main field, weakening the leading tip, and strengthening the trailing one. Rather more distortion will be observed, because with forced commutation we can allow a greater specific loading than in the normal type of machine. Diagrammatically the curve of E.M.F. will be as shown in fig. 151 H, A being the curve with the armature unloaded, and B loaded. If we now superpose upon this curve the E.M.F. curve due to the commutating poles, we will get the curve shown in fig. 151 I.

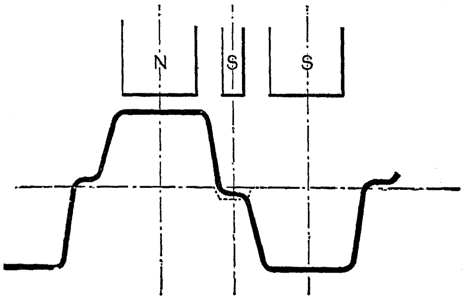


Fig. 151 I

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[1] Experimental confirmation of this can be found in an article by Messrs. Wall and Smith in the *Electrician* of April 6th, 1906.

It will then be noted that immediately under the commutating pole there is a position of constant magnetic field, and this is the whole condition aimed at. This field is what we require to know most thoroughly. We must know its limits and its M.M.F., and when we have found all this accurately, then we can produce a sparklessly running machine.

Below we will consider briefly the conditions governing the stability of this field.

Leakage Fields.—We see that the flux of the commutating poles enters the pole of opposite polarity, and from this it follows that the main flux is not disturbed; that there being equal exciting power on all the main poles, the main fields will be equal. Certainly some of this field has been diverted through the commutating poles, but on examination of fig. 151J it will be seen that the main flux of that pole threads through the armature coils before being diverted to the commutating poles.

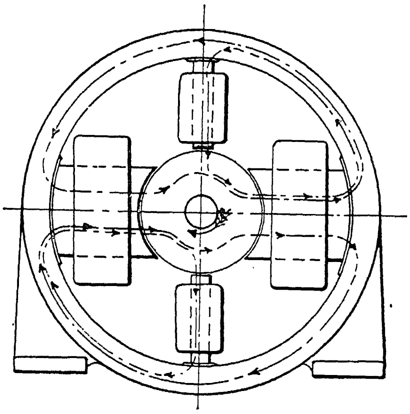

Fig. 151J

So far we have considered the magnetic circuit as if all the magnetism generated therein was conducted through the circuit without loss—in other words, we have not taken into account the leakage field, which in this type of machine is considerable.

The leakage field in the case of ordinary machines has been clearly defined, and experimental research has determined its limits with great exactness. With this abnormal form of machine we will find that we have only to make certain corrections to our formulæ to make it suitable for practical calculations.

The leakage field which has been found to be of greatest magnitude is that which leaks from pole-tip to pole-tip, and as we have now placed a magnetic conductor in the path of this leakage field, we must expect the leakage to be considerably increased. It will be easily understood that such being the case, if great care is not taken in designing the poles, the leakage flux entering the commutating poles from the main field may even exceed the normal flux in the commutating pole, saturating the field so that it is not able, except under greatly increased magnetizing force, to give a sufficiently strong reversing field; with given magnetizing power, should there be a saturated commutating field, the machine would probably run sparklessly at half load, but with an increase of load and the increased current flowing round the commutating poles, would not correspondingly increase the flux to the required amount, and sparking would begin.

Should the winding be proportioned for this saturation, sparking would commence at less than full load—due to the over-compensating of the field—but at full load the machine might be just right. Many machines will be found to be running in this manner due to insufficient care being exercised in the original design.

The leakage factor, or that amount by which the gap flux must be multiplied to give the flux in the pole-pieces, has been found to lie between 1.3 in small machines, say of 5 horse-power or thereabouts, to as low a figure as 1.12 in machines of considerable output and size; 1.2 being a general value for machines from 20 horse-power to about 60 horse-power, and of good design. This applies to machines without interpoles. In standard types of machines it has been usual to make the pole arc 70 per cent of the armature circumference, *i.e.* pole arc divided by pitch of poles = .7, and even slightly larger than this in particular cases, but with commutating poles placed between the main pole-tips it is quite impossible to allow a ratio as large as this. It has been found by experiment that by reducing the distance between the pole-tips by 10 per cent—which is about equal to the reduction in interpolar space brought about by inserting commutating poles—the leakage factor is increased by about 30 per cent.

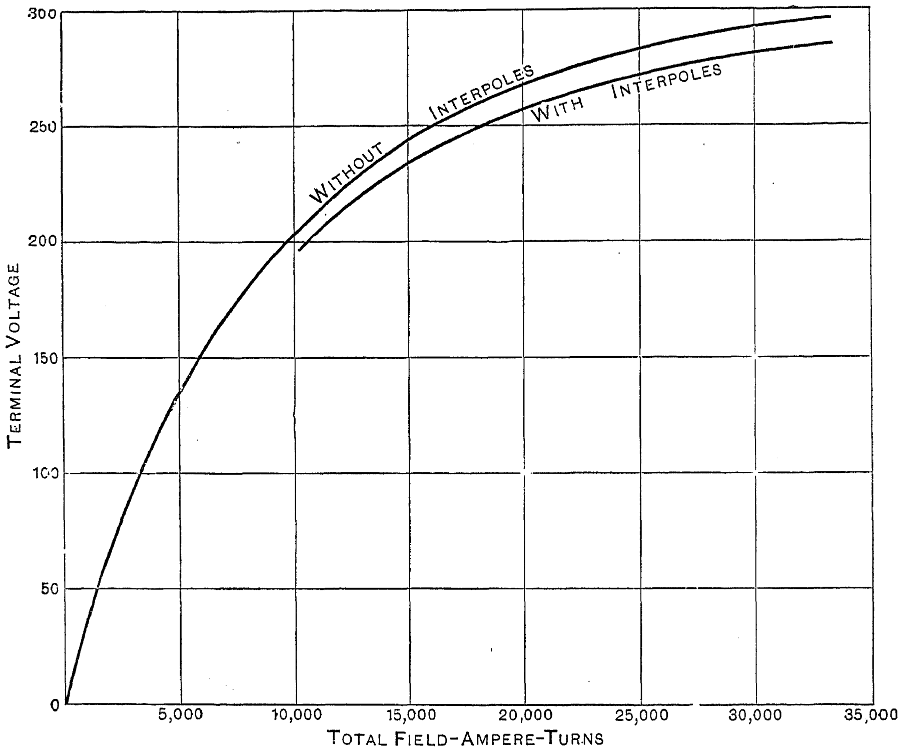

Fig. 151 K.—Excitation Curves of 20 K.W. 4-pole Dynamo

A 20 horse-power motor would have a total leakage factor of about 1.58, or, in other words, keeping the main field of the same dimensions as in the normal machine, the main flux would have to be 30 per cent less for the same exciting power, but there is a more than equal gain by reducing the length of the air-gap. Having neutralized the sparking field, there is now no necessity to keep the air-gaps large; they can be reduced to merely a mechanical clearance. Economic design compels us to have large armature slots, about ½ inch wide, and such being the case, it is necessary that the commutating-pole face should cover nearly two, because there will in all probability be more than one section in a slot and more than one section short-circuited under a brush, and therefore all the sections short-circuited cannot be in one slot during that period, so that the number of sections short-circuited will settle the width of the commutating-pole face. The general method is to make the ratio of polar span to pole pitch of the main field between .5 and .6.

In fig. 151 K are drawn exciting curves of two machines, showing (1) a curve from a motor without commutating poles, and (2) the same motor with interpoles; the difference is very marked.

Having now got a general knowledge of the conditions prevailing in the organs of this type of machine, we will apply them to a standard design, after which we can sum up our altered formulæ. Three designs will be sufficient for our purpose, and will be as follows:—

1st. Standard stock design, say 25 horse-power, protected type, for a 500 volt circuit, running at a speed of 800 revolutions per minute.

2nd. A design for a special purpose, say 25 horse-power motor with intermittent rating, running at a speed of 500 revolutions per minute, and suitable for an E.M.F. of 500 volts.

3rd. A direct-coupled plant of 200 kilowatts 500 volts, running at 500 revolutions per minute.

Calling the output co-efficient ξ,

$$\text{then } \xi = \frac{W}{d^2 l R}$$

where W = Watts output of dynamo or motor
d = Diameter of the armature in inches
l = Length of the armature in inches
R = Revolutions per minute.

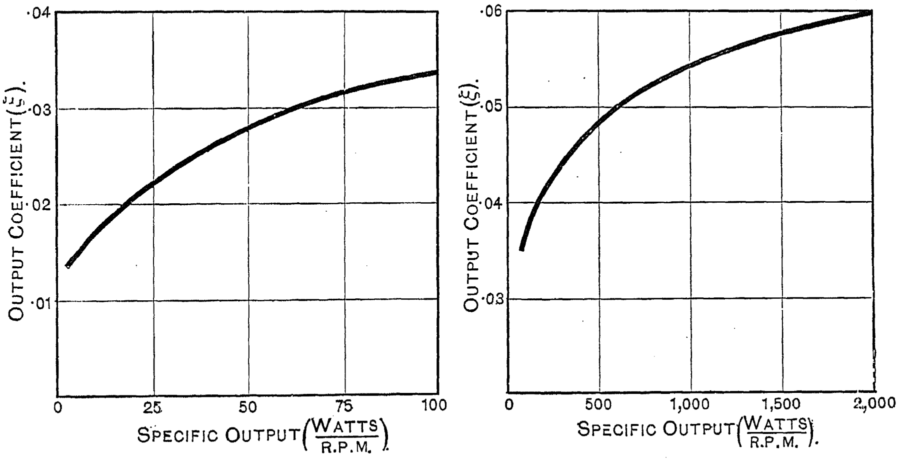

Fig. 151 L

In fig. 151 L are given two sets of output curves which have been found to be sufficiently correct for the new class of machinery.

It will be seen that, in the calculation of the reactance voltage, use has been made of formula ([illegible]) on page 140, but we must bear in mind that we have found $2\pi NL$ the reactance of the coil, and that $2\pi N$ is the maximum rate of cutting a sinusoidal variation assumed.

We must therefore divide the amount found by $\frac{\pi}{2}$, which will give us the average E.M.F., which we must neutralise.

The difficulty we have now to meet is how to satisfactorily predetermine the E.M.F. set up in the short circuited coil while passing under the commutating pole.

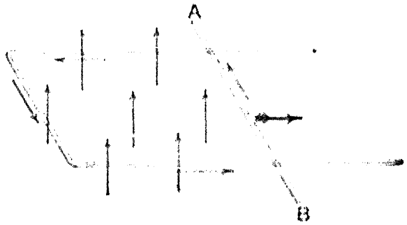

Fig. 151 M

If we assume a circuit such as shown on fig. 151 M, in which the bar AB moves forward along the two parallel bars of the circuit so as to enclose more lines of force in that circuit, then the E.M.F. set up in that circuit by the uniform motion parallel to itself of the sliding-bar is numerically given by the product of the length of the bar, its velocity, and the magnetic flux density of the field in which it moves.

The formula for the E.M.F. produced in the short circuited coil while passing through the commutating-pole field can therefore be deduced as follows:—

$$e = B_g \times V \times L \times 10^{-8}.$$

Where e = voltage required to overcome the reactance voltage.

L = length in inches of circuit cutting commutating-pole field = 2 × length of pole face × number of turns in armature coil.

B_g = Magnetic density in the pole face in lines per square inch.

V = Velocity of periphery of armature in inches per second.

Having found the density in the pole face, it is an easy matter to determine the number of ampere turns required to overcome the reluctance of the gap, teeth, and magnetic circuit, and when found we must remember that before we can utilize it we must overcome the M.M.F. of the armature, which is concentrated immediately underneath this pole; and this amount must be added to the amount already necessary to overcome the reluctance of the magnetic path. The method of calculating these amounts is set forth in the tabulated results. If our estimations are in all cases correct, the machine will run sparklessly at all loads and in either direction with constant brush position.

In figs. 151 N and 151 O are given sectional drawings of the 200 kilowatt generator and the 25 horse-power motor, mentioned above, and in tabulated form below will be found the general particulars of the machines, from which the reader will be able to follow the design.

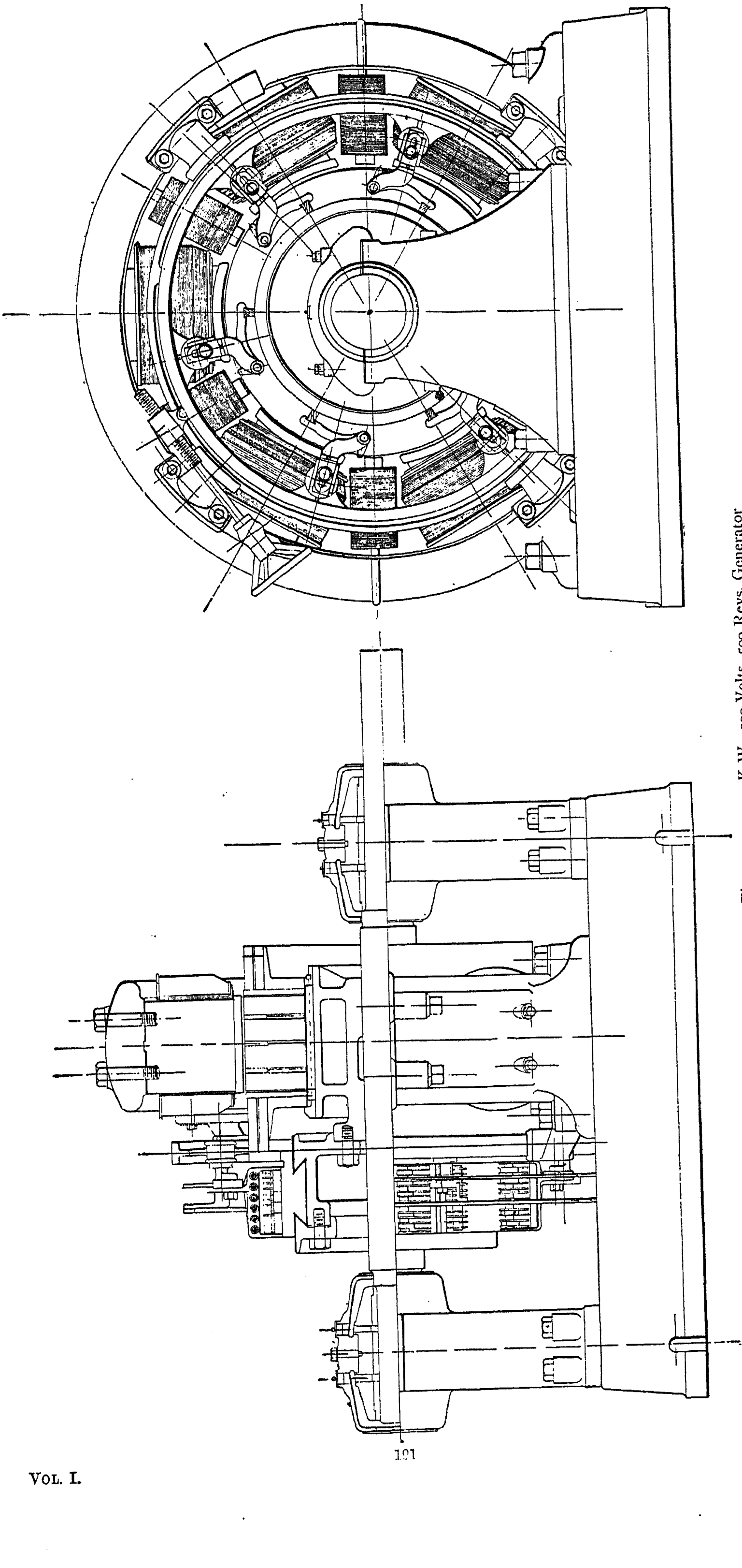

Fig. 151 N.—200 K.W., 500 Volts, 500 Revs. Generator

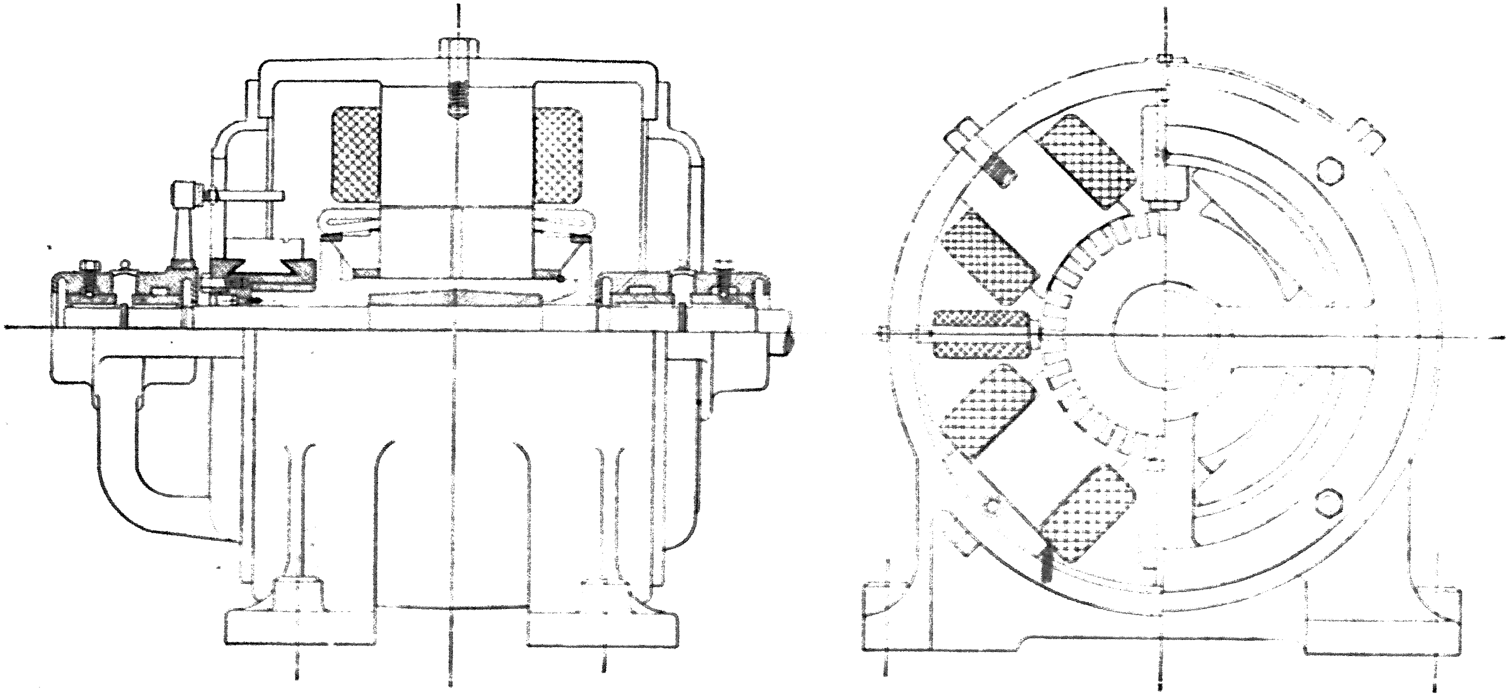

Fig. 131a.—25 H.P. Motor

Rating.	Constant.	Constant.	Intermittent.
Output	200 K.W.	25 H.P.	25 H.P.
Volts	500	500	500
Amperes	400	42	42
Revolutions per minute ...	500	800	500
Number of poles	6	4	4
Armature—			
Diameter over all	30	12.75	12.75
Length over conductors ...	22	19¼	19¼
Diameter of bottom of slots ...	26.5	10.43	10.43
Internal diameter	17	5·5	5·5
Length over iron	10	8	8
Effective length of iron ...	7·9	6.75	6.75
Depth of slot	1.75	1.16	1.16
Width of slot	·5	.64	.64
Number of slots	90	30	30
Width of tooth at root ...	·43	.46	.46
Depth of conductor	.68	.109 diam.	.109
Width of conductor	·043	—	.065
Space factor	·43	·305	·345
Number of ventilating ducts ...	3	1	1
Width of each ventilating duct	·375	·5	·5
Armature core	mild steel	mild steel	mild steel
Spider	cast-iron	cast-iron	cast-iron
Yoke	cast-steel	cast-steel	cast-steel
Pole tips	laminated steel	laminated steel	laminated steel
Brushes	carbon	carbon	carbon
Magnet Core—			
Length of pole face	10	8	8
Length of pole arc	10.25	6.5	6.5
Pole arc ÷ pole pitch	.65	.65	.65
Radial length	8.5	6.5	6.5
Diameter	9¼	—	—
Length of pole core parallel to shaft	—	8	8
Width of pole	—	5	5
Bore of fields	30.25	12.43	12.43
Depth of air-gap per side ...	.125	·093	·093
Outside diameter of yoke ...	55	28.75	28.75
Commutator—			
Diameter	21	8	8
Rubbing length	6½	3	3
Number of segments	180	59	89
Width of segment at top ...	·336	·395	·253
Thickness of mica insulation ...	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$

Brushes—

Rating.	Constant.	Constant.	Intermittent.
Number of sets	6	2	2
Number per set	4	4	4
Size of brush	1½ × ¾	¾ × ¾	¾ × ¾
Contact area of each brush ...	1.125	.562	.562

Electrical Data—

Number of conductors ...	1080	708	1068
Conductors per slot	12	24	36
Style of winding	multi-circuit single	two circuit single	two-circuit single
Mean length of conductor ...	28.75 in.	20 in.	20 in.
No load voltage	500	500	500
Resistance between brushes ...	.02 cold	.0262 cold	.05 cold
Current densities in conductors	2300	2250	3000
Average voltage between commutator bars	8.3	34	22.5
Armature ampere turns per pole	6000	1860	2750
Armature flux load	5.7	2.67	2.67
Leakage coefficient (main poles)	1.33	1.4	1.4
„ (commutating poles)	1.5	1.6	1.6
Flux in main poles	7.6	3.75	3.75
Flux in commutating poles ...	.35	.21	3.45
Section of armature, sq. in. ...	75	33.2	33.2
Magnetic length through armature	16	9	9
Section of poles, sq. in. ...	67.2	36	36
Magnetic length through poles	9.375	6.5	6.5
Section of yoke, sq. in. ...	78	50.5	50.5
Magnetic length through yoke	27	21.75	21.75
Section of teeth, sq. in. ...	37.5	18.5	18.5
Magnetic length of teeth ...	3.5	2.32	2.32

Commutating-Pole Calculations—

Number of segments short-circuited under 1 set of brushes	3	2	3
Number of turns per commutator segments	3	6	6
Embedded length	7.9 × 2	6.75 × 2	6.75 × 2
Lines per inch of embedded length	10 15.8 × 10 = 158	10 13.5 × 10 = 135	10 13.5 × 10 = 135
Number of turns undergoing commutation	9 × 2	12 × 2	18 × 2
Flux linked with these turns ...	158 × 18 = 2850	135 × 24 = 3250	135 × 36 = 4850
Free length	41.7	26.5	26.5
Lines per inch of free length ...	2	2	2
Number of turns undergoing commutation	9	12	18
Flux linked with these turns ...	83.4 × 9 = 750	53 × 12 = 635	53 × 18 = 950

Commutating-Pole Calculations (*Continued*)—

Rating.	Constant.	Constant.	Intermittent.
Total flux linked with turns, undergoing commutation	2850 750 3600	3250 635 3885	4850 950 5800
Inductance in henrys	3600×10^{-8}	3885×10^{-8}	5800×10^{-8}
Circumference of commutator	66 in.	25 in.	25 in.
Revolutions per second	8.35	13.3	8.35
Peripheral speed inches per second	550	332	210
Arc of brush contact	·75 in.	·75 in.	·75 in.
Current completely reversed	$\frac{\cdot 75}{550} = .00136$ sec.	$\frac{\cdot 75}{332} = .00225$ sec.	$\frac{\cdot 75}{210} = .00356$ sec.
Frequency of commutation	$\frac{1}{2 \times .00136} = 367$	$\frac{1}{2 \times .00225} = 222$	$\frac{1}{2 \times .00356} = 143$
Reactance	$2\pi \times .000036 \times 367 \times 3 = .245$	$2\pi \times .0000389 \times 222 \times 6 = .325$	$2\pi \times .000058 \times 140 \times 6 = .305$
Current per conductor	$\frac{400}{6} = 67$	$\frac{42}{2} = 21$	$\frac{42}{2} = 21$
Reactance voltage	16.3	13.6	13
Average voltage to be neutralized	$\frac{16.3}{1.57} = 10.4$	$\frac{13.6}{1.57} = 8.7$	$\frac{13.0}{1.57} = 8.3$
V = velocity of conductors cutting commutating-pole field in inches per second	790	530	334
L = Length of path cutting commutating-pole field = 2 × length of commutating-pole face × number of turns in 1 armature coil	60	96	96
Flux density in commutating-pole face	21,500	16,300	26,500
Width of commutating-pole face	1½	1½	1½
Length of commutating-pole face	11	9	9
Area of pole face	16½	13½	13½
Commutating-pole flux	·35	.21	·345
Number of commutating poles	6	2	2
Armature strength per pole	6000 A.T.	1850 A.T.	2750 A.T.
Ampere turns for c.p. air-gap	$21{,}500 \times .313 \times .125 = 840$	$16{,}300 \times .313 \times 093 = 475$	$26{,}500 \times .313 \times 093 = 770$
Area of commutating pole	$7 \times .875$	$7 \times .875$	$7 \times .875$
Density in "	79,000	48,000	79,000
A.T. turns from curve per in. of length	20	7	20
Length of commutating pole	8.5	6.5	6.5
Density in teeth under commutating poles	50,000	34,000	56,000
Magnetic length of teeth	1.75	1.16	1.16
Ampere turns per inch of length	6	4	6

Total Ampere Turns on Commutating Pole—

Rating.	Constant.	Constant.	Intermittent.
Pole core	170	45.5	130
Air-gap	840	475	770
Teeth	10.5	4.6	7
Ampere turns to overcome armature reaction	6000	1860	2750
Total ampere turns on commutating pole	7020.5	2385	3657
Turns per pole	$\frac{7020.5}{400} = 17.5$	$\frac{2385}{42} = 57$	$\frac{3657}{42} = 87$
Size of wire	1.0 × .067 4 in. parallel	.192 diam.	.134 diam.
Main winding	shunt	shunt	series
Size of wire	.056	.032	.144
Turns per bobbin	2400	3650	85
Total turns on bobbins ...	14,400	14,600	340
Winding room	5.5 × 2.5	5.0 × 2.0	5.0 × 2.0
Shunt ohms total	165	435	—

The great disadvantage to these poles is the weak field in the pole face, amounting in one case to only 16,300 lines per square inch; and it is possible that the current flowing in short-circuited coil might momentarily reverse the impressed field, allowing more powerful current to flow—producing a fluctuating E.M.F. and consequent sparking—but the authors have never noticed it during test. These considerations are outside of the designer's field.

CHAPTER VII

TURBO-GENERATORS

In this chapter the design of machines for high speeds will be considered without any reference to the economic conditions which have produced this type.

In turbo-driven dynamo-machinery it is not only necessary to have a high rotative speed, which makes the mechanical design of the bearings a matter of the greatest importance, but very high peripheral velocities are required.

In slow-speed machinery—by which is meant dynamo-machinery having a comparatively low number of revolutions per minute—peripheral velocities of 4000 feet per minute are general, and velocities of 6000 feet per minute exceptional; but when we come to look into the dimensions necessary to fulfil given conditions in turbo-driven dynamos, we find that velocities of 10,000 feet per minute are considered a minimum in the case of A.C. generators, and a little higher velocity is considered to be the maximum for D.C. generators. These velocities can be exceeded where the conditions are such as not to allow of a machine of mere normal dimensions, and velocities of 20,000 feet per minute can be considered the limit in alternators, and 17,000 feet per minute in D.C. generators. Below is given a table of centrifugal forces, from which it will be seen that an immense amount of care must be taken in the mechanical design of very high-velocity machinery.

CENTRIFUGAL FORCE IN LBS. WEIGHT PER LB. AT VARIOUS SPEEDS

Dia. of Path.	Revolutions per Minute.								
In.	1000	1100	1200	1400	1600	1800	2000	2500	3000
12	170.25	206.00	245.15	333.75	435.90	551.65	681.00	1064.00	1532.25
18	255.40	309.0	367.7	500.6	653.8	827.4	1021.6	1596.2	2298.8
24	340.50	412.0	490.3	667.5	871.8	1103.3	1362.0	2128.0	3064.5
30	425.60	514.9	612.9	834.4	1089.8	1379.0	1543.6	2660.2	3831.05
36	510.80	618.0	735.5	1001.3	1307.6	1654.9	2043.0	3192.0	4596.7
42	595.90	721.0	858.0	1168.2	1525.8	1931.0	2383.4	3724.0	5362.8
48	681.00	824.0	980.6	1335.0	1743.6	2206.6	2724.0	4256.0	6129.0
54	766.20	927.0	1103.3	1502.2	1961.5	2482.8	3065.0	4789.0	6897.0
60	852.00	1039.9	1235.8	1768.8	2179.6	2758.0	3087.2	5320.4	7662.1
66	936.40	1133.2	1348.5	1835.8	2397.7	3034.2	3745.5	5852.5	8427.8
72	1021.50	1236.0	1470.9	2002.5	2615.2	3309.9	4086.0	6384.0	9193.5

Until recently it was not found possible to make large turbo D.C. generators that would run quite satisfactorily. In slow-speed generators great care is exercised in the choice of constants limiting their general performance; the extra care needed in the successful design of turbo-generators is about in proportion to the differences in the speeds. To take an example:—The smallest power at which a turbine can be made to compete successfully with a high-speed engine is (not below) 500 K.W. In the high-speed engine case the revolutions per minute would be about 300, and the generator armature dimensions would be about 55 inches in diameter by 11 inches long, with say, 10 poles. By contrast the turbo-generator set would run at about 1500 revolutions per minute, or five times as fast as the H.S. engine set, and the armature dimensions would be about 30 inches in diameter by 18 inches long, with say, 4 poles.

The losses being in each case about the same, the turbo set will have to dissipate twice as much energy loss per square inch of radiating surface as the H.S. engine set, therefore the ventilation will have to be twice as good in the turbo set as in the H.S. set.

It would be better to discuss D.C. and A.C. turbos separately. For, although the general mechanical principles underlying the designs are in each case similar, the methods of treatment are radically dissimilar.

D.C. generators will be taken first, as much that has been said in the previous chapters will be required in the study of this type of generator.

Below is given a table of powers and speeds adopted by turbine makers, and these will be seen to be enormously faster than the fastest high-speed engine set, and will mean to the designer a radical change in his formulæ, if not their wholesale abandonment, and a reconstructed system based on experimental results. As a guide for students a table is given showing the constants adopted by designers of D.C. generators, and opposite, a table of suggested constants for use with turbo-generators.

Rated Output in K.W.	Speed in R.P.M.	No. of Poles.
100 to 1000	3000	2
1500 to 3000	1500	4
4000 to 6000	1000	6

A.C. Turbo-Generators, 50 ~.

Rated Output in K.W.	Speed in R.P.M.
50 to 300	3000
500	2000
750	1500

D.C. Turbo-Generators.

	Slow-speed Machinery.	High-speed Machinery.
Peripheral velocity (armature)	4000 to 5000 ft. per min.	12,000 to 20,000 ft. per min
„ (commutator)	2600 to 3000 ft. per min.	5000 to 6000 ft. per min.
Specific loading in ampere wires per inch of armature circumference ...	500	800
Watts per square inch bobbin surface	·5	·45
Watts per square inch commutator surface ...	3.0	4.0
Watts per square inch armature surface ...	3.0	5.0
Magnetic densities in lines per square inch, yoke (steel)	80,000	80,000
Poles in lines per square inch	110,000	110,000
Air-gap „ „	50,000 to 60,000	40,000
Teeth „ „	130,000 to 160,000	125,000
Core-plates „ „	65,000 to 80,000	40,000 to 50,000
Copper densities—		
Armature, amperes per square inch	2300	1500–2000
Carbon, amperes per square inch	35 carbon	50 to 60 graphitic carbon.
Ventilators	1 – ½ every 3 inches	1 – ½ every 1½ inch.

It would be better to take each member of the dynamo separately and apply the suggested constants, and in this way it will be found possible to get a good working idea.

Armatures.—The first suggestion which presents itself is that the armature should be made stationary as in an A.C. generator, and thus at once get over the difficulty of keeping the insulation of the wire good under the enormous strain which must be thrown upon it during running, and the protection of the end connections against the abrasion or cutting of the insulation, and the very difficult problem of commutator design; but the difficulties pale into insignificance by the side of those introduced by rotating the field-magnets and brush-gear round a stationary commutator.

The authors are in favour of as straight ahead a job as possible, without complication, and are confident that the design which has been found good in the past, with proper modifications, will be found equally good in the future.

D.C. turbo armatures, then, will be very similar in structure to the normal type, but it is now impossible to run at a peripheral velocity as low as 4000 to 5000 feet per minute—a velocity of at least double this will have to be arranged for. Take the 500 K.W. generator as an example (we might have taken 5000 K.W., but all that applies to this large size applies to the smaller with equal force), running at 1500 revolutions per minute. Below is given an output coefficient curve for this type of machine, from which it will be seen that the coefficient used is smaller than in a machine of similar output but of slower speed. The reason of this difference will be seen later. We have assumed a diameter corresponding with the maximum peripheral speed which we have previously settled, say 30, and from the output curve we find the length of the armature to be 18. We can be early on the look-out for commutation troubles, and we know that without some form of armature compensation it will be impossible to commutate the current. We have only four poles, and, of course, two collecting paths through the armature; also, we cannot use carbon brushes on account of the high peripheral commutator velocity, so that the good features which carbon brushes introduce must be foregone, unless we can keep the commutator velocity within reasonable limits; and the authors believe that it is possible to do so, and run carbon brushes at a higher velocity than has been usually thought possible.

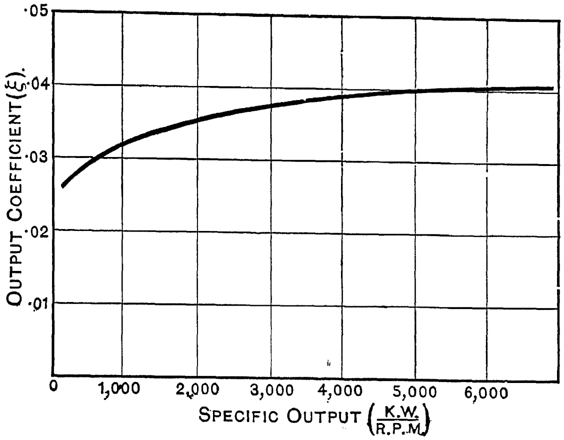

Fig. 151 P.—D.C. Turbo-Generators

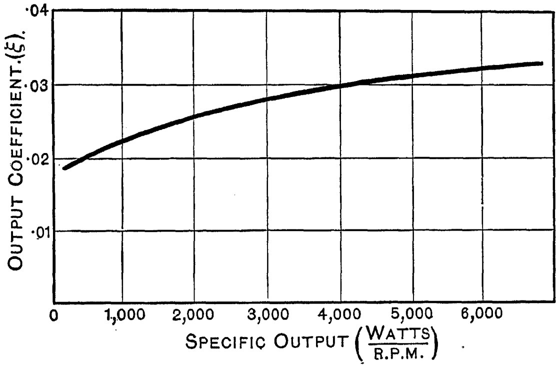

Fig. 151 Q.—50 ~ Turbo-Alternator

We will decide to use commutating poles as the simplest method to overcome some of the objectional features found in commutating large currents with high commutator speeds, and later we can consider some of the methods adopted by various manufacturers to the same end.

Commutating poles assumed, a much smaller ratio of polar span must be used, say 30 per cent less than the usually accepted standard, which would make the ratio somewhere in the neighbourhood of 50 per cent. The area of the pole face would be about 225 square inches, and to keep down the number of armature conductors we must have as large a field as possible, so we may fix the density in the pole face at say about 40,000 lines per square inch, and this would give us a flux of 10.2×10^6; this is near enough to go on with. Taking the above flux and using the formula $N = \frac{E \times 100}{\phi \times n} \times \frac{P_1{}^*}{P_2}$, we find we require 208 inductors, which we can quite conveniently arrange for round the circumference of the armature. So this flux can stand as fixed. As we noticed earlier, ventilation will have to be carefully looked to, and on the armature alone the watts to be dissipated amount to about 60 per cent of the total loss, and assuming a total efficiency of 94 per cent, the watts to be dissipated total up to 15 K.W., an extravagant amount for an armature 30 inches by 18 inches if not carefully ventilated. We will therefore have to use every possible means to ventilate the armature. Ventilating strips will have to be introduced, say every $1\frac{1}{2}$ inch at least, and great attention must be given to their design. The best types are those with a gently-curving vane, similar to fan vanes, and placed opposite every slot, as shown in fig. 151 R. This is a very costly design, but the authors believe the extra cost is justified. The effect of increasing the ventilating spaces is to deepen the core, if the length of the armature is not to exceed the length just fixed (and this point requires careful thought). If we increase the number of air-spaces, we decrease the net length. This would be an advantage as far as sparking is considered, as any decrease in the embedded length of a conductor tends to decreased reactance; but as it is in our power to obliterate the effect of reactance by the judicious use of commutating poles, this does not matter so much. Again, assuming a greatly deepened core, this would tend to restrict ventilation; we have to bear in mind that in no way must we constrict the ventilation spaces, but allow a free passage to the surface of the core. Right-angle bends and constricted openings, &c., will require power to drive the air through them, and some external means must be employed, such as a fan, which is a disadvantage, as it requires too great an expenditure of power for the resultant gain. In fig. 151 R are given examples of ventilating plates. In the authors' opinion B is the best method, the other not being so good.

The method adopted for bolting the armature to the spider should be positive. Dove-tail pieces on the core-plates should enter similar slots on the spider, the whole being bolted together by massive bolts. The end connections are a matter that must be carefully looked to. It is a great advantage that it is not likely that anything but strip-wound armatures will ever be required, and they do not present any great difficulties in

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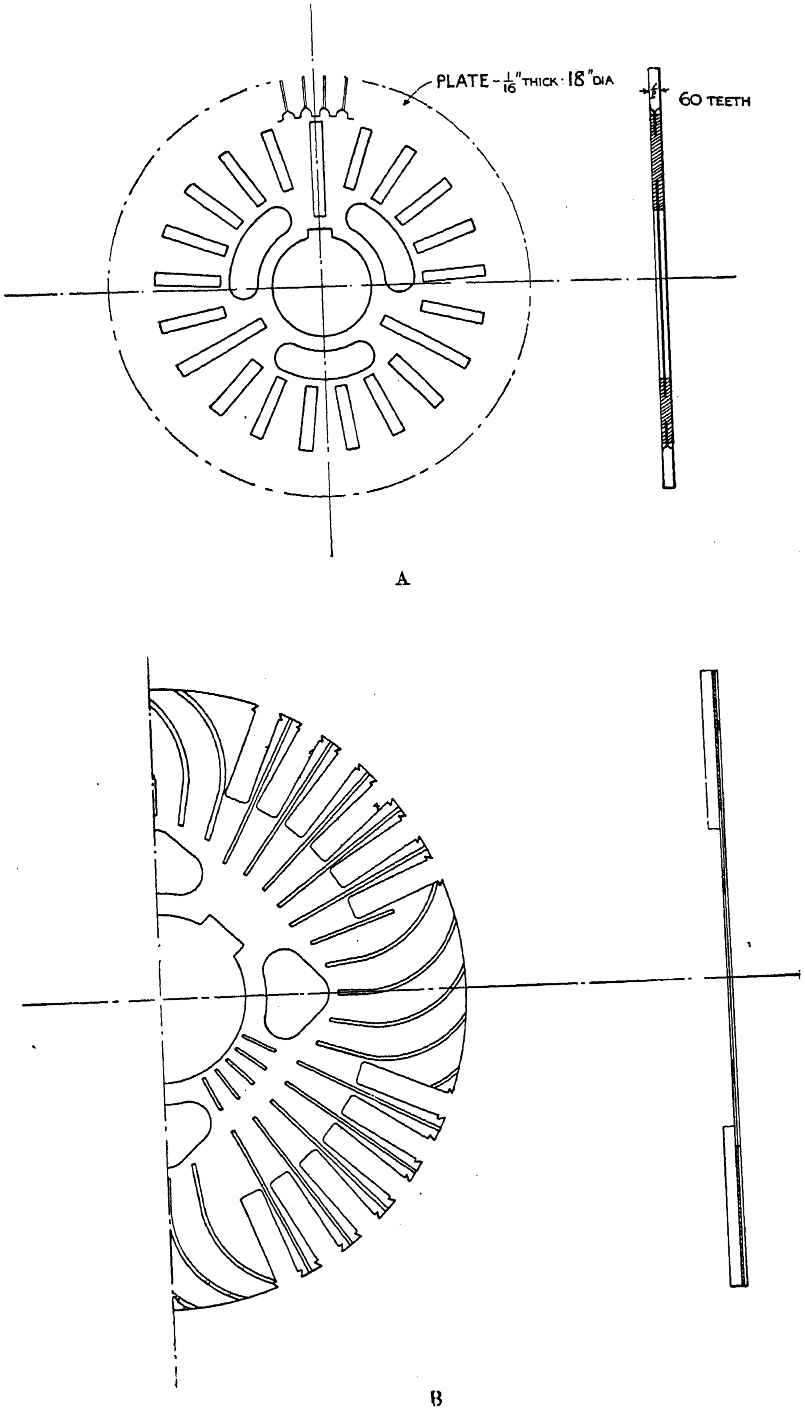
* For meaning of symbols see chapter on continuous generators, page 132.



A

B

Fig. 151R

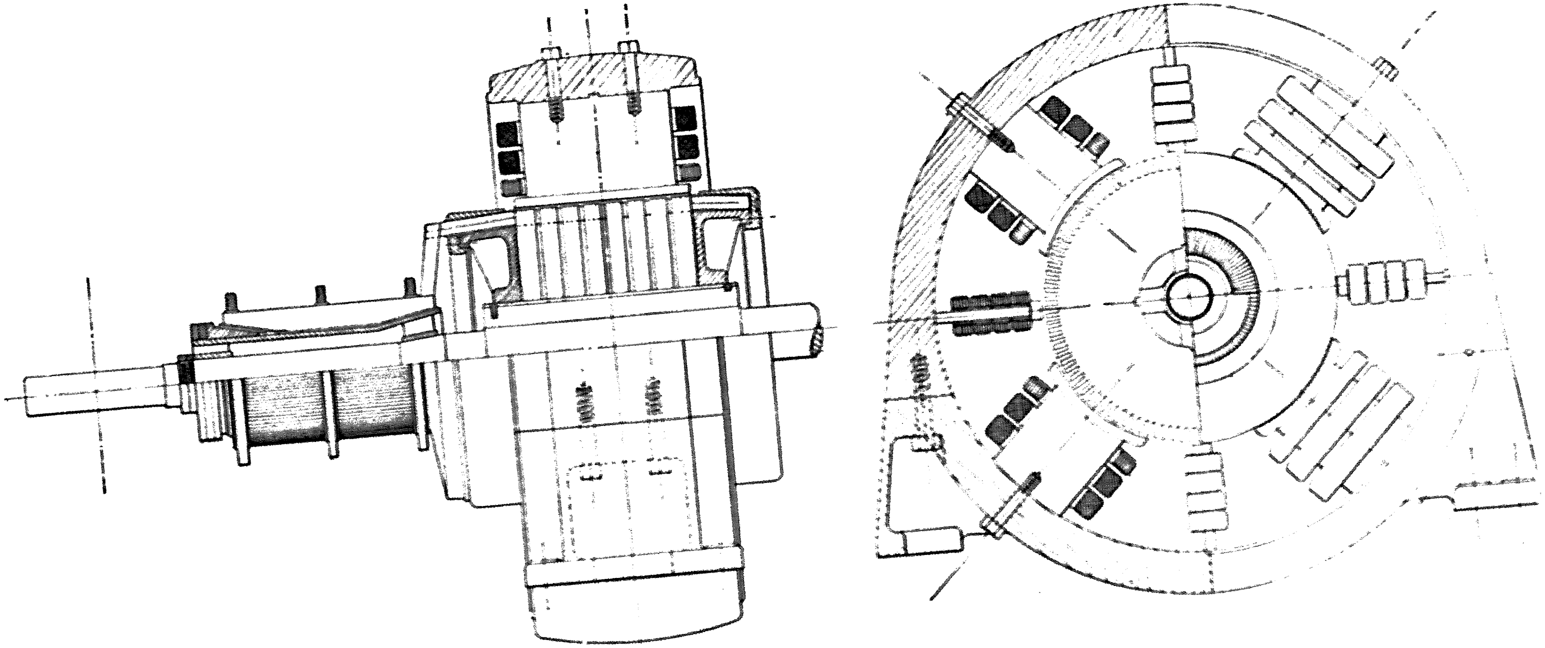


Fig. 151 S

insulating and binding. The end connections should be brought out over an end barrel, this barrel having a slight camber, as shown in fig. 151S, a layer of hard mica being interposed between the upper and lower windings, and a tightly-fitting cap drawn over all. This cap should be arranged to draw up on the wedge principle. Fig. 151S shows this method as adopted by the authors' firm.

Commutator.—We now come to the most difficult part of the armature of which to make a satisfactory mechanical job. The maximum velocity usually considered good practice in slower-speed machines is about 3500 feet per minute, but adopting as low a speed as this would mean a commutator of such a length as to be quite out of the question. A velocity of 5000 feet per minute could be arranged for by a radical change in design, and carbon brushes could be used quite satisfactorily at this speed. The authors know of machines in which carbon brushes are doing good service at 4000 feet per minute; and there is no material reason why brushes of suitable quality should not be run perfectly at a velocity of 5000 feet per minute. The carbon would necessarily be of a graphitic character, and the current density would be a little higher than in the ordinary carbon brush. The successful running of carbon brushes would depend more on the design of the brush-holder than upon the brush itself. The diameter of the commutator would be 12½ inches, and the collecting length 13 inches. In machines of much larger output the commutator might have to be cooled artificially.

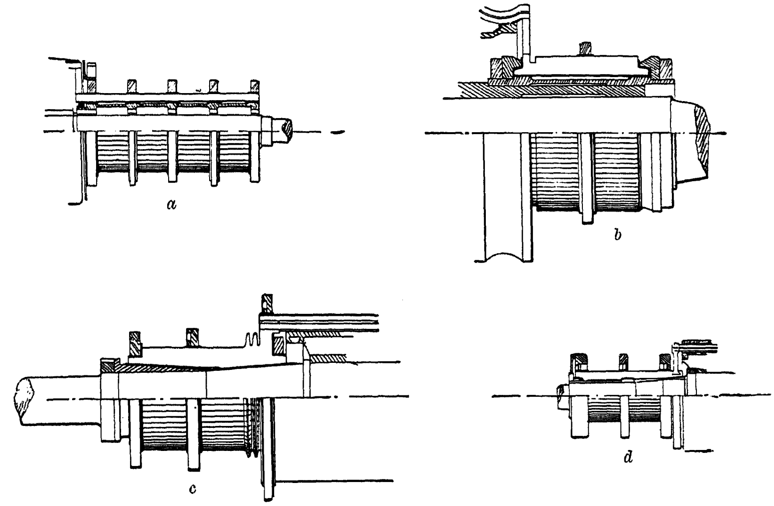


Fig. 151 T

In fig. 151 T different methods of fixing the bars are sketched, and that shown at *a* is a suitable design for very high speed, but they are all good enough for fairly high speeds.

Brush-Gear.—As before mentioned, a great deal more depends upon the design of the brush-holder than upon the brush itself. It is imperative that the brush must have as little inertia as possible, and must be allowed free radial movement. It must also be so placed in the holder that binding against the sides is an impossibility. The Morgan Crucible Co., who have spent a great deal of thought upon the matter, have brought out a brush-holder which embodies the above conditions to a remarkable degree, and is working very well at extremely high speeds. At the same time, designers always keep simplicity of construction in the foreground, but it can easily be overdone — the general tendency is to keep the brush-gear as simple as possible.

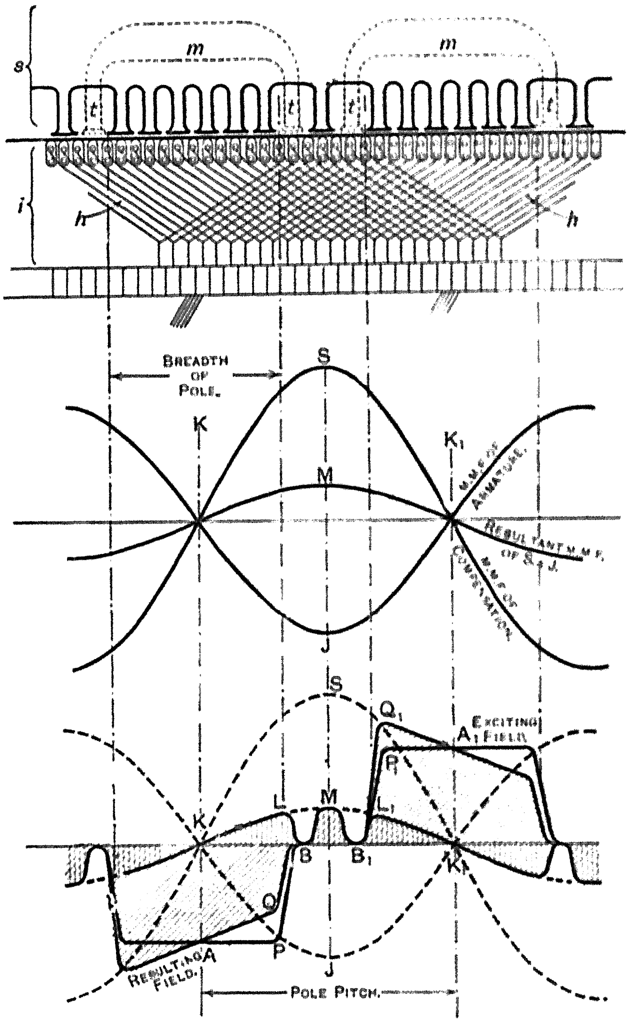


Fig. 151 u

Field - Magnets. — Field-magnet design has been taken lastly, as there is no great change in the construction for this type of machine as compared with those used for slower-speed machines, as will be seen from the tabulated figures given of generator at the end of the chapter; but a brief description of compensating devices may be given under this head.

The Deri system of compensation will be described, as nearly all other systems are only modifications of this type. This winding was patented about the year 1898, and to-day finds favour with Continental engineers. Many modifications of this winding are in general use, and give every satisfaction. In this method of compensation the armature field is balanced, and the commutating field is arranged in a similar manner to the interpole method.

Generally the machine is arranged as follows:—

The armature is in all respects similar to the ordinary armature, there being no special winding placed upon it. The field system, however, is totally different to the standard type of machine. In the first place, there

definite poles in which a winding is placed. The field system is to
pearances similar to the stator of an alternating-current motor. It is
up of punchings with teeth cut in the internal diameter. These teeth
enly spaced except mid-way between each pole—the neutral position
re one tooth is omitted on either side of the central tooth, as shown
n fig. 151U. This tooth acts as the reversing pole for the short-
ed windings. As the armature field is neutralized there can be no
ing or demagnetizing force, and the air-gap can be made merely a
nical clearance. In the slots on the inner circumference the exciting
gs are placed, and also the windings for neutralizing the armature
on.

e main exciting coils are placed in the larger slots formed by omit-
tooth. These windings form poles, as shown at P in the diagram,
51 U. Of course it will be understood that the exciting power
ed will be little as compared with a generator having definite poles,
ade of different types of material, requiring many ampere turns to
me the reluctance of joints, large air-gap, and considerable length
gnetic path. In the Deri machine the gap is merely a clearance,
ould be as small as $\frac{1}{16}$ of an inch per side. Again, the magnetic
are as short as possible, and of the best medium for conveying
etism.

e compensating winding consists of a number of conductors distri-
in the stator slots in a similar manner to the armature windings.
turns must equal the armature turns, or rather, the ampere turns
npensation must be slightly greater than the armature turns by that
it necessary to produce a field of sufficient strength to overcome the
nce of the short-circuited coil.

ere will be no other voltage drop on the machine than that produced
ohmic loss.

e brushes will remain in the same position at all loads.

the compensating winding is in series with the armature wind-
he field produced will exactly counterbalance the armature field,
he machine will generate sparklessly no matter what the load
be. This winding is therefore eminently suited for high-speed
nes.

ternators.—When we come to the design of turbo-alternators, an
ly new problem has to be faced. All that has been said with regard
ocities still holds good, and in this particular A.C. and D.C. genera-
resent the same defects. The points that more particularly interest
esigner are electrical rather than mechanical. There is, of course,
choice of speeds with a given frequency than in a D.C. machine.
umber of poles will be either two, four, or six, two-pole machines
used for moderate powers, say up to 500 kilowatts, and six-pole
nes for large powers, say over 2000 kilowatts. The greatest difficulty
overcome is perhaps voltage regulation. Many methods have been
ed to this end with excellent results. However, each designer must
the problem in his own way. There is not much published data to
and that which there is does not always fit in with the circumstances

under consideration. A 500-kilowatt 500-volt generator will be described, in which the principal difficulties, and the methods adopted to overcome them, will be outlined.

Below is given a table of speeds and number of poles, from which it will be seen a large choice is not given to the designer.

SPEEDS OF ALTERNATORS

No. of Poles.	40 ~	50 ~	60 ~	No. of Poles.	40 ~	50 ~	60 ~	No. of Poles.	40 ~	50 ~	60 ~
2	2400	3000	3600	30	160	200	240	58	83	103	124
4	1200	1500	1800	32	150	187	225	60	80	100	120
6	800	1000	1200	34	141	176	211	62	77	97	116
8	600	750	900	36	133	166	200	64	75	94	112
10	480	600	720	38	126	158	190	66	73	91	109
12	400	500	600	40	120	150	180	68	71	88	106
14	343	428	514	42	114	143	171	70	69	86	103
16	300	375	450	44	109	136	163	72	67	83	100
18	266	333	400	46	104	130	156	74	65	81	97
20	240	300	360	48	100	125	150	76	63	79	95
22	218	273	327	50	96	120	144	78	61	77	93
24	200	250	300	52	92	115	138	80	60	75	90
26	185	231	277	54	89	111	133				
28	171	214	257	56	86	107	129				

The first thing, then, that must be done is to fix the peripheral speed, as on this quantity everything depends. Should the velocity be kept down, say to about 9000 feet per minute, it would be found impossible to get on a sufficient amount of magnet-winding, and should the peripheral velocity be taken to the other extreme, the care which would have to be taken to guard against the enormous centrifugal forces that would be set up would put this design out of court.

The velocities usually adopted lie between 12,000 feet per minute and 20,000 feet per minute, the latter being considered the outside limit. A great deal of ingenuity has been expended on the design of field-magnets. Of course we presume revolving field-magnets; but without discussing this matter any further, it may here be said that the authors are of opinion that in small-powered machines it is better to adopt the core-plate type of magnet frame—that is, a tooth form of magnet built up after the manner of a D.C. armature, and fitted with exciting coils sunk in these slots,—and to adopt the definite pole type for larger powers.

In fig. 151V is given a sectional arrangement of the above machine, and at the end of the chapter are given the constants and sizes of the various parts which the reader is asked to consider. The peripheral velocity was fixed at 11,000 feet per minute, and the core-plate type of magnet centre decided upon. The reasons which led the authors to adopt it were as follows:—It is a fairly simple matter to thoroughly ventilate the machine. Every portion of exposed surface is bathed in a current of air circulating with a good velocity. The air is easily introduced into the very centre of the machine, the magnet plates can be put together without difficulty and the maximum strength of the material used to overcome the centrifugal forces, the winding can be fixed in the slots without great labour and by a simple device kept there under huge centrifugal forces. It is a simple matter to fix the overhanging ends of the winding, and to protect them from damage. The material is known to be homogeneous throughout, and the finished armature requires very little balancing.

Against these good points we have the drawback that there is not a definite pole. Such being the case, it is impossible that the E.M.F. curve can be sinusoidal. It is also difficult to arrange for a good shape of excitation curve, and it would be expected that the curve would be a hog-backed curve, showing no definite knee or bend as in the general type. This being so, it was thought better to let the air-gap play a larger part than ordinarily, that is, the air-gap ampere turns were made large enough to quite obliterate the effect of the iron ampere turns on the curve, and it very nearly approxi-

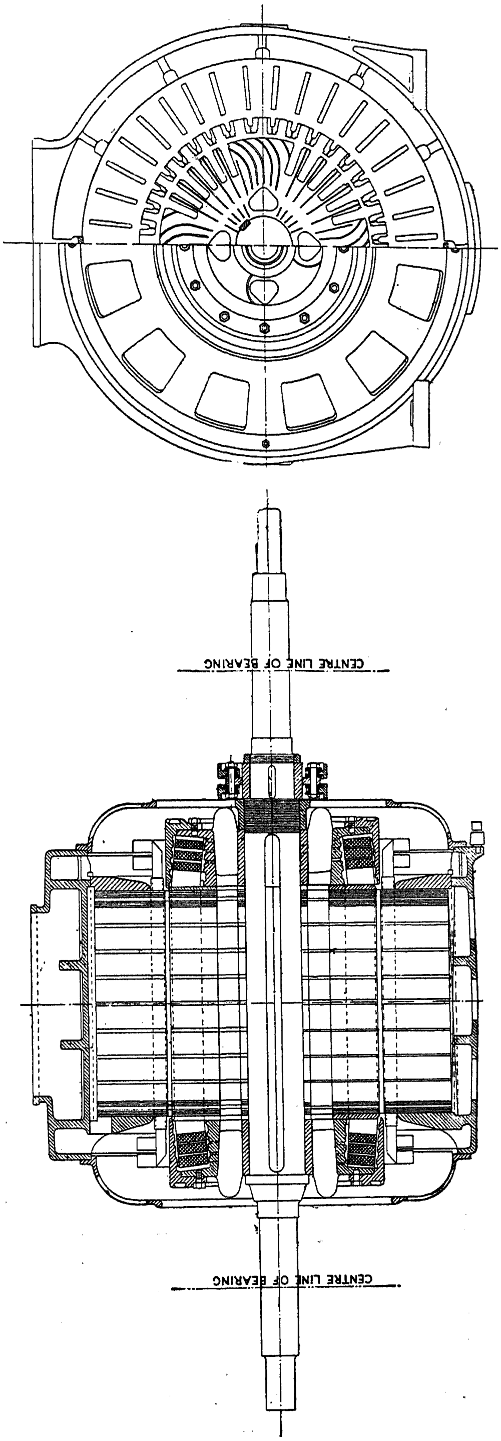


Fig. 151 v.—Section of Turbo-Alternator

mates to a straight line. The usually called-for voltage regulation would be complied with if the

$$\frac{\text{No-voltage field A.T.}}{\text{Full-load armature A.T.}} = 2$$

It will be seen by reference to the tabulated results given below that the armature turns per phase are 12.

A.T. on the armature = number of turns in series per phase × current per phase $\sqrt{2}$ × 1.5. As the rising and falling of the armature current does not follow a true sine wave, the multiplier $\sqrt{2}$ will be greater, probably by about 15 per cent. In fig. 151 W is given the excitation curve of the machine, showing the slope of the curve as compared with that of a slower-speed machine.

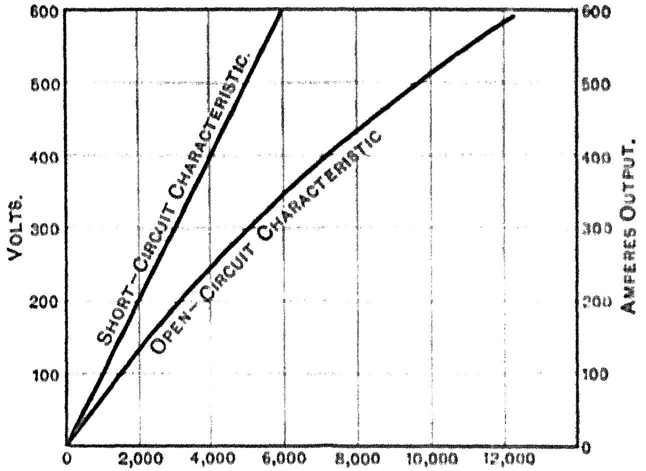


Fig. 151 W.—Test Curves of 500 K.V.A. Turbo-Alternator

500-KILOWATT THREE-PHASE ALTERNATOR

587 K.V.A. Revolutions, 1500. Number of poles, 4. Volts, 550.
Amperes, 615. Cos ϕ .85.

Outside diameter of stator core	45 inches
Inside diameter of stator core	27 "
Gross length of stator core	27 "
Number of ventilators	9
Size of each ventilator	½ inch
Net length of armature	20.2 inches
Number of slots	36
" " per pole and phase	3
" " per phase	12
Size of slot, 1.625 inch deep × 1 inch wide	
Conductors per slot, 2 made up of 4 copper strips	
Size of strip, 1.2 × .07 inch	
Total conductors in series per phase	24
Length of one turn	2.8 yards

Magnet core—

Length of magnet core	86 inches
Number of ventilators	9
Width of each ventilator	½ inch
Number of slots	24
Number of slots per pole	6
Depth of slots	4 inches
Width of slots	1 inch
Number of turns per slot	39
Wound, 13 deep and 3 wide	
Size of wire	.27 × .25
Total turns on four bobbins	468

Magnetic densities—

Air-gap	46,000
Armature core	41,000
Armature teeth	80,000
Magnet centre	127,000
Coefficient of leakage	1.25

Copper densities—

Armature copper	1450 at cos ϕ .85
Magnet copper	1800 at cos ϕ .85
Light load ampere turns on all bobbins	9600
Armature ampere turns	5000

500-KILOWATT D.C. TURBO-GENERATOR

500 volts. 1000 amperes. 1500 revolutions per minute. 4 poles.

DIMENSIONS

Armature—

External diameter	30 inches
Internal diameter	10 "
Gross length	18 "
Number of ventilating ducts	7
Width of each duct	½ inch
Net length of core	13.1
Peripheral velocity	11,800

Slots—

Total number	104
Depth	1.6
Width	.40
Slot pitch at face	.906
Slot pitch at root	.81
Width of tooth at face	.506
Width of tooth at root	.41

Inductors—

Total number	208
Number per slot (2 in parallel)	2
Type of winding	M.C.S.
Size of inductors	.57 × .13
Mean length of one inductor	1.4 yard
Voltage drop in armature (cold)	3 volts
Voltage drop at brushes	2 „
Internal voltage on load	530
Flux in armature (load)	10.2
Mean reactance voltage	10.0
Interpole flux	.425

Commutator—

Outside diameter	14 inches
Length over all	21½ „
Rubbing length	16 „
Number of segments	104
Width of segment and insulation at periphery	.424
Peripheral velocity	5500

Brushes—

Number of brush spindles	4
Brushes per spindle	10
Area of each brush	1½ × ¾ inch
Contact resistance per square inch	.02 ohm
Friction coefficient	.15
Pressure per square inch	2.5 lbs.

Magnets—

Leakage coefficient	1.32
Flux in magnets	13.5
Pole pitch	23.6 inches
Pole arc	12½ „
Ratio of pole arc to pole pitch	.53
Length of air-gap	.25 inch
Net area of pole face	225 square inches
Area of pole core	122 „ „
Area of yoke	78 „ „
Size of interpole face	13.1 × 2
Net area of interpole face	26.2 square inches
Size of interpole core	9 × 1
Area of interpole core	9 square inches
Flux in interpole core	.64
Leakage coefficient	1.5

Shunt windings—

Size of shunt wire	.048
Turns per pole	3750
Mean length of one turn	1.66 yard
Resistance per pole	83 ohms
Current	1.51 ampere

Series windings—

Size of series strip	1.5 × .24
Turns per pole	2½
Current	360 amperes
Density	1000

Interpoles—

Bobbins per pole	4 (in parallel)
Turns per bobbin	7½
Turns per pole	30
Size of strip	.8 × .15
Current	117 amperes
Density	976

Densities—

Air-gap (average)	41,200
Pole face	45,300
Pole core	110,000
Yoke	86,600
Armature core	46,300
Teeth (maximum)	126,500
Teeth (corrected)	121,000
Interpole core	71,000
Interpole face	16,200

Copper—

Armature inductors	1690 amp. per sq. inch
Brushes	44.5 „ „
Shunt bobbins	840 „ „
Series „	1000 „ „
Interpole „	976 „ „

LOSSES

Armature—

Copper, C_2R (hot)	3,300 watts
Iron (core)	5,870 „
Iron (teeth)	6,000 „
Total	15,170 „

Watts per square inch of periphery, 5.0

Commutator—

Brush contact, C_2R	1,780 watts
Friction	2,100 „
Total	3,880 „

Watts per square inch of periphery, 4.1

Main bobbins—

Shunt, C_2R	753 watts
Series, C_2R losses	147 „
Total	900 „

Watts per square inch of periphery, .43

Interpole bobbins—

C_2R loss (total)	235
Watts per square inch, .22	

Total armature losses	15,170	watts
„ commutator losses	3,880	„
„ magnet bobbin losses	900	„
„ interpole bobbin losses	235	„
Friction and windage	5,000	„
Total	25,185	„

Percentage of full load	5.0
Efficiency at full load	95.0

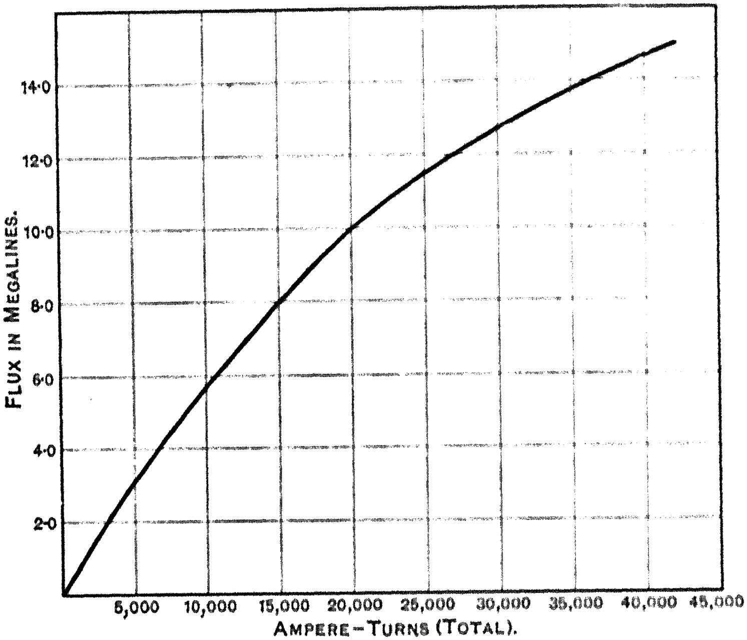


Fig. 151 x.—Open-Circuit Characteristic Curve

4. Alternating-Current Generators

CHAPTER I

GENERAL PRINCIPLES

In alternating-current generators an E.M.F. is induced in the armature windings by causing them to cut a magnetic field, the principle being exactly the same as in the case of D.C. generators; but whereas in the latter case a commutator is required in order to rectify the alternating current produced in the armature winding, and to convert it into a continuous current before it passes to the external circuit, in the case of the alternator the current as generated in the armature coils is led directly to the terminals.

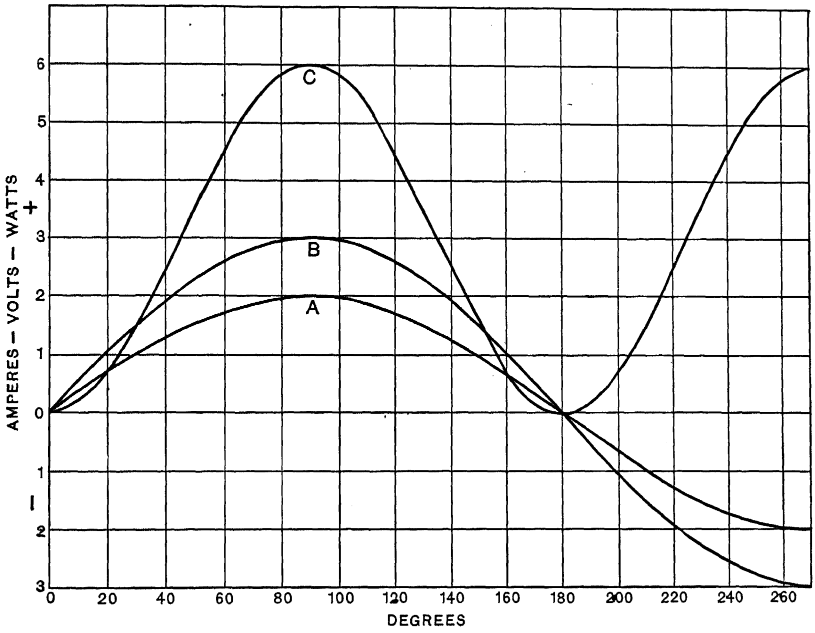


Fig. 152

The necessity of having a commutator makes it necessary, in the case of direct-current machines, that the armature should be the rotating member, the magnets and the brushes bearing on the commutator being fixed; in the case of the alternating-current generator this restriction does not exist, and we are free to make either the magnets or the armature the rotating member.

There are some obvious advantages attached to the plan of making the armature stationary: the current can then be led directly to the outer circuit without having to pass through any sliding contacts; the insulation of the coils can also be more easily carried out if they are not subjected to any centrifugal force. These points, whilst always an advantage, become of paramount importance in machines built for a high voltage; and thus we find that whilst a continuous-current machine usually consists of a rotating armature and a fixed field-magnet, in the case of alternators the reverse is the case, and we more usually find the magnets rotating and the armature stationary.

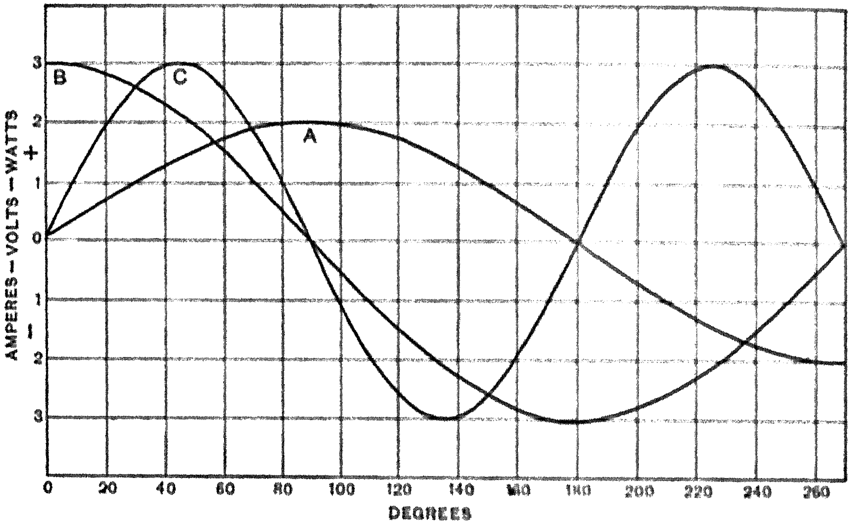


Fig. 153

In the case of continuous-current generators the power delivered by the machine is obtained by multiplying the current by the E.M.F. at the terminals, but in the case of alternators another factor has to be taken into account, namely, the phase relation of the current and E.M.F. In fig. 152 the current is shown by the line A, the corresponding E.M.F. by B, and it can be easily seen that by multiplying together the instantaneous values of the current and E.M.F. we obtain the instantaneous values of the power given out, plotted in the curve C. In this instance the E.M.F. and the current are in phase; they reach their maximum values at the same time, and pass through their zero values at the same time. The mean value of the power is obtained by simply multiplying the R.M.S. value of the current by the R.M.S. of the E.M.F. In fig. 153 the current (A) is shown lagging 90° behind the E.M.F. (B), and the change in the character of the power-curve (C), obtained as before by plotting the product of the instantaneous values of A and of B, is at once apparent; instead of being entirely above the zero line, as in fig. 152, the curve C is now partly above and partly below the zero line, showing that the power delivered is at some times positive and at others negative, *i.e.* the generator is at some instants delivering power to the external circuit, at others absorbing power from it.

When the difference of phase is 90°, the area of curve C below the zero line is equal to the area above this line, showing that over a whole period as much power is absorbed from the circuit as is delivered to it; the generator would in this case be doing no work, the current being entirely out of phase with the E.M.F.

By drawing a similar figure with intermediate values of lag, that is, with the current say 30°, 45°, and 60° out of phase, it will be readily seen that the power delivered to the circuit decreases as the angle of lag increases, and it can be proved that the power delivered to the circuit is obtained by multiplying the effective current by the effective E.M.F. and by cos ϕ where ϕ is the angle of lag.

By making different arrangements of the armature coils of a generator and connecting them up suitably, we can make it either into a single-phase or into a polyphase alternator. For example, a generator having all its armature coils connected in series and brought out to two terminals will be a single-phase alternator; if the coils are divided into two sets, such that the coils in each set are 90 electrical degrees apart from those in the other set, and each set of coils are connected together and brought out to a pair of terminals, we get a two-phase generator. The term "electrical degree" is a useful one in this connection, and its meaning is as follows:—The angular distance from one pole to the next similar pole on the machine will vary according to the number of poles on the machine. This angle, however, whatever it is in actual geometrical degrees, we always call 360 electrical degrees; thus the value of any angle on a dynamo in electrical degrees is obtained by multiplying the number expressing it in geometrical degrees by the number of pairs of poles on the machine. If the machine has only one pair of poles, that is, in the case of a two-pole machine, the electrical coincide with the actual degrees, and it is therefore usual, in diagrams of connections and in explanations of windings involving coils displaced through certain angles from one another, to assume a two-pole machine for the sake of simplicity.

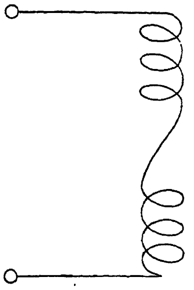


Fig. 154

In fig. 154 is shown diagrammatically a single-phase alternator. The diagram represents a two-pole machine, and the coils are connected in one circuit so as to give one alternating current from one pair of terminals.

In fig. 155 is shown a two-phase, or, as it is sometimes, and with greater accuracy, called, a quarter-phase alternator. The two windings are displaced through 90° to one another, and thus give two currents which differ in phase by 90°.

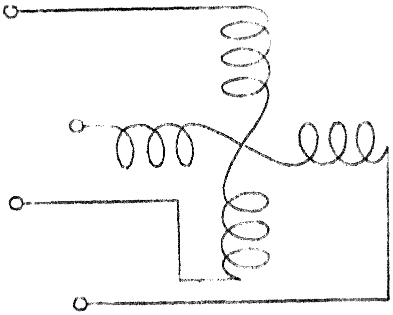


Fig. 155

One of the terminals in each phase may be connected together, making one common terminal to the two circuits, as shown in fig. 156; but with this arrangement of connections the current flowing from the common terminal differs in value from that at the other terminals. Its maximum value is $\sqrt{2}$ times as great as the maximum values of the other currents, and thus in a two-phase system with a common terminal the line common to the two phases must be proportioned to carry approximately 1.4 times the current carried by the other two lines.

A three-phase generator may be connected as represented in fig. 157, and then consists of three separate circuits displaced from one another by 120 electrical degrees.

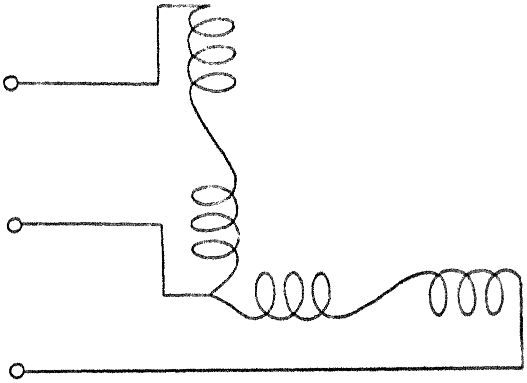


Fig. 156

More usually, however, the circuits are connected together, so that only three terminals and three lines are required. This may be done either as shown in fig. 158 and diagrammatically in fig. 159, which is called the "star" or "gamma" method of connection, or the three windings may be coupled together, as shown diagrammatically in fig. 160, where each forms one side of an equilateral triangle. This is called the "triangle" or "delta" method of connections.

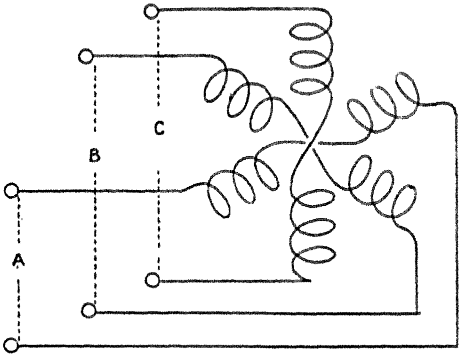


Fig. 157

In the "star" system of connections the three circuits have one common junction. This is frequently called the star-point, and it will be noticed that no connection is required to this point, as the sum of the currents flowing to or from it is always equal to zero. This is readily seen by plotting three sine curves displaced from one another by 120 degrees, and finding the algebraic sum of their ordinates at different points. This sum will everywhere be found to be zero.

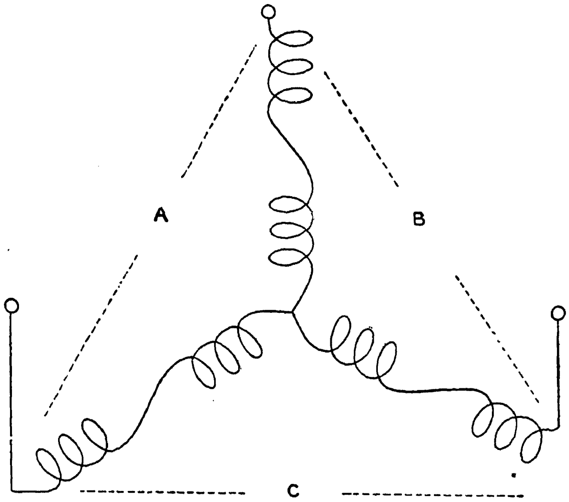


Fig. 158

In figs. 152 and 153 the currents and electromotive forces are graphically represented at each instant, each passing through zero value in the positive direction at a certain instant, attaining maximum value a quarter period later, and passing through zero in the negative direction at half period. If E and C be the maximum values, and if $\omega = 2\pi\sim$, and θ be the phase angle of difference between electromotive force and current, the values at any time t after E.M.F. passes through zero are

$$\begin{aligned} e &= E \sin \omega t \\ c &= C \sin (\omega t - \theta). \end{aligned}$$

Now the mean value of either of them is $2/\pi$ of the maximum value; and the effective value, or the root-mean-square (R.M.S.) value, is $1/\sqrt{2}$ of the maximum value.

To get the activity of the circuit, or the mean power given to the circuit, we have

$$\begin{aligned} ec &= EC \sin \omega t \sin (\omega t - \theta) \\ &= \frac{EC}{2} [2 \sin \omega t \,.\, \sin (\omega t - \theta)] \\ &= \frac{EC}{2} [\cos \theta - \cos (2\,\omega t - \theta)] \end{aligned}$$

Now the mean value of $\cos (2\,\omega t - \theta)$ over a complete period is obviously zero, as the values in the second and third quadrants are negative, and neutralize the positive values in the first and fourth quadrants. Hence the activity or power in a single phase generator

$$\begin{aligned} P &= \tfrac{1}{2} EC \cos \phi \\ &= E\ I \cos \phi, \end{aligned}$$

where P = watts output,
E = effective E.M.F. generated in the circuit,

I = effective current flowing through the circuit,
ϕ = angle between current and E.M.F.

In a two-phase generator, where each phase gives an E. M. F. of E volts and a current of I amperes,

$$P = 2\,E\,I\cos\phi$$

In a three-phase generator

$$P = 3\,E\,I\cos\phi$$

Since, however, the output of a three-phase generator is usually referred to the terminal E. M. F. and the line current, we obtain the following expression—

$$P = \sqrt{3} \times I_1 \times E_1 \cos\phi$$

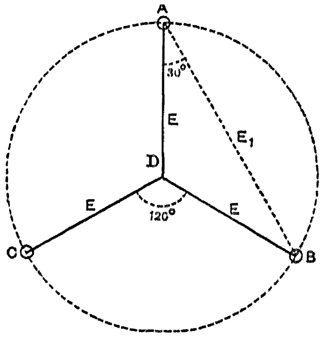


Fig. 159

To prove this, consider a generator in which the three windings are joined at one point, and the three collector rings connected to the remaining three ends of the winding, as fig. 158. In such a system the voltage between the extremities of two windings, as B C, fig. 159, is greater than that generated in each phase—that is, the voltage between A B, B C, and C A is greater than between A D, B D, and C D.

Referring to the diagram we find—

$$E_1 = 2\,E\cos 30 = 2\,E\,\frac{\sqrt{3}}{2} = E\sqrt{3}$$

The current leaving any of the collector rings is the same as flows in the windings, thus $I_1 = I$.

The total power of the three phases is obviously three times as great as that of one phase, thus—

$$P = 3 \times E\,I\cos\phi$$

(ϕ always referring to the phase relation between current and E.M.F. in each phase).

Inserting in this equation the values of line current and E.M.F., that is, substituting for I and E values of I_1 and E_1 as obtained from above equations, we get—

$$P = 3 \times \frac{E_1}{\sqrt{3}} \times I_1\cos\phi = \sqrt{3}\,E_1\,I_1\cos\phi$$

which is the equation given above.

There are, however, several ways of connecting circuits to establish three-phase relation of E.M.F.s, so, for instance, three single-phase circuits may be connected as in fig. 160.

In this case B and C are connected to one collector ring, D and E to another, and F and A to a third.

In this case the voltage between each terminal is obviously that of each phase, in other words, $E_1 = E$.

Any current drawn from a collector ring is, however, made up of the two currents of adjacent phases, and since the currents generated in each phase are displaced from that in the other, the resultant current is not the sum of the currents in each phase, that is, not $2 \times I$, but, as can be ascertained from the diagram, fig. 161.

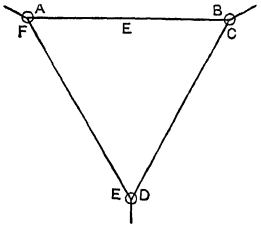


Fig. 160

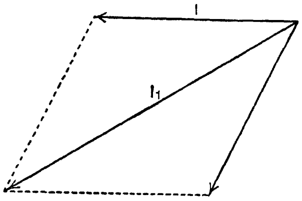


Fig. 161

$$I_1 = \sqrt{3}\, I$$

Substituting these values the values of I_1 and E_1, as above, we get as before—

$$P = 3 \times E \times I \cos \phi$$

$$= 3 \times E_1 \times \frac{I_1}{\sqrt{3}} \cos \phi = \sqrt{3}\, E_1\, I_1 \cos \phi$$

Still a third way would be to connect up two windings as in Fig. 162.

In this case also the power is expressed by

$$P = \sqrt{3}\, E_1\, I_1 \cos \phi$$

But a slightly larger machine is required to give the output, as is seen by the following reasoning.

With the same line current, the carrying capacity of the wire in each of the two windings must be the same as for the line current, that is, each armature conductor must carry I_1 amperes.

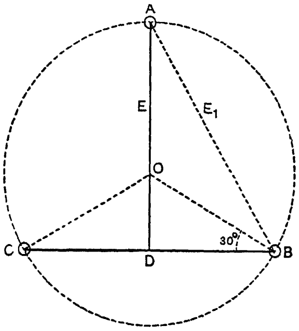


Fig. 162

Phase A D must be wound for:

$$E_{AD} = E + O\,D \text{ volts} = E + E \sin 30$$

$$= E + \frac{E}{2} = 1.5\, E$$

Hence one winding has to be made to give 1.5 E I power and the other $\sqrt{3}$ E I, or both together $(1.5 + \sqrt{3})\, E\, I$

$$= 3.23\, E \times I$$

Whereas if more favourable windings had been used the joint capacity of the three phases would have been $3 \times E \times I$.

CHAPTER II

CONSIDERATIONS ENTERING INTO THE DESIGN OF ALTERNATING-CURRENT GENERATORS

As a rule the designer is confronted with the following problem:—

To design a generator of specified capacity, frequency, and speed, so as to give a certain efficiency at the best possible regulation, or of given regulation with the highest possible efficiency.

The relation between the number of poles, the number of revolutions per minute, and frequency, is obviously:—

$$\text{Frequency} = \frac{\text{Revolutions per minute}}{60} \times \frac{\text{Poles}}{2}$$
$$= \frac{\text{Revolutions per minute} \times \text{Poles}}{120}$$

The efficiency is dependent upon the I^2R in armature and field, core loss in armature, and friction and windage loss.

The regulation depends upon the relations between armature and field magnetomotive force and upon the saturation of the magnetic circuit.

Although elaborate theories have been worked out, giving in a "ready-made" fashion equations which determine all these factors, as a matter of fact machines are designed almost entirely from experience with other machines of more or less similar nature.

The diameter of the armature is often determined quite arbitrarily —at least for the first design—by stipulating a given maximum peripheral speed, which experience shows gives good results with present methods of insulation and mechanical construction.

So, for instance, we may conclude that it is well to run at a peripheral speed of about 6000 feet per minute. That would give the following diameters for various number of revolutions per minute:—

Rev. per min.	Diam. in inches.	Rev. per min.	Diam. in inches.
100	230	900	25.5
300	76.6	1200	19.2
600	38.3	1800	12.8

Experience shows further that, with a given thickness and kind of insulation and a given mechanical construction, the current density in the armature copper should not exceed certain limits. This knowledge, then, gives the first approximation to the cross-section of wire to be used. This current density is, however, frequently governed by the guaranteed losses, but nevertheless for the first design it is wise to start out from the current density. This varies between 1000 and 3500 amperes per square inch, and largely depends upon the voltage and size of coil.

The amount of insulation to be allowed around the coils, and between layers of wire in the same coil, varies also very much, depending upon voltage and material, and it is quite impossible to lay down definite rules; the only safe way for the student is to investigate the practice in similar machines which are a commercial success.

The core loss can be determined with reasonable accuracy from preliminary tests on the iron used. It must, however, be borne in mind that, unless the method of assembling the laminations is correct, the eddy current losses may be as great as the true hysteresis losses.

In modern machinery, however, the eddy losses as a rule are small, and may not amount to more than 20 per cent of the hysteresis losses.

The windage and friction losses are most difficult to estimate. It is possible that these can be predetermined mathematically with new types of machines, but so far it has not been done. The problem is very intricate, to say the least. The manufacturers are, however, able to determine this with reasonable accuracy by building machines of known types—types which have been subjected to extensive tests to determine this feature.

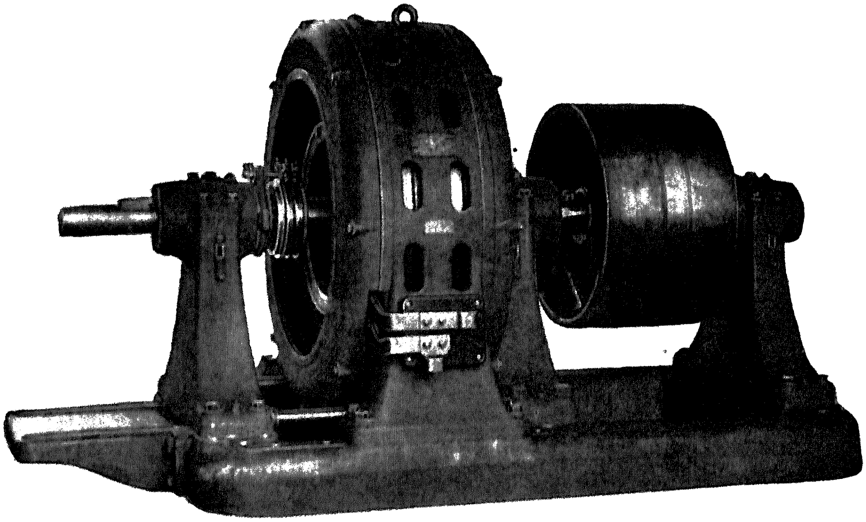


Fig 163.—Single-Phase Generator, revolving-field type

The heating of machines of standard types is fairly uniform, but when a departure from these forms is made this is a very uncertain feature, depending, as it does, so much on the thoroughness of the ventilation and the direction of the flow of air through the active material. The change in shape of a flange, or the addition of some small plate which will alter the direction of the air forced outward by centrifugal force, will often cause a machine, in which the rise in temperature was at first excessive, to run as cool as can be desired. For machines which have good ventilation it is safe to assume that the revolving fields when wound with bare copper will not rise more than 40° Centigrade per watt per square inch radiating surface, and 50° when wire-wound, while for stationary fields a rise of as much as 80° should be allowed for. The rise in armature windings may be taken as from 80° to 120° per watt per square inch, according to the thickness of the insulation, and the rise in the core may be taken as being about 40° per watt per square inch.

By using these constants an approximate idea of the rising temperature of any machine may be obtained, subject, of course, to large variations due to a more or less perfect ventilation.

The regulation can be calculated after the design is made and the saturation curve plotted, but although the reactance of an armature may be predetermined by calculations, these calculations involve the use of so many constants which can with certainty be determined only by experiments, that they can almost be said to be empirical. It is therefore considered best to give specific designs of representative machines of the various types, and from them let the student draw conclusions as to the constants entering.

Fig. 163 gives a half-tone reproduction of a 300-kilowatt 60-cycle single-phase alternator of revolving field type, of the following constants:—

Number of poles	16
Kilowatts	300
Revolutions per minute...	450
Terminal volts, full load	110
Amperes, full load	2720
Peripheral speed, feet per minute	5650
Frequency	60
Armature, diameter at face	48 inches
External diameter core	62 „
Internal diameter core	52.5 „
Length of core between heads	16 „
Per cent insulation between sheets	10
Number of ventilating ducts	5
Width of each	½ inch
Effective length of core	11.15 inches
Number of slots	32
Depth of slot	2¼ inches
Width of slot	1⅛ inch
Width of slot at armature face	1⅛ „
Minimum width of wide tooth	6 inches
Minimum width of narrow tooth	1.14 inch
Size of conductor	.45 × .35 sq. inch
Description of conductor	cable
Size insulated conductor	.485 × .385 sq. inch
Total number of face conductors	320
Arrangement of conductors per slot	2 × 5
Number in parallel per slot	1
Section of series conductor	1.9 square inch
Thickness of wedge	$\frac{5}{16}$ inch
Gap, length in centre	$\frac{3}{16}$ „
„ length, maximum	⅜ „
„ bore of field	47⅝ inches

Pole-piece, length parallel to shaft	15.5 inches
,, length of arc, maximum	6 ,,
,, length of arc, minimum	3 ,,
Magnet core, length, radial	6 ,,
,, length parallel to shaft	15.5 ,,
,, width or diameter	3¾ ,,
Spool, length over all	6 ,,
,, main conductor, size	No. 5 B and S
,, turns in series per spool	156
Yoke, outside diameter	33⅝ inches
,, inside diameter	27.99 ,,
,, thickness	2.8175 ,,
,, length parallel to shaft	18.5 ,,
Collector, number of rings	2
Armature, E.M.F. per circuit, full load	110
,, E.M.F. per circuit, no load	100
,, number of circuits	16
,, mean length of turn	48 inches
,, amperes per square inch conductor	1430
Main field, total resistance ohms	3.78
Megalines in armature, full load	4.13
,, in field, full load	5.4
Armature, section, square inches	106
,, length per pole (magnetic)	5 inches
,, density, full load per square inch	39,000
,, ampere turns, per pole, full load	1700
Teeth, under one pole-piece	1
,, section, square inches	67
,, length	2¼ inches
,, density, full load per square inch	62,000
,, ampere turns, full load	20
Gap, section at pole face	93 square inches
,, length, each side	.23 inch
,, pole face density, full load per square inch	44,500
,, ampere turns, full load	3260
Magnet core, section	55 square inches
,, length	6 inches
,, density, full load per square inch	98,000
,, ampere turns, full load	330
Yoke section	105 square inches
,, length	4 inches
,, density, full load per square inch	51,000
,, ampere turns, full load	50
Total magnetic ampere turns, full load volts	3720

Reactions—

Armature, reactive ampere turns per pole	1700
Field ampere turns, no load, no load volts	3720
,, ampere turns, full non-inductive load	4350
,, ampere turns, full inductive (90°) load	5420

Losses—

Armature, I^2R full load	1460 watts
" core loss, total full load	7284 "
Main field, I^2R total full non-inductive load ...	2915 "
Friction and windage	3000 "
Total losses, full load	14,659 "
" no load	10,481 "
Full-load efficiency...	95.34 per cent.

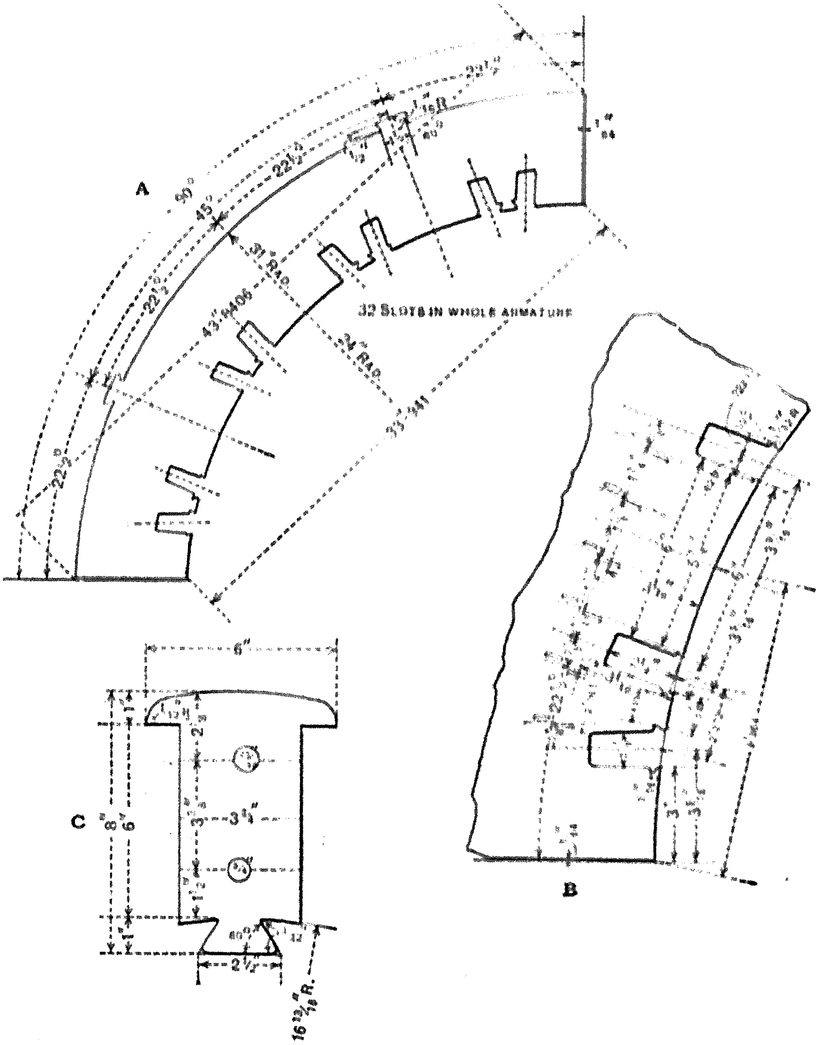


Fig. 164.—Armature and Field Punchings of Single-Phase Generator, revolving field type

Heating—

Armature coil radiating surface	4500 square inches
Watts per square inch	.324
Degrees C. rise per watt per square inch ...	80
Degrees C. rise	26

Heating (*continued*)—

Armature core radiating surface	12,800 square inches
Watts per square inch	·57
Degrees C. rise per watt per square inch	40
Degrees C. rise	23
Field coil radiating surface	5400 square inches
Watts per square inch	·54
Degrees C. rise per watt per square inch	50
Degrees C. rise	27

Sketches of armature and field punchings of this generator are shown in fig. 164.

Fig. 165 gives a half-tone reproduction of a 90-kilowatt 60-cycle single-phase generator of compound type and revolving-armature construction. The principal constants of this machine are:—

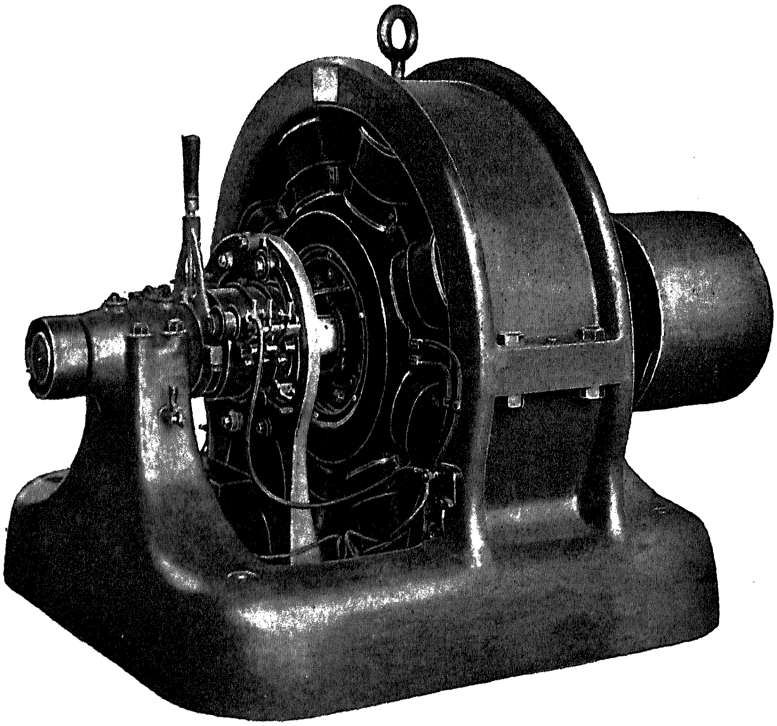


Fig. 165.—Single-Phase Generator, revolving-armature type

Number of poles	8
Kilowatts	90
Revolutions per minute	900
Terminal volts, full load	2300
Amperes, full load	39·1
Peripheral speed, feet per minute	5650
Frequency	60

Dimensions—

Armature, diameter at face	24 inches
,, external diameter core	18¼ ,,
,, internal diameter core	12½ ,,
,, length of core between heads	12 ,,
Per cent insulation between sheets	10
Number of ventilating ducts	5
Width of each	½ inch
Effective length of core	8.55 inches
Number of slots	16
Depth of slot	2⅞ inches
Width of slot	¾ ,,
Width of slot at armature face	¾ ,,
Maximum width of tooth	5.48 inches – 2.38 inches
Minimum ,, ,,	4 ,, – 1.64 ,,
Size of conductor	.0163 square inch
Description of conductor	No. 10 B and S, 2 in multiple
Size insulated conductor	.112 × .214 square inch
Total number face conductors	1280
Arrangement of conductors per slot	4 × 20
Number in parallel per slot	2
Section of series conductor	.0326 square inch
Thickness of wedge	$\frac{7}{16}$ inch
Gap, length in centre	$\frac{3}{16}$,,
,, bore of field	24⅜ inches
Pole-piece, length parallel to shaft	12 ,,
,, length of arc, maximum	4¾ ,,
,, length of arc, minimum	4¾ ,,
Magnet core length, radial	8¾ ,,
,, length parallel to shaft	12 ,,
,, width or diameter	4¾ ,,
Spool, length over all	8 ,,
,, main conductor, size	No. 11 B and S, .091 sq. in.
,, turns in series per spool	390
Yoke, outside diameter	48 inches
,, inside diameter	43 ,,
,, thickness	2½ ,,
,, length parallel to shaft	17 ,,
Collector, number of rings	2
Armature, E.M.F. per circuit, full load	2300
,, E.M.F. per circuit, no load	2080
,, number of circuits	1
,, mean length of conductor	42 inches
,, amperes per square inch conductor	2400
Main field, total resistance	13.8
Compound field, type	series
,, ,, number of turns in series per spool	64
,, ,, mean length of turn	3.38 feet
,, ,, total resistance	.755 ohm
Megalines in armature, full load	2.7

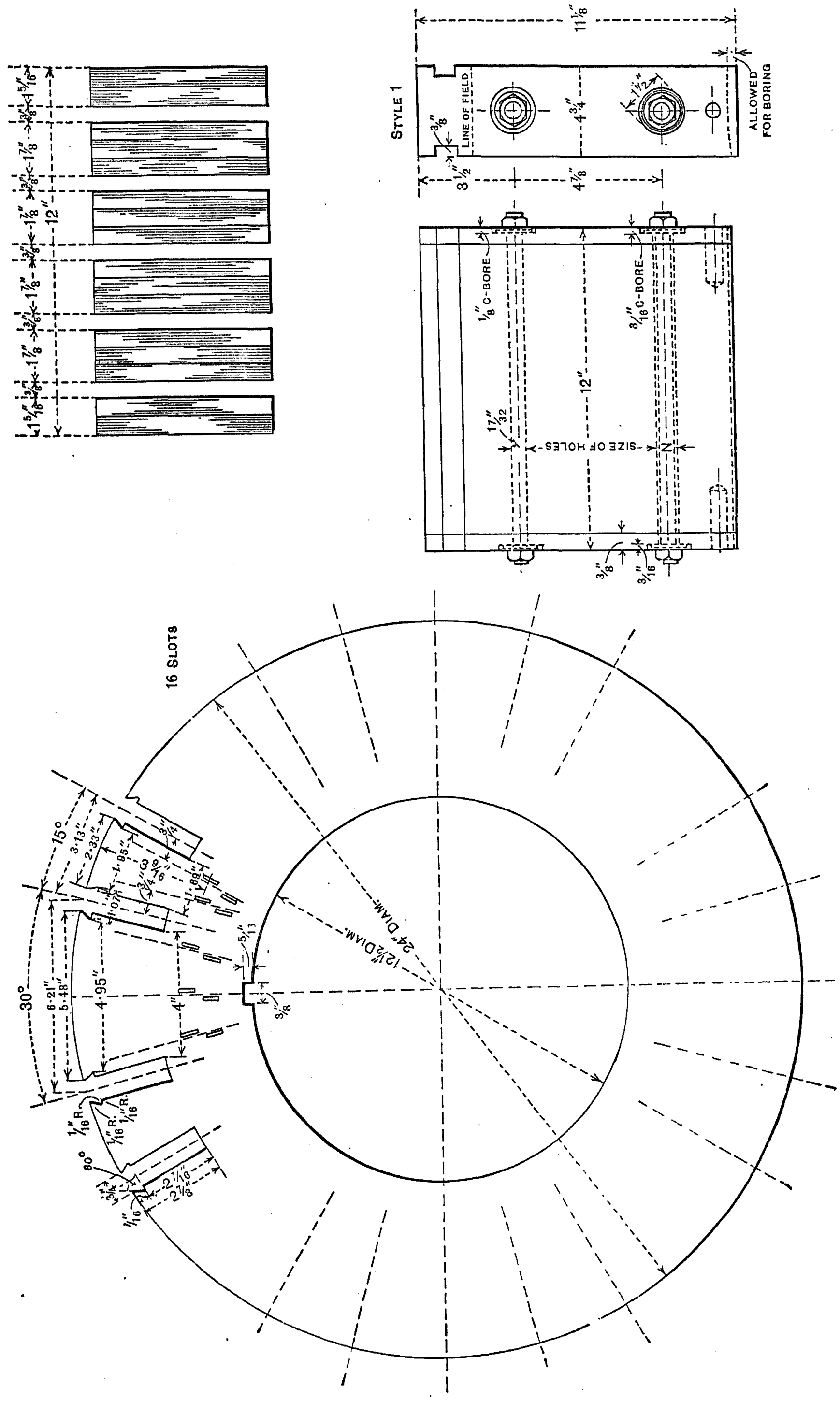


Fig. 166.—Armature and Field Punchings of Single-Phase Generator, revolving-armature type

Dimensions (*continued*)—

Megalines in field, full load	3·5
Armature, section	49.2 square inches
„ length (magnetic) per pole	3.02 inches
„ density, full load per square inch	54,600
„ ampere turns, full load	31
Teeth, number under one pole-piece	1
„ section	34.2 square inches
„ length	2.875 inches
„ density, full load per square inch	78,800
„ ampere turns, full load	60
Gap, section at pole-piece	57 square inches
„ length, each side	$\frac{3}{16}$ inch
„ pole face density, full load per square inch	47,250
„ ampere turns, full load	2770
Magnet core, section	51 square inches
„ length	8¾ inches
„ density, full load per square inch	68,400
„ density, no load „ „	62,500
Yoke, section	100 square inches
„ length	9 inches
„ density, full load per square inch	33,000
„ ampere turns, full load	436
Total magnetic ampere turns, full load volts	3220
Armature, reactive ampere turns per pole	1565
Field, ampere turns, no-load, no-load volts	2850
Field, ampere turns, full non-inductive load	3900
Field, ampere turns, full inductive (90°) load	5612
Armature, core loss, total full load	4000 watts
Main field, $I^2 R$ total full non-inductive load	738 „
Compound field, $I^2 R$ total full load	478 „
Friction and windage	1000 „
Total losses, full load	7400 „
„ „ no load	5320 „
Efficiency, full load	91.78%

Sketches of the armature and field punchings of this generator are shown in fig. 166.

For the instruction of the student the principal calculations of the first-mentioned machine are given below.

The relation between E.M.F., flux, frequency, and ampere turns in series is expressed as—

$$E = \frac{\pi \sqrt{2} \times M \times n \times t}{10^8}$$

where E = E.M.F. generated in the winding,
M = flux per pole in armature,
n = frequency,
t = turns in series per circuit in armature.

In this machine, by substituting proper values in above equation, we get—

$$110 = 4.44 \times \frac{M \times 60}{10^8} \times \frac{160}{16}, \text{ or } M = 4{,}130{,}000$$

Due to leakage between the field poles, it is necessary to supply a higher magnetomotive force than corresponds to this flux, the amount varying in different types of machines with the ratio of length of poles to the space between them, the length of gap, &c. The "leakage coefficient" can be calculated without much difficulty in much the same manner as the distribution of currents in multiple circuits. It varies between 10 per cent and 30 per cent in revolving field machines of modern construction. Assuming therefore 30 per cent, we have in the field poles a flux of 5.4 megalines, in the armature a flux of 4.13 megalines.

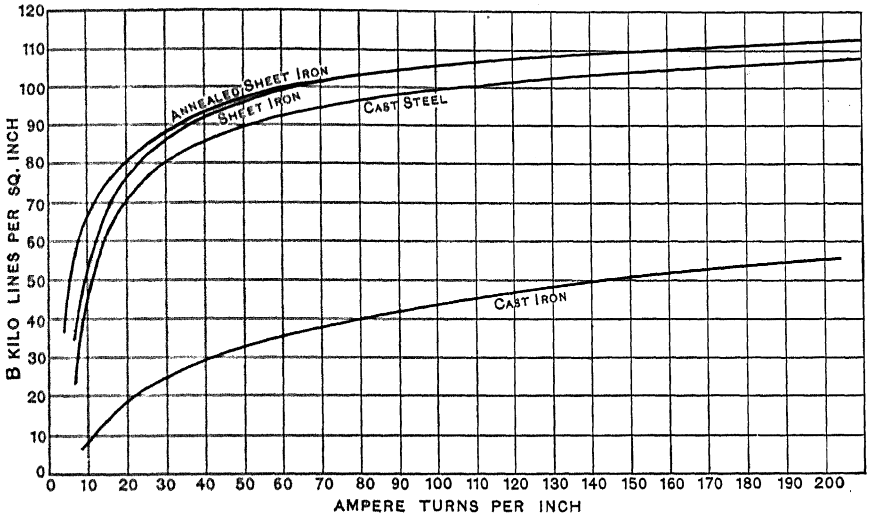


Fig. 167

The magnetic density, that is, the flux per square inch cross-section, can readily be calculated from the dimensions given; and the corresponding magnetomotive force obtained from saturation curves of the magnetic material used.

Fig. 167 gives what might be called standard saturation curves, being averages of a number of tests on different kinds of commercial iron and steel. The abscissæ give the M.M.F. in ampere turns per 1 inch length of magnetic circuit, the ordinates the corresponding density in kilolines per square inch. From these curves the necessary no-load M.M.F. is seen to be 3720 ampere turns.

The reactive ampere turns per pole in the armature of a single-phase generator may be expressed as the product of current and armature turns per pole, thus—

$$2720 \times \frac{10}{16} = 1700$$

At non-inductive load these ampere turns are demagnetizing in a direction at right angles to the field ampere turns, therefore the total number of ampere turns at full non-inductive load should be—

$$A\,T = \sqrt{(\text{no-load } A\,T)^2 + (\text{armature reactive } A\,T)^2}$$

Due to the effect of armature self-induction, however, a greater number of ampere turns are required. In this particular type experience has shown that a close approximation can be obtained by substituting for the reactive ampere turns a synchronous reactance one-third greater, therefore the excitation at full non-inductive load should be—

$$\sqrt{3720^2 + (1.33 \times 1700)^2} = 4350$$

The field excitation at full inductive load is in this particular type the sum of no-load ampere turns and armature reaction, or 3720 + 1700 = 5420 A T.

It is possible to calculate this in a more theoretic way, but such investigations lead to very long and complex equations, involving constants which eventually have been derived from experiments on similar machines.

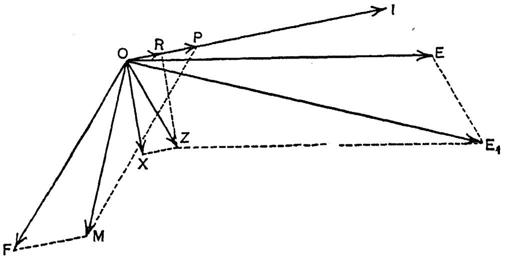


Fig. 168

Assuming that the internal reactance, resistance, equivalent armature reaction, or rather demagnetizing effect of the armature M.M.F. and the no-load ampere turns are known, the resultant M.M.F. for any phase relation of current and E.M.F. may be obtained graphically as follows:—

Let, in fig. 168, O E be the E.M.F. at the terminal of the generator;

O I the phase of current, which in this case is shown lagging behind the terminal E.M.F.

O R is the E.M.F. consumed by the resistance of the armature winding;

O X the E.M.F. consumed by the reactance of the armature winding, and therefore 90° ahead of the current;

O Z the total E.M.F. consumed in the armature.

The E.M.F. which must be induced in the armature to give O E at the terminal is therefore O E_1.

From the no-load saturation curve is found the M.M.F. O M, which is required to give this E.M.F. (O M is obviously 90° ahead of O E_1, since the M.M.F. is in phase with the flux, and the flux 90° ahead of the E.M.F. induced by the flux).

This M.M.F. O M is the resultant of the field and armature M.M.F., that is, of O P and O F. In which case O P is the M.M.F. of the armature, that is, the reactive ampere turns per pole in the armature, which, of course, are in phase with the current, and O F the field M.M.F., that is, the number of ampere turns necessary to give the proper terminal voltage.

The regulation of an alternating-current generator is defined as the percentage rise in voltage when full non-inductive load is thrown off at full-load excitation. It is obvious that this can readily be determined after the corresponding field excitation (as given above) and saturation curve are determined. In this case the regulation is seen to be 11 per cent.

The short-circuiting current, that is, the current flowing in the armature, if short-circuited at full-load excitation, is determined in a similar way. In that case the armature has to demagnetize the field to such an extent, that only the flux required to give sufficient E.M.F. to overcome the internal drop is left. The ratio of short-circuit current to full-load current can therefore approximately be expressed as $\frac{\text{Full-load field excitation}}{\text{Armature reaction}}$.

Neglecting the effect of change in saturation, the relation between armature current and terminal voltage at constant field excitation can be readily found by the following equation:—

$$E = \sqrt{(e + ir + i_1 x)^2 + (i_1 r - ix)^2}$$

where E is the no-load E.M.F. corresponding to the field excitation,
e the terminal voltage,
i the energy current,
i_1 the wattless current (which is positive if the wattless current is lagging, negative if leading),
r the armature resistance,
x the armature reactance.

Neglecting the armature resistance, we find—

At short circuit when $e = 0$ and $i = 0$

$$E = i_1 x, \text{ or } x = \frac{E}{i_1}$$

Thus, if short-circuiting current is known, a value of synchronous reactance x can be obtained.

The field characteristic of the machine which gives the relation between terminal voltage and current at constant excitation can readily be calculated from the equations given above.

In the case of non-inductive load, negligible armature resistance, this relation is expressed as—

$$E^2 = e^2 + i^2 x^2$$

which is the equation of an ellipse.

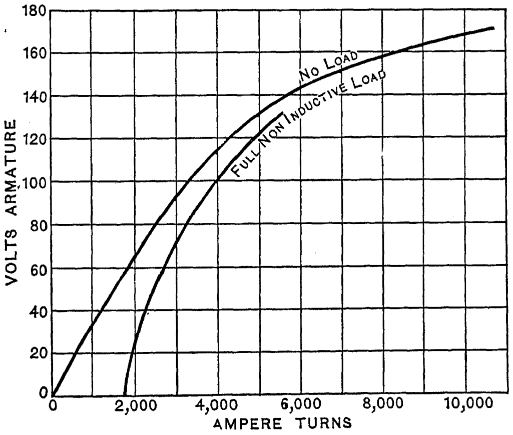


Fig. 169

As an instance:—The machine given above requires at full load 4350 ampere turns per pole. Its armature reaction is 1700 ampere turns. Thus, short-circuit current at full-load excitation is 2.56 times full-load current, or 2.56 × 2720 = 7000 amperes.

From the no-load saturation curve of the machine, fig. 169, we find that corresponding to 4350 ampere turns is 122 volts; thus—

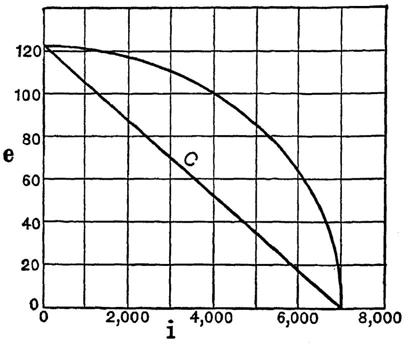


Fig. 170

$$x = \frac{122}{7000} = .0173 \text{ ohm}$$

and $122^2 = e^2 + i^2 .0003$

Substituting various values for i in this equation, and solving same in reference to e, we are enabled to plot the characteristic curve as given in fig. 170.

With a full inductive load it is readily proven that the characteristic is a straight line, as shown in line C, same diagram.

Corresponding curves for other power factors can readily be calculated from the equations given; in general, it will be found that as soon as the power factor is less than 80 per cent, the characteristic approaches closely that for full inductive load.

CHAPTER III

CALCULATION OF LOSSES AND EFFICIENCY

Calculation of I^2R Armature Losses.—The armature has 32 slots, and each slot has 2 × 5 conductors, consisting of pressed cable (to avoid eddy losses). Area of cable = .45 × .35 = .1575 square inch. Assuming 75 per cent to be actual copper, this would make an effective area of .75 × .1575 = .118 square inch. Mean length of armature turn is 48 inches. Therefore, the resistance per circuit = .00315 ohm. Amperes per circuit $= \frac{2720}{16} = 170$. I^2R per circuit $= 170^2 \times .00315 = 91.2$ watts; thus, total I^2R armature losses = 91.2 × 16 = 1460 watts.

Core Losses.—As stated above, to determine the hysteresis losses of the iron used, it is necessary to test each individual lot. Assuming, however, an average hysteresis coefficient of .0033, and with the use of Steinmetz's law, the following curve, giving the relation between watts lost per cubic inch of magnetic material at one cycle per second and corresponding density, has been plotted.

Calculation of Core Loss in Teeth.—

Volume of teeth = 3330 cubic inches. B = 62,000
Volume armature core = 9470 „ B = 39,000

Thus, from fig. 171 the watts loss per cubic inch at one cycle are—

For the teeth = .0127. For core = .0062
Hysteresis loss, teeth = 3330 × 60 × .0127 = 2540 watts
Hysteresis loss, core = 9470 × 60 × .0062 = 3530 „

Making a total hysteresis loss of 6070 watts. Allowing, as stated above, 20 per cent for eddy losses, the total core loss would be 7284 watts.

I^2R loss in field winding is calculated in exactly the same way as I^2R armature loss. Each spool is wound with 156 turns of No. 5 B and S wire. Mean length of turn = 50.5 inches, or 4.2 feet. Total length of turn = 156 × 4.2 = 655 feet. Wire No. 5 B and S at 60° C. has a resistance of .357 ohm per 1000 feet.

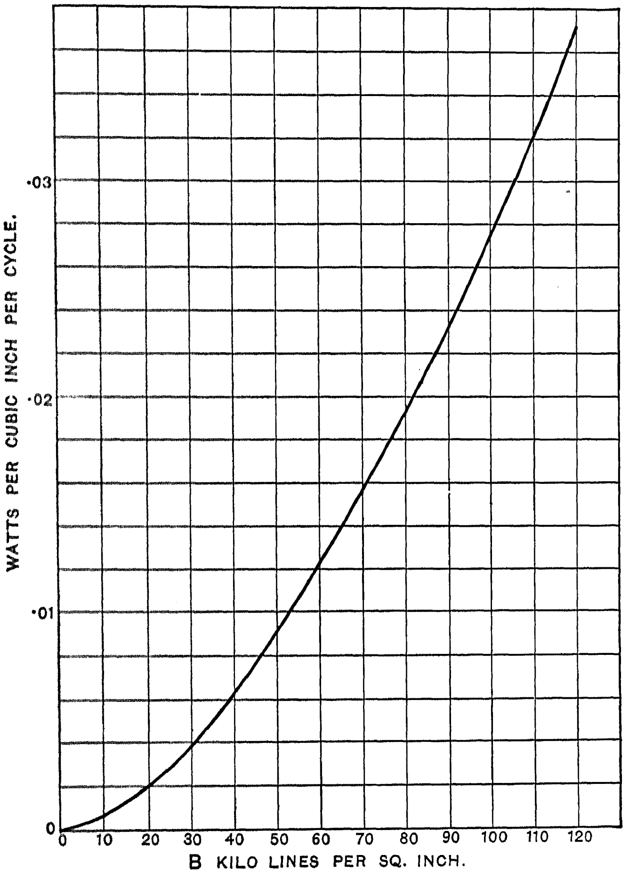


Fig. 171

Thus, resistance per spool = .357 × .655 = .234 ohm, or total resistance of all 16 spools = 3.74 ohms.

$$\text{Exciting current} = \frac{4350}{156} = 27.95 \text{ amperes}$$

$$I^2R = 27.95^2 \times 3.74 = 2915 \text{ watts}$$

Friction and windage are assumed to be 1 per cent, or 3000 watts Thus, total losses at full load = 1460 + 7284 + 2915 + 3000, or 14,659.

$$\text{Efficiency at full load} = \frac{300}{314.659} = 95.34 \text{ per cent.}$$

CHAPTER IV

TWO-PHASE GENERATORS

There need be very little said about the design of this type of machine, since it is essentially a single-phase machine with two independent windings placed 90° apart, that is, the two windings are displaced one-half of the distance between one pair of poles.

In designing such a generator each phase is treated independently, that is, for instance, if it is desired to lay out a 100-kilowatt two-phase generator which will give a potential of 2000 volts, each phase is designed for 50 kilowatts and 2000 volts, and the determination of flux, losses, excitation, &c., is made exactly in the same way as with single-phase machines.

In fig. 172 is shown the arrangement of a two-phase inductor alternator as commonly constructed. It has two stationary external armatures, each having two sets of coils corresponding to each of the two phases. Since all poles for each armature have the same magnetic potential, the armature coils, as the field sweeps by, are only subjected to a change of magnetism corresponding to a given maximum value and zero; thus, for a given flux per pole, the E.M.F. induced must be one-half of that in an ordinary machine, or *vice versa*; twice as large a flux and twice as much magnetic material is required for the same armature reaction.

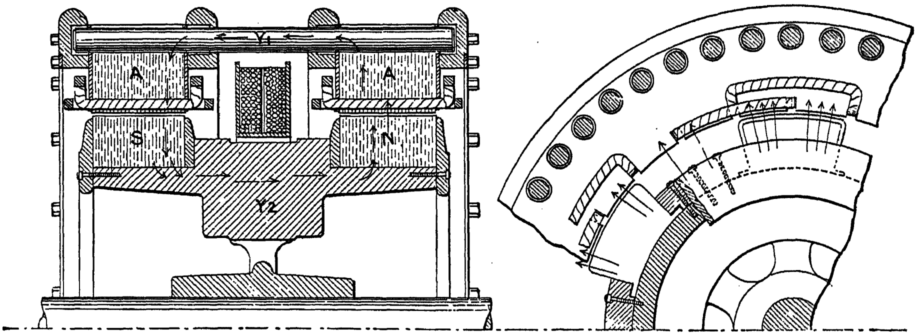


Fig. 172.—Two-Phase Inductor Alternator

To reduce the amount to the least possible, and also to reduce the eddy losses introduced by pulsations in the main magnetic circuit, very high densities are usually employed. This obviously requires a large field excitation for the iron part of the magnetic circuit; therefore, in order to keep the total M.M.F. as low as possible, the machines are usually built with small clearance.

These generators are often supplied with a switching device by which the number of armature conductors per circuit can be varied, and thereby the voltage changed. This is somewhat more complicated than by changing the field excitation, but it is warranted by the high densities employed.

The field excitation is supplied from a single coil placed between the two sets of poles and fastened to the stationary armature structure.

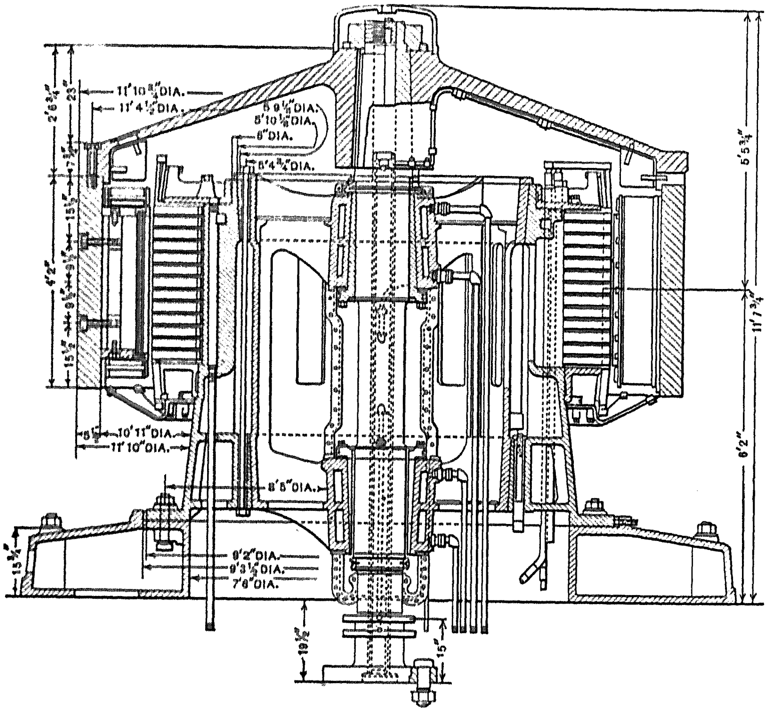


Fig. 173.—Two-Phase Generator

This has the advantage that no moving wires are used, but the disadvantage that if the field coils for some reason or other are punctured repair is difficult.

The plant of the Niagara Falls Company offers a good illustration of a type of two-phase generator, as is shown in fig. 173. It was built and installed by the Westinghouse Company of Pittsburg in 1895, and is of the external revolving-field type, having a capacity of 5000 horsepower. The speed is 250 revolutions per minute, which, with twelve poles, gives a frequency of 25 cycles. The outside diameter is 11 feet 10 inches, thus the peripheral speed is extremely high, necessitating a very strong and expensive nickel-steel ring for supporting the poles, and requiring very careful workmanship in order to secure a balance of the revolving parts. Although the efficiency of these machines is high, water cooling becomes a necessity, as the loss on such a large unit is about 100 horse-power. The most noticeable electrical feature is the high armature reaction employed, so that the regulation of these machines permits a rise of 32 per cent in voltage if full non-inductive load is removed at full-load field excitation. Later machines, furnished by the same company, showed some improvement in the methods of carrying off the heat generated. The first radical change in design appears in the next addition of generators, furnished by the General Electric Company of Schenectady, U. S. A., which, while still holding to the original type, have greatly improved electrical constants. The regulation, instead of being 32 per cent, is reduced to 10 per cent through a diminution of the armature reaction accompanied by an increase of flux and field excitation. The full-load efficiency, taken from actual test, is 98.2 per cent, which is probably the highest efficiency ever attained in an electrical generator. The last step towards the present design comes with the substitution of the internal for the external revolving field. This change, while giving machines of the same electrical constants, greatly lowers the cost, principally by making unnecessary the huge nickel-steel revolving ring. It is of interest to note that this type has been adopted for the latest extension of this company, and orders have already been placed with the General Electric Company for three 10,000 horse-power units.

CHAPTER V

THREE-PHASE GENERATORS

By far the greater number of alternating-current generators belong to this class, the chief reason for this being that by the use of three-phase power a saving in the copper of the transmission line is secured. The characteristics of the machines do not differ from those of the two-phase type, nor are the methods of calculation of the constants different.

The three windings are displaced in position on armature one-third of the distance between two poles of same polarity, that is, two-thirds the distance between one set of poles, or 120° in phase. The windings are either connected **Y** or delta. The connection used is immaterial, and governed only by the convenience of winding.

The **Y**-connection obviously requires less number of conductors of larger cross-section than the delta-connection, since, if the line current is I amperes, and the potential between any two lines is E volts, each winding, in the case of the **Y**-connected machine, has to give $\frac{E}{\sqrt{3}}$ volts and carry the full-line current, whereas, with the delta connection, each phase has to generate the full-line potential, but carries $\frac{I}{\sqrt{3}}$ amperes, as is shown in figs. 159, 160, and 161.

As an instance of three-phase generators, we may design a 200-kilowatt, 50-cycle, 1000 volt, revolving-armature machine as follows:—

Number of poles, 16,

Speed, therefore, $\frac{120 \times 50}{16} = 375$ revolutions per minute.

$$\text{Amperes in line} = \frac{\text{watts}}{\sqrt{3} \times \text{E.M.F. between lines}} = \frac{200{,}000}{1000 \times \sqrt{3}} = 116.$$

At a peripheral speed of about 5000 feet per minute, which might be considered fair for such unit, we find a diameter of 51.4 inches. For the sake of convenience we may therefore choose 48 inches diameter. This gives 9.42 inches between poles at armature surface, and therefore, if we choose a construction of one slot per pole for each phase, a distance between the slots of 3.14 inches.

Experience has furthermore shown that an armature reaction of about 1500 ampere turns per pole in the armature gives good relation between regulation and efficiency. The student can easily determine these values by assuming different armature reactions and making complete designs, which afterwards are compared.

This determines the number of conductors per slot.

Assume the armature to be Y-connected; the current per conductor is therefore 116 amperes, and, since armature reaction per pole in a three-phase machine is expressed as $1.5\sqrt{2} \times I \times t$, where t is the number of turns per pole, $t = \frac{1500}{1.5\sqrt{2} \times 116} = 6.1$ turns per pole and phase, or 12.2 conductors.

Obviously we must choose an even number of conductors per slot, therefore we may settle on 12 conductors per slot, which gives a convenient grouping of the wires.

At 2000 amperes per square inch in the armature copper, we would require a conductor between No. 1 and No. 2 B and S gauge-wire. No. 1 B and S would give a current density of 1770 amperes per square inch, and No. 2 a current density of 2230 amperes per square inch. Both of these are allowable as first approximation, but for the sake of higher efficiency we may choose No. 1 B and S. For mechanical reasons this size of wire is too large to handle, so it is advisable to substitute four No. 7 B and S wires wound as a double-twin wire. Assuming a double cotton covering, the size of this wire would be .30 × .30 square inch. The cross-section of copper is the same as No. 1 B and S, or .0656 square inch. Arranging the conductors 2 × 6, and allowing ⅛-inch insulation all around, and a $\frac{3}{16}$-inch wedge at the top of the slot, we get the size of slot as .85 inch × 2.25 inches.

Calculation of Flux.—The number of turns in series per circuit $= 6 \times 16 = 96$. Since the machine is Y-connected, each phase generates $\frac{1000}{\sqrt{3}}$ volts.

Therefore $e = \frac{1000}{\sqrt{3}} = \frac{4.44 \times M \times 50 \times 96}{10^8}$, or M = 2.72 megalines. Armature flux = 2.72 megalines. The field flux may be assumed 30 per cent greater, therefore field flux = 3.54 megalines.

As a first approximation we may decide to have the ampere turns consumed in the gap 2.5 times as great as the armature reaction, which gives 3700 ampere turns.

For mechanical reasons the average air-gap is here assumed to be $\frac{5}{16}$ inch; therefore, since in air the relation between M.M.F. density and length of gap is expressed as $AT = .313 \times B \times L$, we have $B \times L \times .313 = 3700$, or $B = 38{,}600$, which would correspond to an area of 70.5 square inches; making the pole arc 6 inches, or 64 per cent of the pitch between two poles, gives a depth of the field poles of 11.75 inches. For the sake of convenience we make the field 11.5 inches, and the armature 12 inches wide.

Assuming that we supply the armature with four ventilating ducts, each $\frac{3}{8}$ inch wide, and that there is 10 per cent insulation between the laminations, this would bring the effective width of iron to 9.5 inches.

Calculation of Total No-Load Ampere Turns.

	Section.	Length.	Density.	Amp. Turns.	Material.
	Sq. Ins.	Ins.	10^3 Lines per Sq. In		
Armature core	71.2	5.6	38.2	25	Soft Iron.
" teeth	43.5	2¼	62.5	20	"
Pole face	69	.3125	39.4	3850	"
Magnet core	39	6	91.2	210	"
Yoke or Field Spider ...	81.5	3	43.4	30	Cast-steel.
				4135	

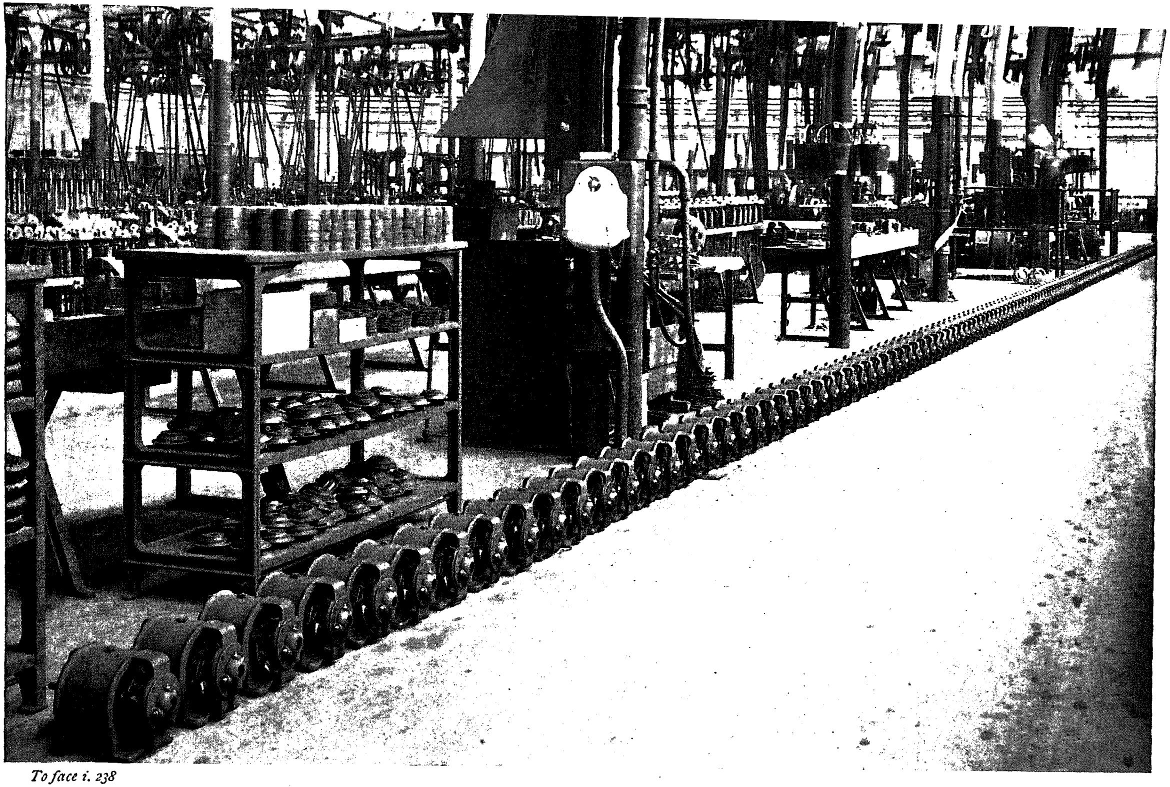
Thus no-load ampere turns at 1000 volts = 4135. Armature reaction $AR = \sqrt{2} \times 1.5 \times 116 \times 6 = 1475$ ampere turns. Therefore the ampere turns at full non-inductive load

$$= \sqrt{4135^2 + (1.33 \times 1475)^2} = 4540 \text{ ampere turns}$$

The ampere turns at full inductive load = 4135 + 1475 = 5610. The no-load voltage corresponding to 4540 ampere turns is 1075 volts, thus regulation = 7.5 per cent.

Short-circuited current is approximately $\frac{4540}{1475} = 3.08$ full-load current, or 358 amperes.

Calculation of Field Winding.—Assuming the exciter voltage to be 60 volts, it is generally customary to allow 20 per cent margin in rheostats; thus, 48 volts are available for field, or 3 volts per spool. Assuming that we ran the field copper at a density of 1200 amperes per square inch, it would require $\frac{4540}{1200} = 3.78$ square inches copper in each spool.



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MOTOR TESTING. SMALL-MOTOR TESTING-SHOP (Crompton & Co., Limited, Arc Works, Chelmsford)

It is customary, when possible, to wind the field with copper strips. If the strips are assumed to be .8 inch wide × .06 inch thick, we find that there are required $\frac{3.78}{.048} = 79$ turns, which, with necessary insulation, will just nicely fill the spool space. Generally two or three different sizes have to be tried before the right combination is found.

Thus we may decide to wind each spool with 79 turns of copper strip .8 × .06 square inch. Resistance of all spools in series $= 9.3 \times \frac{l}{a}$, where l is the length in feet and a = area in square inches. The mean length of turn is 39 inches.

$$r = \frac{.0093 \times 79 \times 39 \times 16}{12000 \times .048} = .8546$$

I R full non-inductive $= 57.5 \times .855 = 49$ volts, $I^2R = 2820$ watts
I R full inductive $= 71 \times .855 = 60.6$ volts, $I^2R = 4300$ watts
I R no load $= 52.3 \times .855 = 44.5$ volts, $I^2R = 2330$ watts

I^2 R Armature Loss.—This is figured in the same way as before and calculated to be 2560 watts, the mean length of turn being 56 inches.

Core Loss.—Calculating the hysteresis loss as before, and adding 30 per cent for eddy losses, gives the loss in core to be = 2450
„ „ teeth = 2160
4610

Friction we assume to be 2000 watts, or 1 per cent.

Calculation of Efficiency.—Core loss and friction are practically constant at all loads. The field excitation changes somewhat, but not enough to materially change the efficiency; so we are safe when assuming it constant at the lower outputs.

Losses at full load = 9430 + 2560 = 11.990 watts
„ ¾ „ = 9430 + 1440 = 10.870 „
„ ½ „ = 9430 + 640 = 10.070 „
„ ¼ „ = 9430 + 160 = 9.590 „

Thus efficiency at full load $= \frac{200}{211.99} = 94.3$ per cent.

„ „ ¾ load $= \frac{150}{160.87} = 93.4$ „

„ „ ½ load $= \frac{100}{110.07} = 90.75$ „

„ „ ¼ load $= \frac{50}{59.59} = 83{\cdot}9$ „

Effect of High and Low Armature Reaction.—To see more clearly the effect of this, let us assume exactly the same machine as above, but with just half the armature reaction—that is, each slot has 2 × 3 conductors instead of 2 × 6. Size of slot is then .85 × 1.35 square inch.

The magnetic flux per pole is just double what it was before, or 5.44

in armature and 7.08 in field. For the sake of comparison, we keep the densities in the different parts the same.

	Section.	Length.	Density.	Amp. Turns.	Material.
	Sq. In.	Ins.	10^3 Lines per Sq. In.		
Armature core, ...	142	5.6	38.4	25	Laminations
" teeth, ...	86.4	2½	63	20	"
Pole face, ...	141	·3125	38.6	3830	"
Magnet core, ...	79	6	90	200	"
Yoke or Field Spider,	163	3	43·5	30	Cast-steel
				4105	

Armature reaction = 2 × 1.5 × 116 × 3 = 737 ampere turns
Ampere turns full non-inductive = 4160
" " " inductive = 4842
Regulation at full non-inductive load = 1½ per cent.

Losses and Efficiency. — Calculating the losses in the same way as before, we find that the I^2R in the field, assuming that the field is wound with 79 turns 1.125 × .06 and that the mean length of turn is 63 inches, is 2520 watts.

I^2R armature, 1830 watts
Core loss, core, 4770 "
" teeth, 2510 "
7280 "
Friction, 2000 watts

Losses at full load = 11800 + 1830 = 13630, Efficiency = 93·7%
" ¾ " = 11800 + 1030 = 12830, " = 92.1 "
" ½ " = 11800 + 458 = 12258, " = 89.1 "
" ¼ " = 11800 + 114 = 11914, " = 80.6 "

The effect of the lower armature reaction is thus: Better regulation, greater core loss, thus lower efficiency, and a great deal larger machine, which obviously means more expense.

Fig. 174 shows a favourite type of large three-phase alternators. This generator is one of the 3500-kilowatt machines recently built for the New York Edison Company. The cut represents it as it appeared in the power-house before the assembling had been entirely completed. This machine, although rated at 3500 kilowatts, is designed with a capacity of practically 5000 kilowatts, this being necessary on account of the overload guarantees which were specified.

As may be seen, this alternator belongs to the class of direct-connected machines. It runs at 75 revolutions per minute, and has 40 poles; consequently gives power at 25 cycles per second. The potential is 6600 volts. The diameter at the air-gap is 16 feet 8 inches, giving a peripheral speed of 3900 feet per minute. The fields are wound with flat copper ribbons, and designed with a cross-section of copper such as to give ⅓ ohm resistance. The full non-inductive load excitation is 300 amperes. The armature copper loss is 26.5 kilowatts and the core loss 62 kilowatts; therefore the efficiency at full load is 96.7 per cent, at ¾ load 92.2 per cent, and at ½ load 94.8 per cent.

One of the most favoured types of alternating-current generators developed recently is that called the "compensated alternator". These generators are a combination of a regular alternator and a direct-connected exciter, which derives its voltage partly from its own shunt field and partly from the reaction of alternating currents of the main alternator on its field.

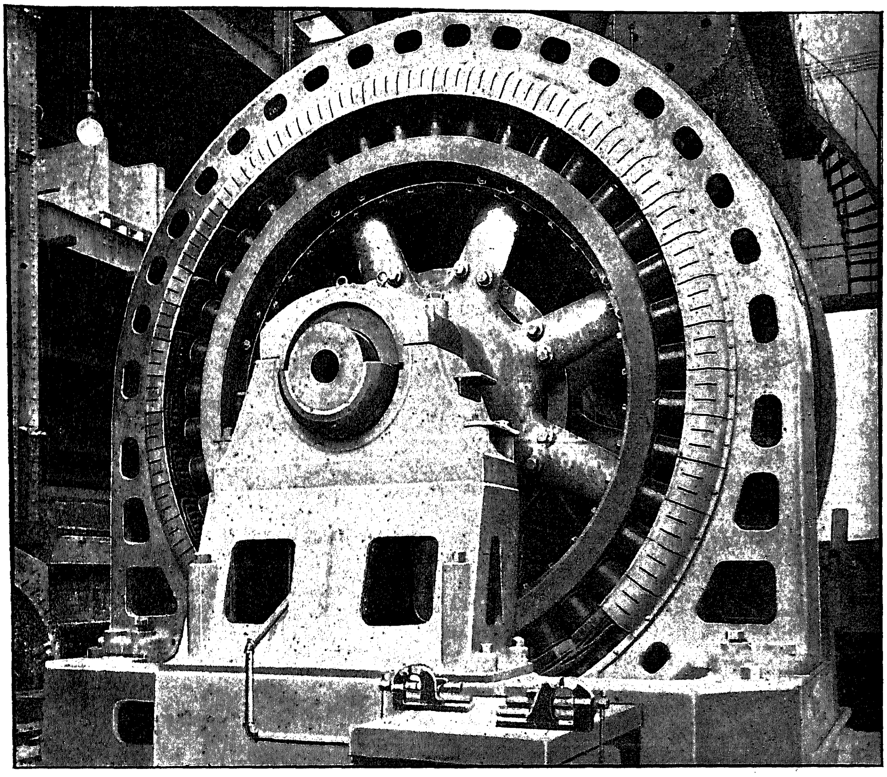

Fig 174.—Three-Phase Alternator

Since it is necessary to run the exciter in exact synchronism with the alternator itself, direct connection or gearing is required. Consequently, as a rule, the cost and the size of the exciter is greater than with the ordinary type. The control of potential is practically perfect, hence it is possible to design the alternator proper with much higher armature reaction than commonly used in hand-regulated alternators. Consequently its cost is relatively less; hence the cost of the complete unit will not exceed that of the usual combination of an alternator and a belted exciter.

To explain the theory of this method of compounding leads necessarily to somewhat complicated equations—at least if the problem is solved without the use of complex quantities.[1] With the use of the latter it is, however, relatively easy, and therefore given below:—

In an alternating-current generator of r ohms internal resistance per phase, x ohms reactance per phase, e terminal voltage per phase, and a current of $i + ji_1$ amperes, the following relation exists:—

$$e_0 = e + (i + ji_1)(r - jx)$$

where e_0 is the induced E.M.F.—that is, the no-load E.M.F.—corresponding to the resultant flux due to the combined armature and field excitation.

As this E.M.F. is 90 degrees behind the magnetism, the resultant magnetism can be expressed as follows:—

$$\mathrm{M}_r = -jke_0 = -jk\,[e + (i + ji_1)(r - jx)]$$

The armature M.M.F. is in phase with the resultant current, and can, therefore, be expressed as $n(i + ji_1) = \mathrm{M}_a$.

Therefore the field excitation, which is the difference between resultant magnetization and armature magnetization, is—

$$\mathrm{M}_f = \mathrm{M}_r - \mathrm{M}_a = -jke_0 - n(i + ji_1) =$$
$$i_1kr - in - ikx - j\,[ke + ikr + i_1kx + i_1n]$$

or the absolute value of M_f =

$$\sqrt{[i_1kr - i(n + kx)]^2 + [ke + irk + i_1(kx + n)]^2}$$
$$= \sqrt{\mathrm{I}^2[r^2k^2 + (n + kx)^2] + k^2e^2 + 2ke[irk + i_1(n + kx)]}$$

where $\mathrm{I} = \sqrt{i^2 + i_1^2}$

Or, since the voltage across the fields is proportional to the M.M.F., we can express it as follows:—

$$ef = a\sqrt{\mathrm{I}^2[r^2k^2 + (n + kx)^2] + k^2e^2 + 2ke[irk + i_1(n + kx)]}$$

The voltage across the alternator fields at no load is found by substituting $\mathrm{I} = \mathrm{O}$; thus $ef_0 = ake$. Therefore, for a given load I, the voltage across the field has to increase from ef_0 to ef, or—

$$e_x = a\,[\sqrt{\mathrm{I}^2[r^2k^2 + (n + kx)]^2 + k^2e^2 + 2ke[irk + i_1(n + kx)]} - ek]$$

and neglecting terms of r^2, since the armature resistance is very small compared with its reactance, we have—

$$e_x = a\,[\sqrt{\mathrm{I}^2(n + kx)^2 + k^2e^2 + 2ke[irk + i_1(n + kx)]} - ek]$$

in which equation e_x is the direct-current voltage which has to be generated as the load on the alternator changes, and a the ratio of E.M.F. across the fields to the corresponding M.M.F. Also—

$\mathrm{I} = \sqrt{i^2 + i_1^2}$ = resultant current per phase of alternator;
i = energy component of current,
i_1 = wattless component of current,
$\frac{i_1}{i} = \tan\omega$. $\cos\omega$ = power factor.

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[1] See page 676, vol. xxxvii, *Electrical World and Engineer* for April 21st, 1901, article by Ernst J. Berg on "Compounding of Alternators by Compensating Exciter".

n equals ratio of the armature reaction expressed in ampere turns to the resultant current.

k equals ratio of resultant M.M.F. of field and armature to the corresponding induced E.M.F.

e equals E.M.F. per phase at terminal of alternator.

r equals resistance per phase of armature winding

x equals reactance per phase of armature winding.

Perfect control of generator voltage requires, therefore, a device which gives this value of e_x for any current I, and this E.M.F. should be algebraically added to that which the exciter gives at no load on alternator.

A close approximation to this can be obtained by the arrangement invented by Mr. E. W. Rice, jun., which consists of directly connecting, gearing, or otherwise operating an exciter synchronously with the generator, and exciting this exciter by the current of the alternator (with or without the use of current transformers).

It is obvious that an alternating current supplied to such exciter armature will magnetize the machine and cause it to give any desired direct-current voltage, and that the voltage depends not only upon the magnitude of the current, but also upon its phase relation to the field poles.

In practice, one exciter is often used not only to give the required voltage across the alternator fields at no load, but also to increase the voltage with the load; part of the M.M.F. is obtained by a shunt field, as in a regular direct-current generator, and the increase in voltage with the load by the alternating current entering the armature and magnetizing its field.

The M.M.F. of this alternating current is expressed by

$$M_a = c \mathrm{I} \sin (\omega + \phi)$$

and the corresponding voltage as $e_a = b c \mathrm{I} \cos (\omega + \phi)$, where c is the numerical ratio between the M.M.F. and the current, b is the ratio between the E.M.F. and corresponding M.M.F., I is the total current of the alternator, and ω is the angle corresponding to the power factor of the load, $\tan \omega = \frac{z_1}{z}$, ϕ is the angle between field poles of alternator and exciter; in other words, the angle which has been chosen in displacing the field of the exciter to give the best results all round.

Were it not that even at full non-inductive load the voltage across the alternator fields has to be higher than at no load, it would be best not to offset the poles of this exciter relative to the poles of the alternator, since then with, say, full inductive load the magnetizing effect of the armature current is a maximum in one direction and with full condenser load a maximum in the other direction, and at non-inductive load no effect takes place, which obviously is just what is wanted. Since, however, considerable compounding is necessary, even at non-inductive load, the armature has to be offset from this position, and thereby the control made less accurate. All commercial loads are, however, inductive, and therefore it is not of great importance to be able to compound for condenser loads or loads requiring leading current, at least not for such loads of very poor power factors.

Let us assume, for simplicity, that the excitation is obtained by two exciters, one direct-current generator giving a constant voltage corresponding to the voltage across the alternator fields at no load, and the compounding being obtained by another synchronously driven exciter with its direct-current circuit connected in series with the main exciter, and its excitation obtained solely from the alternating current supplied from the main alternator through its armature winding.

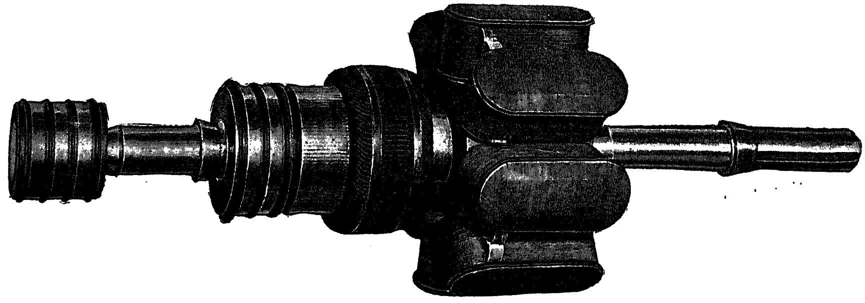


Fig. 175

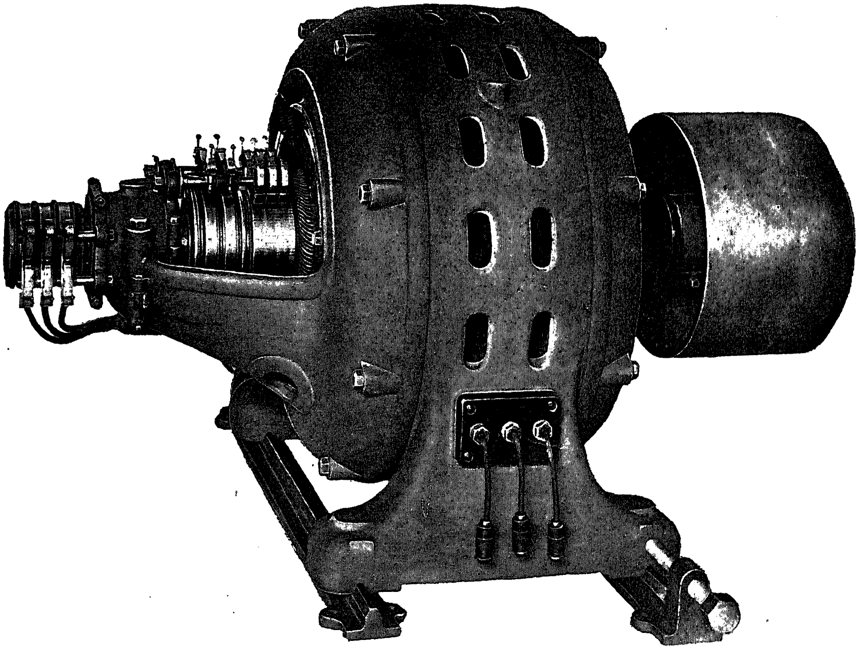


Fig. 176

The constants of this exciter can be ascertained from the previously given equation by substituting numerical values. For instance, if it is desired that the potential of the alternator shall be exactly the same at no load, full non-inductive load, and a load of 70 per cent power factor, we have—

At full non-inductive load,

$$e_x = e_a$$

$$a[\sqrt{I^2(n + kx)^2 + k^2e^2 + 2k^2eIr} - ek] = bcI\cos(\omega + \phi)$$

At 70 per cent power factor,

$$i_1 = i,\ \omega = 45$$

$$a[\sqrt{I^2(n + kx)^2 + k^2e^2 + 1.42ke.I(rk + n + kx)} - ek]$$
$$= bcI\cos(45 + \phi)$$

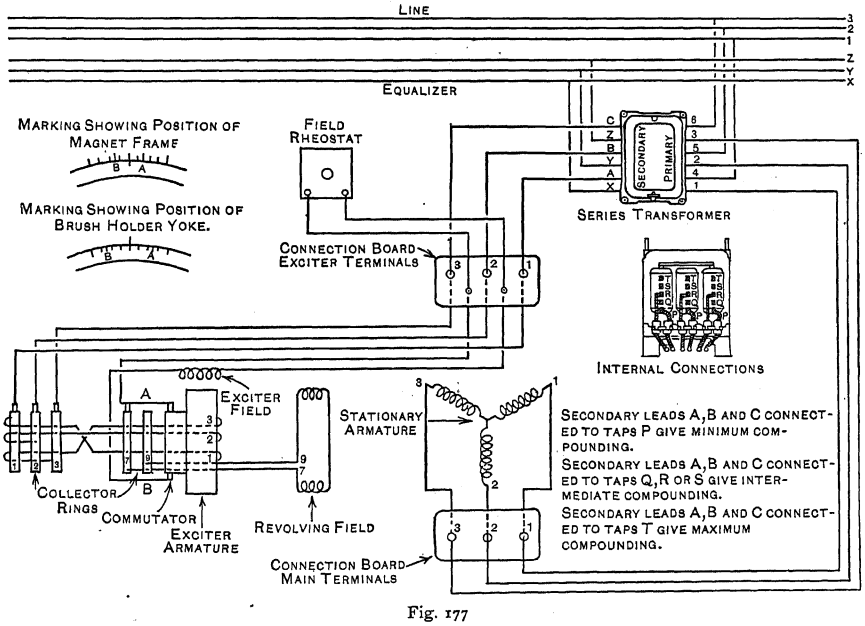
To take a numerical example—

Let $e = 1$ ...... $I = 1$ ...... $Ir = .01e$ ...... $Ix = .25e$
$k = 1$ ...... $n = 25$ ...... $a = 1$ ...... $b = 1$

Solving these equations in reference to ϕ and c, we find—

$$\phi = 20^\circ \qquad c = .128$$

By substituting different values of current it will be found that the voltage of the alternator remains practically constant for non-inductive loads and inductive loads of all power factors.



SECONDARY LEADS A,B AND C CONNECTED TO TAPS P GIVE MINIMUM COMPOUNDING.
SECONDARY LEADS A,B AND C CONNECTED TO TAPS Q,R OR S GIVE INTERMEDIATE COMPOUNDING.
SECONDARY LEADS A,B AND C CONNECTED TO TAPS T GIVE MAXIMUM COMPOUNDING.

Fig. 177

To appreciate what is gained by this control, we find from the expression of ef that disregarding the effect of saturation of alternator fields with the usual separately excited alternator, the voltage would, for instance, rise 13 per cent when full non-inductive load was thrown off, would rise 31 per cent if the same kilovolt-amperes at 90 per cent power factor were thrown off, 40 per cent if same kilovolt-amperes at 70 per cent power factor were thrown off, &c.; whereas with the present control, as can be ascertained by inserting proper values, the voltage in these cases is constant within a small fraction of a per cent.

The practical arrangement of this exciter and alternator is illustrated in figs. 175, 176, and the connections are diagrammatically shown in fig. 177.

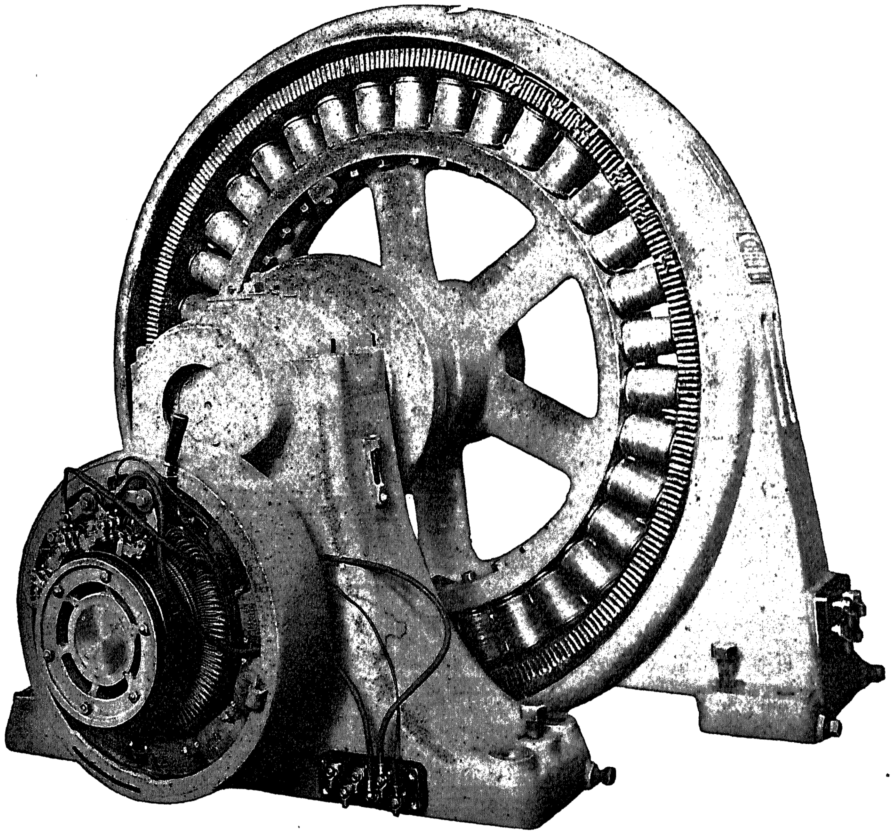


Fig. 178

The alternator itself is a standard machine of revolving-field type. The exciter armature with commutator is placed on the same shaft as the fields of the alternator, and this shaft carries, in addition, two sets of collector rings. Depending upon the system used, one set has three or four rings for a three-phase and two-phase alternator respectively, which carry the alternating current from the series transformers to the armature winding of the exciter. The second set, which has two rings only, carries the direct current to the alternator field. This exciter is usually wound for 75 volts, which is a convenient voltage, enabling the field spools of the alternators to be made of flat copper ribbons.

The size and character of this exciter is the same as any other directly-connected exciter, and, with the exception of the collector rings, it is a standard direct-current generator.

Fig. 178 shows another form of compensated generator. This differs in no essential feature from the other forms, except that its exciter is geared instead of direct connected. This allows an increase of speed of the exciter with a consequent reduction in the number of poles, which otherwise would become prohibitive when used with slow-speed alternators.

CHAPTER VI

INDUCTION GENERATORS

If an induction motor is driven above its synchronous speed, and is connected to a supply of alternating current of large enough capacity to give the excitation as induction motor, it becomes a generator, generating current at the same frequency as the frequency set by the machine supplying excitation; therefore such a generator is not a self-contained unit, but will always require a synchronous machine as exciter. This type has some advantages over the ordinary alternator in the fact that the revolving part can be made of very simple mechanical construction, as, for instance, a squirrel cage winding, and therefore permit of being rotated at very high speeds—speeds which would not be considered safe if the ordinary distributed winding was employed. On the other hand, it does not permit of any convenient method of potential control, and requires an alternator of from 20 to 50 per cent of its capacity as an exciter. This does not mean, however, that an induction generator requires in power 20 to 50 per cent for excitation, but only in capacity of exciter, since the current taken for excitation is largely wattless.

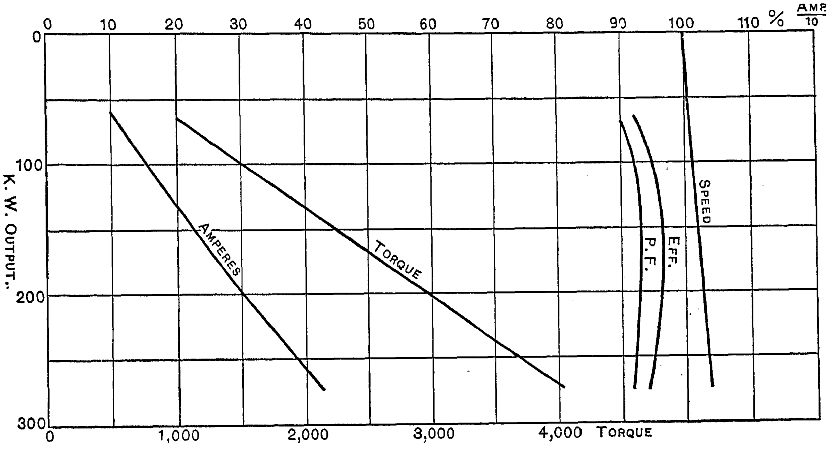


Fig. 179

In order to regulate fairly it must have large enough exciter, so that not only the excitation, but also the drop in voltage in the machine is taken care of. The reactance of such an induction generator is high, so that with lagging load the voltage tends to drop very quickly.

Assuming, therefore, that a certain induction generator is excited from an alternating-current generator of 30 per cent of its own capacity, in almost all cases it will be necessary to vary the voltage on the exciting alternator more than 100 per cent with the load on the induction generator.

Characteristic curves of such a machine are given in fig. 179.

CHAPTER VII

INVERTED CONVERTERS AND DOUBLE-CURRENT GENERATORS

These types of alternators are, in their general appearance, very much like the regular rotary converters, that is, they consist essentially of a direct-current generator with two or more collector rings tapped to proper points on the armature wiring. The electrical constants, however, are quite different in the two, since in the inverted converter, which receives direct current and supplies alternating current the resultant current is less than that in a double-current generator, due to the interaction of the currents in the armature windings, and therefore the size wire used for a given output is smaller. Its armature reaction is also usually decidedly greater and its speed higher.

The double-current generator, which is a machine called upon to deliver its full output in direct or alternating current, or in both simultaneously, must be so designed that it will carry its load under each condition without undue heating. Since there is no compensation between alternating and direct currents in the winding, the resistance of the commutator coil, as well as the armature reaction, must be made low. The result of this is that, as a rule, the double-current generator is decidedly larger and more expensive than the inverted converter.

Usually these converters are built for low frequency, since with low frequency the electrical constants become more favourable. A number of inverted converters have been made, however, for as high frequency as 60 cycles, and have proved satisfactory.

The ratio between the direct- and alternating-current voltage of such a machine depends upon the system, that is, the kind of alternating current taken out, whether it is two-phase, three-phase, single-phase, &c., the shape of the pole-pieces, particularly the ratio between width of pole to the distance between the poles, the resistance of the winding, and upon the position in which the brushes are.

With the average design of machines, that is, with pole arcs of about

65 to 75 per cent of the pole pitch, the average ratios at no load are as follows:—

Continuous current	100
Single-phase "	71.5
Two-phase "	71.5
Three-phase "	61
Six-phase, Y or delta	61
Six-phase, diametrical connection	71.5

In the inverted converter, as the load increases the ratio changes, so that at full load, in a machine of average design, it would be:

Continuous current	100
Single-phase "	70.5
Two-phase "	70.5
Three-phase "	60
Six-phase, Y or delta	59.5
Six-phase, diametrical connection	70

Due to the compensating effect of alternating and direct currents in such a converter, the ratio between the output of a direct-current machine and a converter of the same size and speed is as follows:—

Continuous current generator	100
Single-phase converter	85
Two-phase "	164
Three-phase "	134
Six-phase "	196

It must be borne in mind, however, that this increase in output with multi-phase converters over that as direct-current generator, can only be counted on if the alternating-current load is non-inductive. With an inductive load, the compensation of alternating and direct currents is not so marked, so that, even with a load of about 85 per cent power factor, the heating is almost the same as direct-current generator or as converter. This means that often there is but a slight change in design between an inverted converter and a double-current generator if they are operated at the same speed. Most frequently, however, the inverted converter is run at a higher speed, since it does not require any belts, and its speed is not limited by that of the steam-engine; therefore its size and cost are less.

The larger the number of phases the better is the compensation between alternating and direct current, and therefore the less heating for the same output is obtained. Consequently these machines are frequently made of six-phase design, which, however, has the disadvantage that it requires six collector rings.

When used as converters proper, that is, when converting alternating to direct current, there are more reasons for making the machine of six- rather than two- or three-phase design, since with a six-phase connection their stability is increased, that is, they have less tendency to hunt.

When operated as inverted converters, that is, from the direct-current

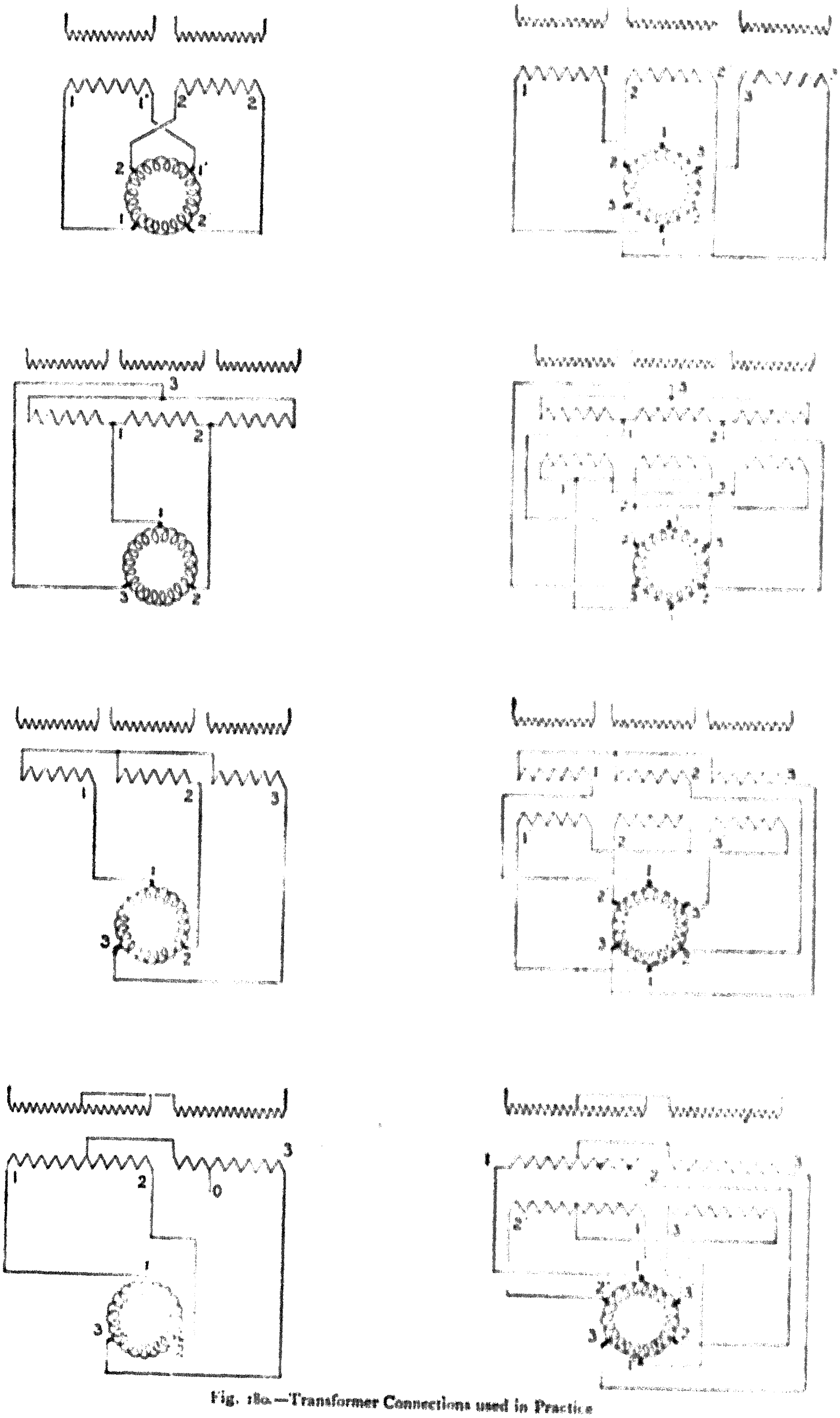
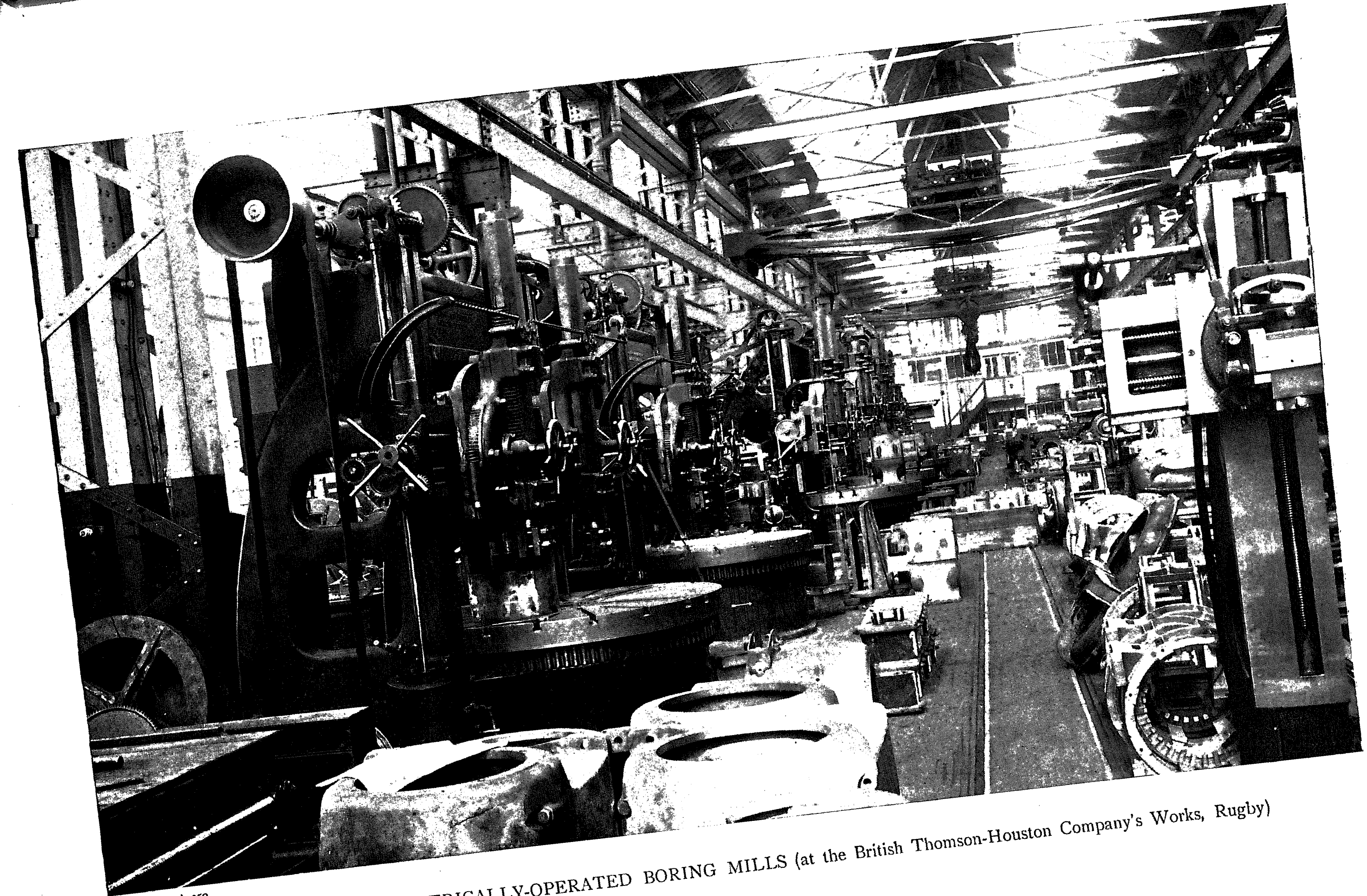


Fig. 180.—Transformer Connections used in Practice



To face i, 250

DIRECT-DRIVEN ELECTRICALLY-OPERATED BORING MILLS (at the British Thomson-Houston Company's Works, Rugby) side, there is rarely any tendency to hunt, therefore three-phase inverted converters are frequently used for units, which would be designed as six-phase machines if they were to be used as straight converters.

In fig. 180 are given a number of transformer connections used in practice.

The double-current generators are usually designed with direct-current series field, so that when they are operated as direct-current machines they will compound.

Inverted converters are usually designed as shunt machines only, and as in the case of all direct-current motors, it is very essential that all precautions are taken so that the field circuit cannot be broken. In a direct-current motor, if the field breaks, the motor will run away; but since usually the motor is belted or direct connected to some load, it cannot, as a rule, reach high enough speed to fly to pieces, whereas with inverted converters, which have no load, in a few seconds' time, if the field breaks, it will reach such a speed as to burst, due to centrifugal force. They are therefore frequently equipped with automatic circuit breakers, which cut off the current in case of an accident, and also with a centrifugal device, or some kind of a magnetic device, which opens the circuit-breakers when the speed increases a certain amount.

CHAPTER VIII

SELF-EXCITED ALTERNATORS

Recently a great deal has been published about various types of self-exciting alternators, notably the Heyland motor used as generator. Therefore, although they have not as yet been brought into general commercial use, it is advisable to say a few words about them.

The claim for the Heyland motor is that it can give its output at practically 100 per cent power factor. This is accomplished by supplying the secondary of such an induction motor with currents from the primary, which currents are led in to the proper points of the secondary winding by means of brushes. The original design of such secondary was to use a squirrel-cage armature with high-resistance end-rings. Later on it was proposed to cut the end-ring into sections and join the sections by high-resistance windings. The latest modification is the use of a regular winding with a commutator in connection with a squirrel-cage winding of the usual type.

The excitation is supplied at low voltage to these brushes instead of at the high voltage of the primary, as in the case of the ordinary motor. The reason why only a relatively low voltage is required, and therefore a low number of volt amperes, is that the secondary winding offers very low reactance to the current, since connected in multiple with it is a short-circuited winding.

Such motor also operates as generator at practically any frequency. Its regulation is, however, poor, and its value as generator therefore slight, unless automatic devices are arranged for compounding. Such devices are introduced by Heyland, and consist of supplying a second set of brushes midway between the first, which give a current to the secondary of the motor proportional to the load, or rather the requirements of the load, for various power factors. This gives a satisfactory control of potential, and the use of this type of generators may receive considerable favour if the present apparently necessary complication can be simplified.

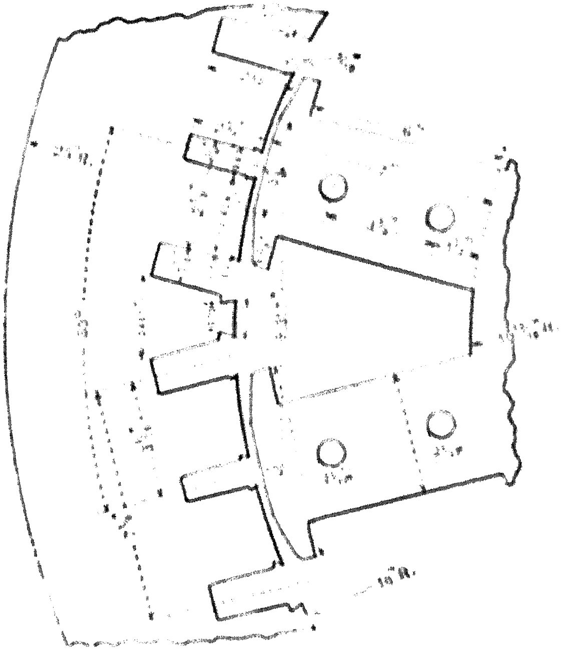


Fig. 181

Effect of Shape of Pole on the Wave Shape of Alternators. In fig. 181 is given a sketch of the shape of poles and slots in a certain monocyclic generator, and in figs. 182 to 189 a number of potential waves from the same generator under various conditions. This particular machine is essentially a single phase generator, which, in addition to the single phase winding, has a winding interposed midway between the main single phase winding, which contains a small number of turns intended to generate an E.M.F. in quadrature to the main E.M.F. which is used in starting motors. This particular machine has been chosen because the armature is somewhat unsymmetrical, and therefore gives a better opportunity to study the effect of pole shape on wave shape.

In one set of experiments the poles were made in exact accordance with the figure; in the other set the faces of the poles were turned so that the chamfer was cut off. The air-gap was still retained at the same length by skimming the poles. Fig. 182 gives the wave of E.M.F. between the mains at no load and also, in dotted lines, the sine wave corresponding to the same maximum value. Fig. 183 gives the no-load E.M.F. between the teaser and the main, that is, the E.M.F. which is obtained between the terminals of one of the main windings and the terminals of one of the auxiliary windings, the teaser, which has only one-quarter as many turns as the main winding. Fig. 184 shows the wave of E.M.F. in the main circuit with full non-inductive load, and fig. 185 the corresponding wave with load between teaser and main. Fig. 186 gives the no-load E.M.F. between the mains after the chamfer has been removed from the poles. Fig. 187, the corresponding E.M.F. between teaser and main. Fig. 188 shows the E.M.F. between the mains at full non-inductive load without chamfered poles, and fig. 189 the corresponding E.M.F. between teaser and main.

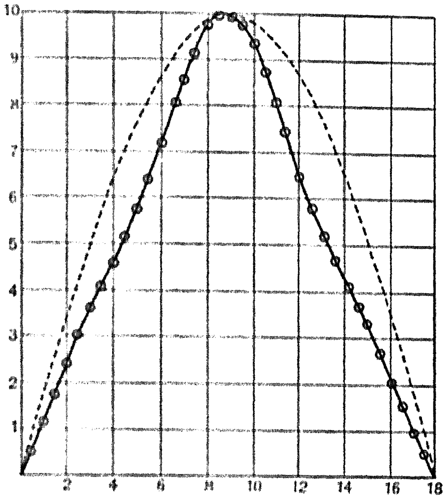


Fig. 182

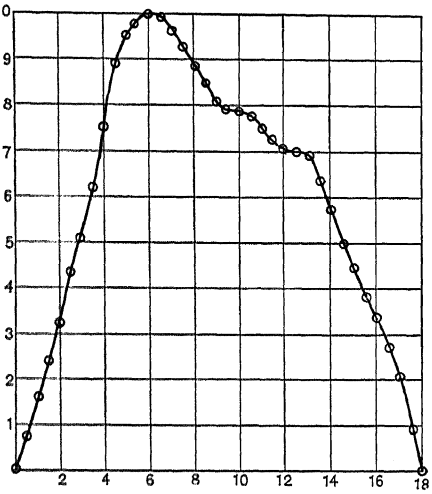


Fig. 183

In looking over these curves it is of interest to see, first, that the effect of chamfering the poles is very marked indeed on the wave shape; and second, that its shape is not greatly changed with the load, but that what change has occurred makes it conform more closely to the true sine curve.

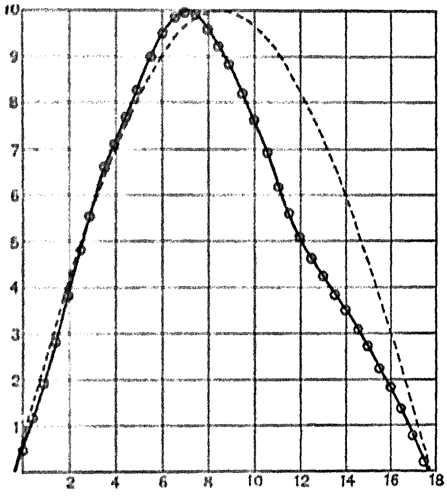


Fig. 184

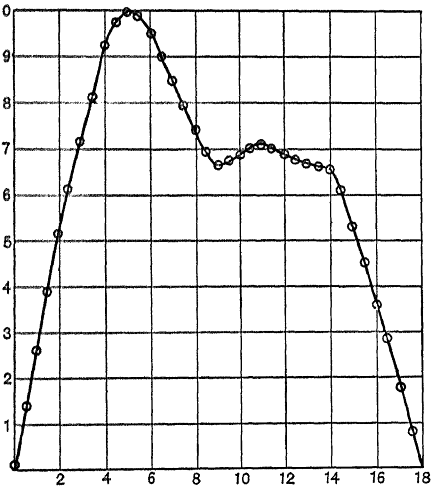


Fig. 185

Parallel Operation of Alternators. — Recently a great deal has been published about parallel operation of alternating-current generators, particularly when directly connected to steam-engines of low speed. As a matter of fact, very little was known about any difficulty in parallel operation until this class of service was introduced.

With belted units it is safe to say that any type of alternators will run in parallel. A single-phase machine may be synchronized to one of the phases of a three-phase generator, and will stay in step. A smooth-core three-phaser will run satisfactorily in parallel with an iron-clad, &c. The difference in wave shape of the machines is quite unessential, although undoubtedly if there is a great difference a slight cross-current will flow.

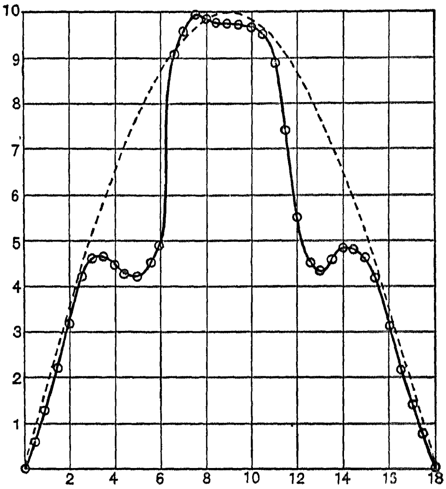


Fig. 186

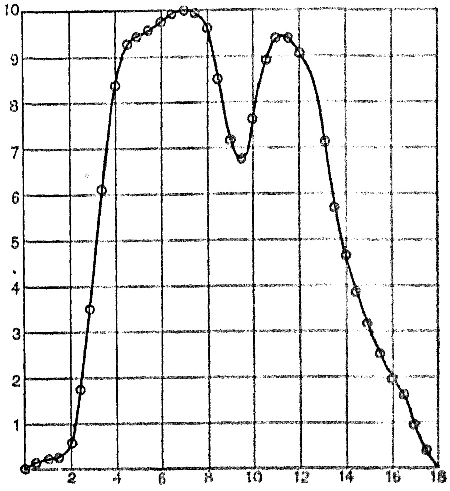


Fig. 187

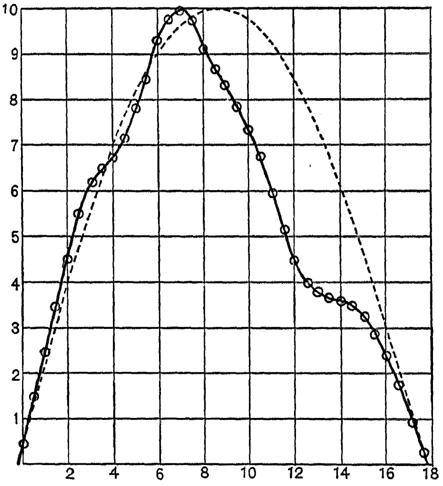


Fig. 188

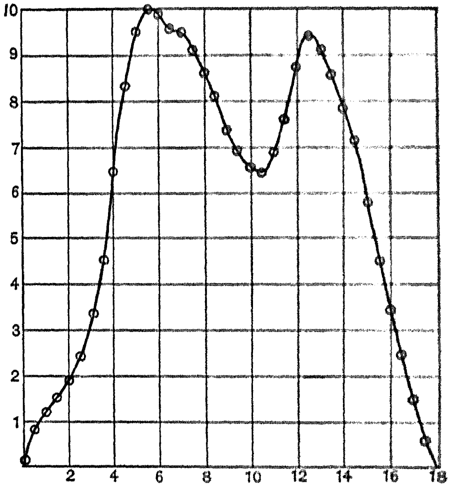


Fig. 189

With direct-connected units, however, some difficulty has been met with in keeping machines in step. It was found that frequently the machines would run satisfactorily in multiple for a few minutes and then gradually a cross-current would appear, which increased until it reached such values that it became prohibitive or the machines dropped out of step. Invariably this difficulty has been overcome by improving the regulation of the steam-engine so as not to permit of rapid fluctuations of speed.

As is known, two alternators in parallel will distribute the power delivered in proportion to the power given them by the steam-engine, that is, in proportion to the steam admitted to the cylinders. Therefore, if the governing device is very sensitive, so that the slightest change of speed will effect a sudden opening or sudden closing of the valves or change of cut-off, one engine will rob the other and a "see-sawing" will be initiated. The remedy, of course, is to make the dashpots of the governing device sluggish, so that this "see-sawing" effect cannot take place.

On the other hand, it is desirable, for other reasons, that the governing device is not sluggish, so that in case of a sudden relief of all load the speed of the engine will not reach prohibitive values. To effect this, compound dashpots of various construction have been designed.

European practice is to build alternators intended for direct connection with Le Blanc's amortisseurs, that is, with short-circuited windings in the poles, and even in the interpolar space. This winding tends to prevent the alternators from hunting; at the same time it involves some loss of energy.

The effect of high or low armature reaction on the parallel operation of alternators is not so important as might be thought. The lower the armature reaction the greater the synchronizing power. At the same time the larger cross-current will flow for a given variation in speed, that is, for a given variation in the angular velocity of the steam-engine.

Judging from experience, it is not advisable, therefore, to have machines of too low armature reaction; and, on the other hand, it is not well to have the armature reaction too high. Under average conditions it would seem as though the regulation should be made from 6 to 10 per cent, that is, the armature reaction so proportioned that, when full non-inductive load is suddenly removed from the machine with full-load excitation, the voltage shall not rise more than 6 to 10 per cent.

CHAPTER IX

MOTOR GENERATORS AND ROTARY CONVERTERS

If a continuous-current generator as described in the last article be directly coupled to an alternating-current motor, we will obviously have a piece of apparatus capable of taking alternating current from the supply mains and delivering instead continuous current.

The properties of alternating currents and alternating-current generators make them specially suitable for the transmission of power to considerable distances, whilst for many purposes, *e.g.* tramway work, motors for use in workshops and factories, &c., continuous-current motors have hitherto been considered the most suitable; hence there arises a need for machinery such as that described above, which can take from the mains energy supplied as alternating current and deliver it as continuous current. The energy is usually generated in one or more large stations, is transmitted as high-tension alternating current to sub-stations, in which is placed the transforming apparatus by means of which it is converted into continuous current for use in the immediate vicinity of the sub-station.

The apparatus used for this purpose is capable of various modifications, but it usually consists of either a "rotary converter" or a "motor generator".

The latter is perhaps the simplest in its theory, and, as its name implies, consists of an alternating-current motor directly coupled to a continuous-current generator. The motor may be either a synchronous or an induction motor, and it is quite common to see both types installed in the same sub-station.

The rotary converter consists of a machine very similar in construction to an ordinary continuous-current generator, but with the addition of slip-rings such as are provided on alternating-current machinery. The armature is wound with the ordinary continuous-current winding, but at equidistant points of the winding, tappings are taken off to slip-rings; the number of slip-rings provided is usually three or six. The alternating-current supply is led into these slip-rings, and continuous current is collected at the commutator.

Present practice in the matter of sub-stations is not at all uniform, and even on the same system "rotary converters" and motor generators, with induction and with synchronous motors, are frequently put down side by side. Still greater is the divergence in the methods of using the apparatus, and in the starting and controlling gear.

If motor generators are used, it is possible to wind the motor for the high voltage of supply and to connect it directly to the bus-bars. The continuous-current generator is, on the contrary, wound for a comparatively low voltage, which it supplies directly to the low-tension mains. Thus on many tramway systems the voltage of supply is about 6000 volts, while the continuous-current system is worked about 500 volts. There is no difficulty in building motors, whether induction motor or synchronous, to run on a 6000-volt circuit. If, however, rotary converters are used, as there is only one winding which is common both to the alternating and the continuous-current system, no such range of voltage variation is possible. The ratio of the alternating-current voltage to continuous-current voltage is fixed within narrow limits by the construction of the machine, and it is therefore essential to use static transformers, which take the current at the high voltage of supply and transform it to a suitable voltage before delivery to the slip-rings of the rotary. The cheaper first cost of the rotary converter (consisting of one machine instead of two) is thus to a certain extent set off by the necessity for using static transformers.

It is, however, in the methods of starting the machinery to work, and in the different ways of regulating the voltage at the continuous-current end, that the greatest divergences of practice are to be found.

In sub-stations forming part of a large system the continuous-current mains are probably always kept live, that is to say, some of the machines are always kept running, and continuous current is always available to start the rotaries or motor generators. In this case the starting up is frequently done from the continuous-current side, that is, the continuous-current generator is temporarily used as a motor. It is connected through a starting resistance to the continuous-current mains, the starting resistance is gradually cut out, and when the set has got up to normal speed the switches are closed which connect the slip-rings to the high-tension bus-bars. In the case of rotary converters and of synchronous motors it is, however, necessary to see that they are in synchronism before they are switched on. They must be treated in exactly the same way as alternators which are being put into parallel.

It will, however, be observed that if any break-down occurs involving the temporary stoppage of all the machinery, this method is not available, the supply of continuous current having been temporarily stopped; and in order that the sub-station may be able to start up again under these circumstances, it is necessary that some at least of the machines may be able to start from the high-tension alternating side. This is easily arranged for in the case of motor generators driven by induction motors. These can be started either by means of auto-starters, that is, small transformers which reduce the voltage and therefore the high starting current which the motor would otherwise take from the mains, or the rotor may be provided with slip-rings and starting resistances in the usual way. This latter method requires a smaller starting current, a point which is frequently of great importance. On the other hand, it requires brushes and slip-rings on the rotor, and does away with the simplicity of a motor connected directly to the mains and operating without any moving contacts whatever.

The rotary converter also can be started directly from the alternating-current mains. In order, however, to avoid excessive starting current, it is necessary to reduce the voltage at starting. This is usually done by having tappings on the transformer secondary, which give a suitable starting voltage. In the case of large rotaries it is more usual to provide a small induction motor directly coupled to the machine to run it up to synchronism before it is switched on to the mains. When rotaries are started up by either of these methods, the friction of the bearings is of course a very important part of the force to be overcome, and in order to reduce the starting current to a minimum, rotaries have recently in some cases been fitted with ball-bearings.

The synchronous motor cannot be started directly from the alternating-current end except it be provided with some arrangement on the field-magnets, which will during the starting period cause it to work as an induction motor. Even with this added complication the motor takes a very considerable starting current from the mains, and the more usual method is either to start from the continuous-current end, or to provide a small induction motor to run up the set to synchronism, as already described in the case of the rotary.

The voltage regulation is perfectly simple in the case of motor generators. The continuous-current machine is independent, and driven at practically a constant speed; it may therefore be a simple shunt-machine provided with hand regulation in the shunt circuit, or it may be a compound wound generator; in fact, the voltage regulation is carried out exactly as it would be on an engine-driven generator.

The case of the rotary is quite different. The ratio of direct-current voltage to alternating-current is fixed by the construction of the machine, and can only be varied by means of shunt resistance if a certain amount of self-induction is introduced in the mains between the high-tension bus-bars and the slip-rings. This may be done either by constructing the transformers so that they have considerable self-induction, or by introducing an inductive resistance between the transformers and the slip-rings.

Another method adopted to vary the direct-current voltage on a rotary is to provide different tappings on the secondaries of the transformers, so that the alternating voltage at the slip-rings may be varied, and thus vary the commutator voltage. The same result can be obtained by coupling to the rotary a small alternating-current booster. The voltage of this can be altered by shunt-regulation, and the voltage of supply is thus under control.

It will be seen from the above that by the time the rotary converter has been supplied with the necessary static transformers, and with one or other of the necessary devices to give control of the voltage, it has lost much of its initial simplicity.

Whilst rotary converters have been very extensively used on the Continent and America, they have been less frequently installed in Great Britain, partly on account of the need of accessories, but chiefly from the opinion held by many engineers in this country, that the rotary is a less stable machine than the motor generator, that is, that it is more liable to surge and to fall out of step with the generators.

Another combination of machines used for the purpose of transforming from continuous to alternating current has lately come to the front in Great Britain. It has been called a motor converter, and is really a combination of the rotary converter and the motor generator. It consists of a continuous-current generator driven by an induction motor, but the machines instead of being merely mechanically coupled are also interconnected electrically, tappings being taken from equidistant points of the continuous-current winding and connected to the rotor winding of the induction motor. This arrangement resembles the coupling of two induction motors in "cascade", that is, feeding the stator of one motor with current generated in the rotor of the other, and the behaviour of this class of apparatus is naturally intermediate between that of the rotary converter and the motor generator driven by an induction motor.

The consideration of the effect of these different kinds of apparatus on the power factor of the central station is of importance. When rotary converters or synchronous motor generators are used, the power factor is under control, and can be varied by varying the excitation, so that the current can be made to be exactly in phase with the E.M.F. or even be made to lead. This is a very desirable feature, as it may materially improve the regulation of the generating sets.

This feature is not present in the case of the motor generator with induction motor, nor in the motor converter.

5. The Continuous-Current Motor

CHAPTER I

DESIGN OF THE ELECTRIC MOTOR

General Principles.—The continuous-current electric motor has become a recognized factor in modern mechanical engineering, fulfilling with pre-eminent success all the functions of the gas- and steam-engine in the driving of factories, machine-shops, printing-presses, and isolated machinery. And further, it has rendered commercially possible many mechanical processes which were difficult or impossible with other driving power. This success lies in the principles upon which the design, construction, and working of the motor depend.

The function of the electric motor is to convert electric into mechanical power. It is therefore the converse of the dynamo, and although the general principles of the electric and magnetic design of the two are identical, the constructional details necessarily vary to suit the different purposes for which each is used. In extreme cases there may be marked differences, even sufficient to prevent reversibility, *i.e.* motors specially designed for traction purposes, for example, would be useless as dynamos, although ordinarily designed dynamos generally make excellent motors.

A dynamo is usually placed in a specially arranged engine-room. It is driven at a more or less constant speed in one direction, and is required to give varying current at a constant pressure. Moreover, the load is usually maintained within well-defined limits, and is not subject to sudden or large variations.

A motor is placed necessarily close to the work it has to do, and is frequently in a position presenting one or more difficulties—it may be exposed to dust or to the weather, be near to inflammable material, or liable to accidental damage, &c. The work it has to do is generally not controlled by any special arrangement, and so may greatly exceed the designed maximum for shorter or longer periods. And the pressure of supply is not always constant, and frequently has to be varied within wide limits to enable the motor to perform its work, for the speed of the driven machines is often variable, and sometimes changes rapidly. Further, the motor may be required to run with equal facility in both directions. These conditions require special designing of the

electric, magnetic, and mechanical details, and may sometimes involve complicated switching and regulating devices.

It is also clear that the commercial "rating" of a motor will be very different from that of a dynamo of similar over-all dimensions and equal weight.

Classes of Motors.—There are three main classes of motors. The first is adapted for driving fixed machinery, and is chiefly shunt wound; the second is the traction type, and is series wound; the third is used for lifts, cranes, &c., and is usually compound wound.

Fundamental Principles and Equations.—The principles of the magnetic circuit of a dynamo have already been fully discussed in the chapter on the dynamo. The methods of determining the ampere turns on armature and field-magnets have also been described, and there is no need to recapitulate them here; but it should be noted that the total E.M.F. of the motor armature is less than that of the applied pressure by the number of volts lost over the armature, and the field-windings, if they be in series with the armature.

Thus if E be the pressure of supply, e_a the back electromotive force of the armature will be given for a series motor by the equation—

$$e_a = \mathrm{E} - i(r_a + r_f) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

This equation is important, as the number of revolutions per minute of the armature vary with the back E.M.F. and not with the terminal pressure.

The fundamental equation connecting the chief variables and constants for a motor of any number of poles may be written—

$$e_a = \mathrm{Z}_a \mathrm{N}_a n \frac{10^{-8}}{60} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Where Z_a = The number of turns counted all round the armature.
N_a = The number of lines of force from one *pair* of poles in the field-magnets.
n = The number of revolutions per minute.

In a motor the torque on the armature shaft is a chief point in the design, but especially in the traction type, for the acceleration of the load to be moved depends directly upon this factor. The torque equation therefore requires special consideration. This may be written—

$$\text{Torque in lb.-ft.} = \frac{\mathrm{Z}_a i_a \mathrm{N}_a}{852.3 \times 10^{-6}} \quad \ldots \quad \ldots \quad (3)$$

i_a being the armature current in amperes.

This shows the torque is directly proportional to the product of the number of turns of wire, the magnetic flux, and the armature current, all expressed in proper units. Since the armature turns are fixed for a given design the torque varies as the product of the current and the magnetic flux, and it is possible to vary the torque by the armature current or by the magnetic flux, or by both in any proportions.

In the series motor both these quantities vary together, but owing to the slope of the magnetic induction curves for iron and steel, the increments of magnetic flux are for low degrees of magnetization much more rapid than the corresponding increments of exciting current; while after the knee of the curve is passed the magnetic flux is practically constant, and the torque simply increases with the current.

This important feature has special bearing on the design of traction motors, and mainly determines the cross-section of the field-magnet cores for a given specification.

In shunt motors, since the field excitation is constant the torque varies directly with the armature current, if the reactions be neglected. In compound-wound motors, with the series coils arranged to assist the shunt winding, a combination of the two effects is obtained which is specially useful in lift motors requiring rapid acceleration at starting with constant speed after the maximum revolutions per minute are attained. Lift motors are frequently wound in compound, the series coils being short circuited after the normal full-speed is attained. In some recent electric lifts shunt motors are used because the control gear is simplified thereby.

Output of Motors.—The law of the magnetic circuit has been stated in detail in the chapter on the dynamo, and its application in designing machines to fulfil definite specifications has been fully developed therein.

The principles governing the design and construction of a motor are essentially the same, due regard being given to the special requirements of the power problem.

The output of a motor depends theoretically on the length and diameter of the armature, speed of rotation, and the density of the magnetic flux. But practically the output for given windings, field-magnets, and armature depends on the sparking and heating limits, and as soon as one of these is reached the maximum working load is obtained, and the motor is "rated" accordingly.

Armature reactions have already been discussed in the dynamo section. For a further study I would refer the reader to a book by C. C. Hawkins, entitled *The Theory of Commutation*, which is the most exhaustive treatise on this complex subject. It contains many practical as well as theoretical deductions, and is amply illustrated with diagrams and tests.

Modern motors are so designed that the heating limit is reached before the sparking limit. Indeed many of the best motors will carry a temporary overload of 100 per cent without serious sparking. The heating limit is therefore the chief factor in "rating" motors, and presents some difficulty unless a strict specification be adhered to; and obviously the specification requires to be adapted to the special class of work for which the motor is required.

For example, a traction motor is usually defined to give a certain load at a certain number of revolutions per minute for one hour with a maximum rise of temperature, at any accessible part of the armature or interior of the casing, not exceeding 75° C., the rise of temperature being measured by a thermometer, say one minute after stopping, and the inspection-covers being kept closed until the end of this period.

This high rise of temperature is never obtained in general working, and is permissible only with a motor used intermittently with varying loads and frequent stops. A stationary motor running continuously should be "rated" with a much lower temperature rise, or the insulation would soon char and the windings would burn out. The general limit for motors of this type is from 21° C. to 27° C.; but this requires to be varied to suit the average temperature of the site in which the motor is to work. A motor driving scrapers on the top of an economizer, for example, would probably be subjected to a higher normal temperature than a motor driving shafting in a factory, and hence would require to be rated at a lower power.

The rise of temperature is also affected very considerably by the ventilation of the site of the motor, and it is found that much better results as regards output and efficiency are obtainable with a given motor in a well-ventilated position than in a confined space with little or no draught of air.

In June of 1902 Dr. M. G. Rasch read a most instructive paper before the International Tramways Union on the "Rating of Motors", with special reference to the German regulations, which give, as a general rule, the following limits of rise in temperature, assuming the atmosphere not to exceed 33° C.:—"For armatures with cotton insulations, 50° C.; for paper insulation, 60° C.; for mica, amiant, or similar preparations, 80° C.; for field-coils, a further increase of 10° C."

These limits are higher than those generally adopted in this country. It should be noted, however, that the rise of temperature in the field-coils is in Germany measured by the increase of electrical resistance, and hence the temperature of the interior of the coil and not merely that of the outside is measured, as is usual in Great Britain. The electrical test is obviously more reliable.

Dr. M. G. Rasch, in connection with his experiments with tram-car motors, suggests the following formula for determining the safe period to test traction-motors in the test-room:—

$$x = \frac{t \times T_1}{t + 28 (T - T_1)}$$

Where x is the useful duration of the trial to be determined, T the time during which the car is to be out of the depôt, T_1 the time during which the motor is to be working ($T - T_1$ therefore gives the time during which the motor is not working), t the required rise of temperature of the motor above that of the atmosphere, and 28 is a constant expressing the cooling effect per hour in degrees Centigrade, and is presumably taken from practical observations.

From this equation, if we take a rise of 70° C. for a day of fourteen hours, and assume the motor to be in actual use during one-half of the time, or seven hours, the time for a useful test at full load of the motor is found to be about two hours. Other assumed conditions would, of course, give a different value for x. The equation appears to agree with practical results, and should be useful in judicious hands.

In making temperature tests great care should be taken to see that carbon brushes do not press unduly hard on the commutator, and that there is no sparking, as the heat generated by one or both of these causes is often considerable enough to make the tests unreliable.

Mr. M. B. Field, in a recent paper read before the Glasgow Local Section of the Institution of Electrical Engineers, gives the following tests of a tram motor illustrative of these important points:—

Test I.—One hour's run; brushes sparking badly, and friction of carbons on commutator excessive.

Final Temperature of	Armature, -	68° C.	Rise,	49° C.
„ „	Field-Coils,	70° C.	„	51° C.
„ „	Commutator,	111° C.	„	92° C.
„ „	Atmosphere,	19° C.		

Test II.—Same motor, same load; one hour's run, with brushes properly adjusted, and no sparking.

Final Temperature of	Armature,	68° C.	Rise,	48° C.
„ „	Field-Coils,	67°.5 C.	„	47°.5 C.
„ „	Commutator,	63° C.	„	43° C.
„ „	Atmosphere,	20° C.		

The difference between the two tests is sufficiently obvious.

Motor Designing.—The mechanical details require most careful attention. The armature shaft must be sufficiently stiff to resist twisting and bending. Messrs. Paterson & Cooper specify the maximum deflection at full load not to exceed 0.004 of an inch. The core plates and end rings must be rigidly keyed to the shaft or to a stout spider, and should be slotted so as to give a positive drive to the windings. The coils should be former wound and be interchangeable. Ventilation ducts should be provided in all but the smaller sizes.

The commutator is now invariably made of hard copper, stamped to the required section and insulated with best mica, preferably Indian. The whole should be mounted on a sleeve keyed rigidly to the shaft.

Messrs. Bruce, Peebles, & Co. build up the sections in a special jig, and then compress them by means of an outside clamping ring forced on hydraulically, the whole being previously heated to a temperature of 150° F. Whilst held thus, the bush is firmly bolted on and the clamping ring removed. On cooling, the whole commutator sets so firmly that striking with a mallet does not affect the bars.

The outer diameter is usually adjusted to give a surface velocity adapted for carbon brushes, not exceeding, say, 2,000 feet per minute, a higher speed being apt to give rise to excessive heat, and to make commutation difficult.

Carbon brushes are practically essential to fix the lead for varying loads in motors built to run in both directions. In no case should the current density exceed 40 amperes per square inch of brush surface at full load.

Also the brush and brush-holders should be light, with an accurate and positive adjustment requiring little attention, and the brush feed should be radial in order to eliminate alterations of "lead". The pressure of the carbon blocks should be maintained constant throughout the wear of the blocks.

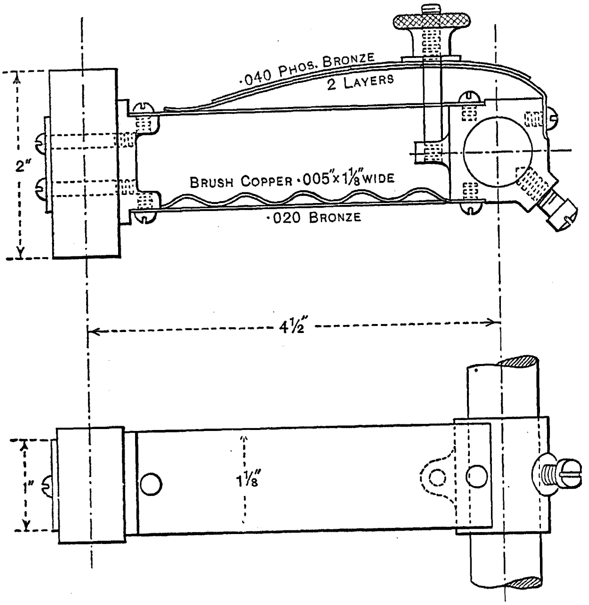


Fig. 190.—Motor Brush-Holder (Blackwell)

In my experience one of the best types of carbon brush-holder is that shown in fig. 190, which appears to embody the points above mentioned. I have found it to give excellent results when other types of brush-holders failed entirely to prevent sparking or heating.

Fig. 191 illustrates the Eck carbon brush-holder. It is of the latest design.

The brush, which has a convex curve *a* that fits a concave curve *b* in the holder, is allowed to turn slightly and to adjust itself to the periphery of the commutator at point *c*.

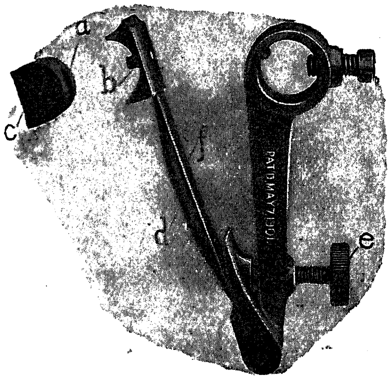


Fig. 191.—Eck Patent Brush and Brush-Holder

A flat tempered steel spring *f* presses the brush-holder against the brush, the tension being adjusted by a screw *e*, made so as to prevent it from turning through vibration. Two or more copper strips *d* running alongside the steel spring carry the current.

The carcass must give efficient mechanical protection to the armature and field-windings; in fact, it should be built so as to enclose these parts more or less completely.

The magnetic circuit must be designed so that the slope of the characteristic curve gives the proper acceleration, and the maximum *rated* output must coincide with a definite part of this curve if the best results are to be obtained.

The heating of the field-coils is practically confined to the C^2R loss, and is readily calculated; that of the armature is not so easily determined.

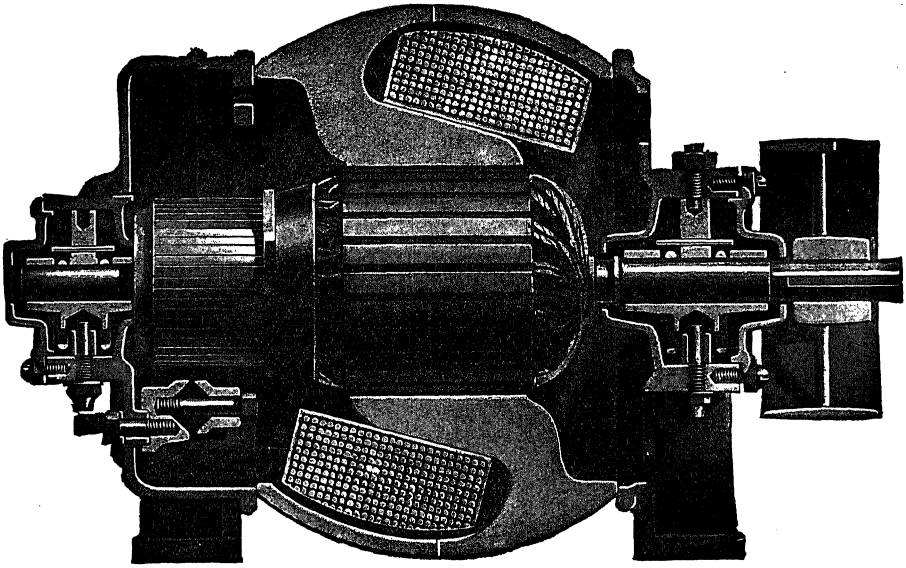


Fig 192.—Sectional View of Two-Pole Lundell Motor

There are four sources of loss of energy in an armature, and all result in heat.

They are, in order of magnitude, the C^2R loss, the hysteresis loss, the loss in eddy currents in iron stampings, end plates (and possibly other parts), and the loss due to the short circuiting of successive sections of the armature during commutation. Eddy currents in the conductors may be neglected in toothed-core armatures of normal design.

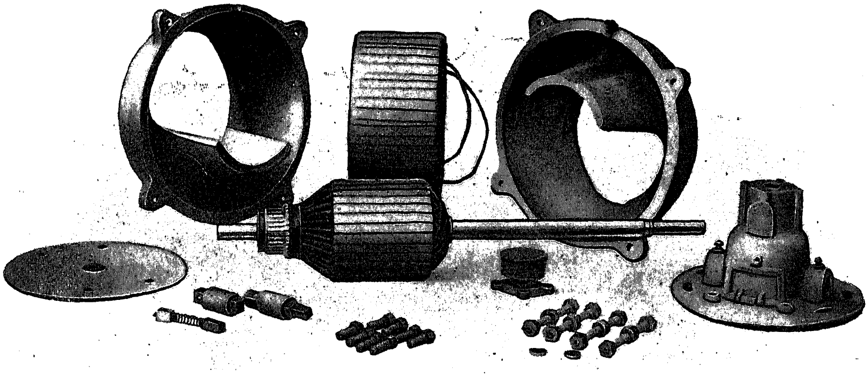


Fig. 193.— View of "Aston Lundell" Motor, showing the formation of the shell

Hysteresis loss varies as the number of cycles of magnetization per second, and eddy currents as the square of the same quantity, which is generally called the frequency or the periodicity. The latter loss can be made sufficiently small by using thin sheet-iron plates, the practical limits varying between 0.06 and 0.03 of an inch. The hysteresis loss, however, depends on the weight and the softness of the iron, the magnetic density, the number of poles in the field-magnets, and the number of revolutions per second.

Practical considerations keep the density in the core too low to cause much heat; but that in the teeth is often pushed to the point of saturation, say between 17,000 and 19,000 C.G.S., with an idea of minimizing the armature reaction, and consequently sparking at the brushes. The same effect, other things being equal, is attained by increasing the length of the air gap, and, it should be noted, without any of the disadvantages attendant on reducing the width of the teeth.

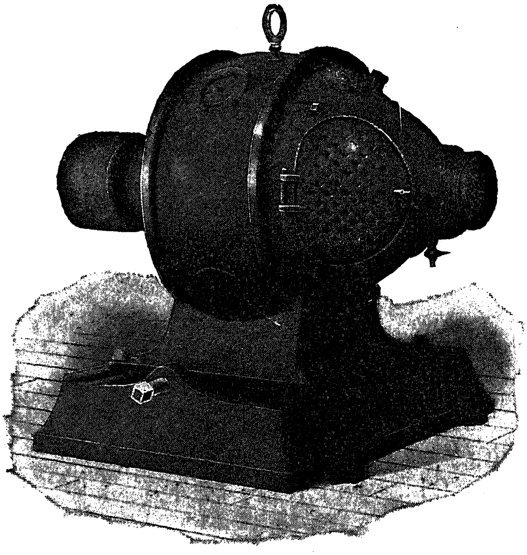


Fig. 194.—Four-Pole Enclosed Motor

The number of field poles varies with the diameter of the armature and the required speed, and in order to keep the hysteresis low, should be as few as the design will allow. The importance of restricting the number of field-poles is obvious when it is remembered that the iron losses usually average from 25 per cent to 50 per cent of the armature loss, and that they vary directly with the number of poles.

Further, in order to ensure a sufficient range of sparkless brush movement it is necessary to reduce the polar angle embraced per field-pole as the number of poles is increased.

In practice the sum of the angles embraced by the poles of a two-pole machine is about 280 degrees, that for a four-pole machine about 270 degrees, and that for a six-pole machine about 260 degrees. For a given armature it follows, then, that the number of teeth embraced per pole is rather less than the inverse proportion of the number of poles, and consequently, for a given total field-magnetic flux the density in the teeth will be greater with a six-pole machine than with a four-pole, and so on.

To counterbalance this effect the radial depth of the teeth should be decreased as the number of poles is increased, for then the width at the roots for the same pitch of teeth is proportionately greater.

Multipolar designs are a feature of modern practice, and have been arrived at chiefly because they permit of a compact, light, enclosed, and slow-speed machine, combining also, I believe, the important factor of maximum output for a given expenditure in material and labour.

The only circular type enclosed two-pole machine with a single exciting coil is the well-known Lundell motor, illustrated in fig. 192, which shows a cross-section and the chief feature, the single field-coil, and in fig. 193, which shows the motor taken apart. This type is built up to 10 horse-power. Larger sizes do not present the same advantages over multipolar designs owing to the relatively smaller cooling effects of the round carcass. Better outputs for weight at given speeds are obtainable from a multipolar design for large machines, and incidental difficulties are avoided.

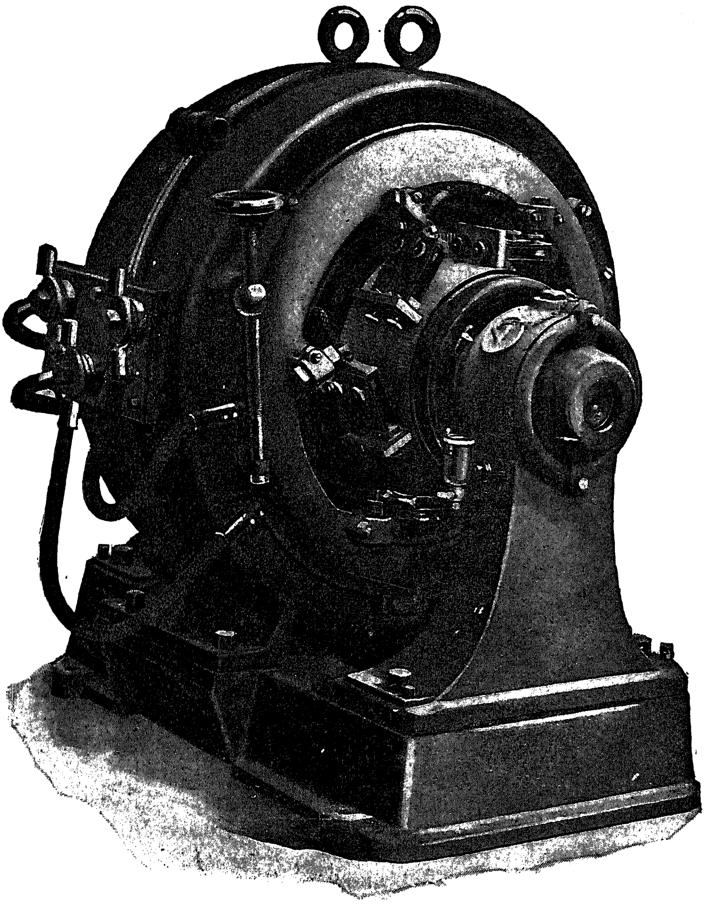


Fig. 195.—Six-Pole M.S. Type (Pedestal Bearings). 10 Horse-Power and Upwards

Makers using less compact and therefore heavier designs for small two-pole enclosed machines are at a relative disadvantage to the Lundell motor of similar output and speed; but all are on equal terms with the multipolar designs having a separate field-winding for each pole.

These points have not been arrived at without much experimenting. The shapes of motor carcasses in the past have been as varied as the schemes of winding and brush details. Now, however, one general shape is found to be practically the best suited for ordinary purposes, and is adopted with but slight variations by all the chief makers.

Leading Features of the Modern Motor.—The present type of the normal stationary continuous-current motor is, for the reasons discussed previously—except in small powers—always multipolar, and the bearings are generally combined with extensions of the field-yoke to enclose more or less completely the armature, commutator, and brushes. When these parts are exposed the motor is said to be of the "open" type. When they are enclosed completely it is said to be of the "enclosed" or "protected" type. When there are openings left for ventilation it is said to be of the "protected ventilated" type. The differences of the three types are illustrated in figs. 194, 195, 196, and 197, which show the leading features.

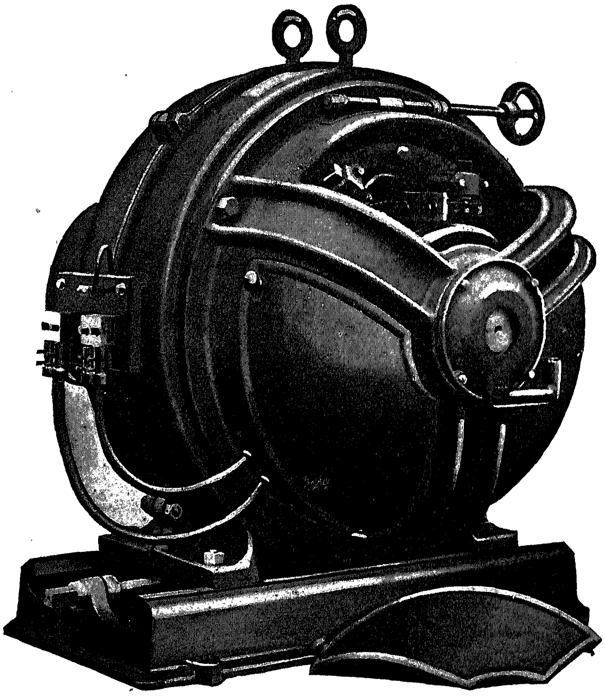


Fig. 196.—Lundell "Protected Ventilated" Six-Pole Motor

This shape of motor appears to demand castings for field-magnets and yoke in order to save costly machining and fitting, and many of the best makers employ them generally. These castings are made in steel on account of weight and space. The best commercial steel castings supplied by several Sheffield firms, and by at least one German firm, have a permeability equal to, or even higher than, that of best wrought-iron; *vide* curves in fig. 198. These castings can be made with a fair approximation to a close magnetic specification, and most of the required shapes are readily obtained free from blow-holes. Wrought-iron, although perhaps somewhat more certain in quality, requires costly machining.

The Johnston-Lundell Electric Traction Company's new designs have laminated field-magnet cores fitted into cast yokes (Plate No. 1), which shows the general arrangement. The field stampings are made of high-permeability iron, and therefore heat less than castings, and further, the design permits of very effective ventilation. It is claimed for this motor that it has a higher output for weight, and a higher efficiency, than any make of motor on the market. Another important feature of these motors is the duplex carbon brush, an end view of which is given (Plate No. 2). The two brushes are respectively high and low resistance carbon blocks combined in one unit. The former minimizes sparking due to the short-circuiting of live commutator segments, while the latter conducts the greater part of the main current. The makers state it is possible, by this means, to obtain a range of speed in the ratio 1 : 3 by shunt-control without the use of interpoles.

Messrs. Bruce, Peebles, & Company build all their motors of from 1 to 15 horse-power, with a framework of best annealed cast-steel, in one piece, round or octagonal in design, with 4 poles, accurately bored, faced, and machined to template, special jigs being used for each operation.

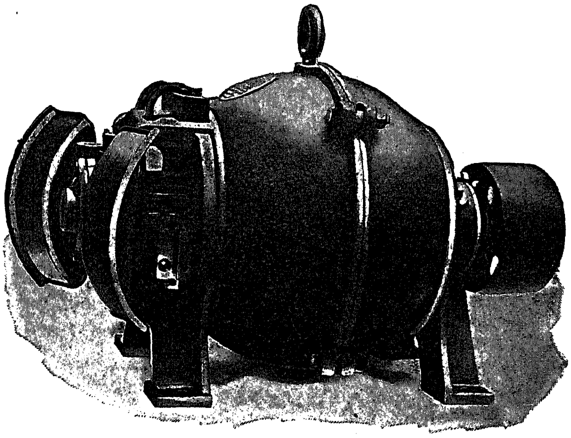


Fig. 197.—Lundell "Enclosed Bi-Polar Motor, with doors open

In motors from 15 horse-power upwards, the framework is circular in design and cast in two portions, so as to allow of the armature being easily removed. There are 4 or 6 poles, according to the size and voltage. Fig. 199 shows one of these motors taken apart. The excellent design and construction are apparent.

Mr. W. B. Esson's well-known designs of circular enclosed and protected types of motors are shown in longitudinal and cross section, and one of Messrs. Johnson & Phillips' 15-horse-power motors in figs. 200 and 201, the details of construction being clearly shown. Special notice should be given to the deep slots in the pole cores to reduce the demagnetizing effect of the armature. The longitudinal ventilating slots through the armature core are characteristic of best modern designs. The yoke-ring is bored out for the magnets and also at both ends to take the covers carrying the bearings, these being held in position by two set-screws. All this is lathe work. The motor can be fixed to the ceiling, or at any required angle, by simply turning round the bearing bracket to suit, and fixing it there by the set-screws. The brush-rocker, which consists of a circular ring with four projections carrying the brush spindles, is supported by the bearing cover, thus ensuring large bearing surface and rigidity.

The brush-holders are illustrated in fig. 202, and are so designed as to allow a variation in the pressure of from 0.14 lb. to 3 lbs. by placing the steel spring in different notches provided for it. The weight of the moving parts has been reduced as much as possible, and the carbon slides in the brush-holder box at such an angle as to reduce the friction to a minimum.

Accordingly the carbon follows very quickly any slight unevenness of the commutator surface. The current is conveyed from the carbon to the spindle by a flexible copper strip.

An important exception to the use of cast-steel is seen in Messrs.

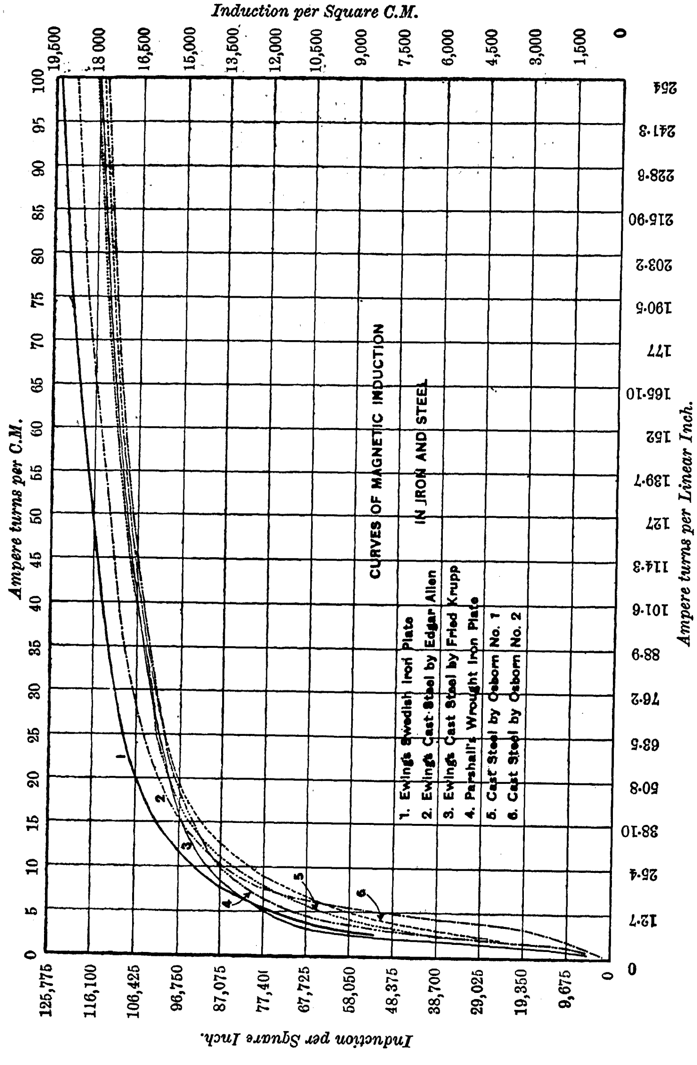


Fig. 198.—Curves connecting Excitation and Induction in Recent Samples of Iron and Steel

Robert W. Blackwell & Co.'s "Universal" Motor, in which the circular yoke is made of soft Norway wrought-iron.

The field cores are also wrought-iron turned and fitted into bored holes under hydraulic pressure, *vide* fig. 203. This illustration shows also this firm's method of reducing speed by back-gearing.

The list price, speed, and output for the enclosed type of these motors are all about the same as for the Lundell two-pole single-coil motor; but the weight is from 30 to 40 per cent heavier, and the probable cost of machining certainly appears to be greater.

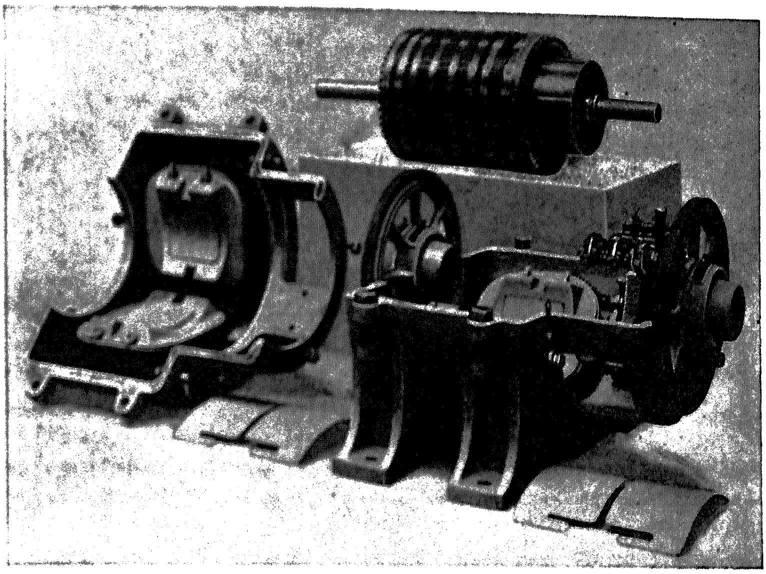


Fig. 199.—Bruce Peebles Motor, taken apart

It should be noted here that considerable improvements have been made in the quality of soft-iron castings for field-magnets, and not a few makers use cast-iron for stationary motors when the weight and over-all dimensions are not important.

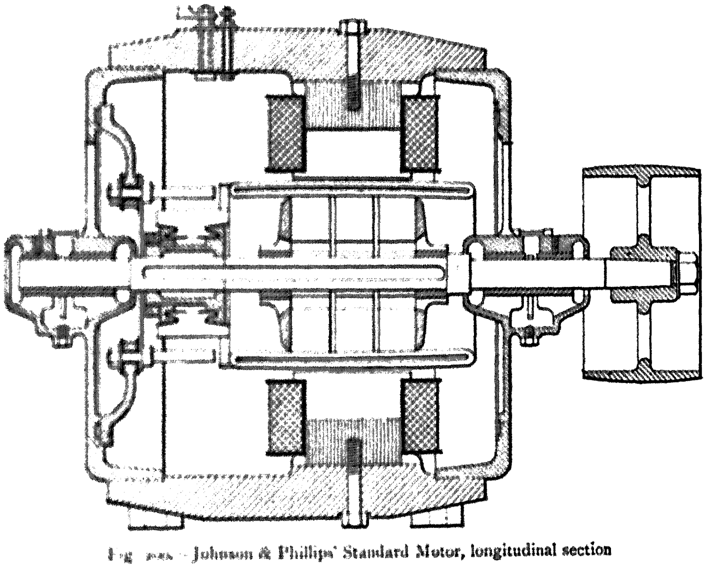


Fig. 200.—Johnson & Phillips' Standard Motor, longitudinal section

Cast-iron is readily made to almost any shape and is easily worked.

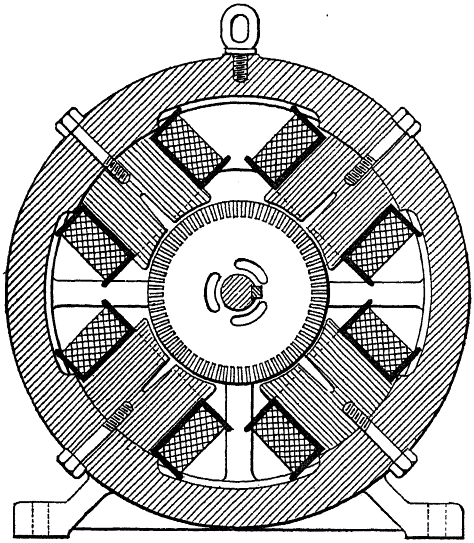


Fig. 201.—Johnson & Phillips' Standard Motor, cross-section

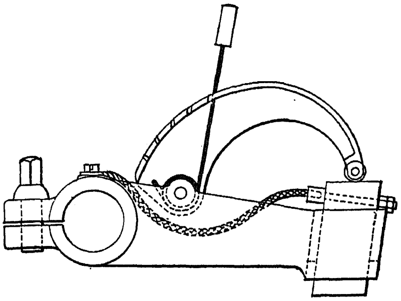


Fig. 202.—Johnson & Phillips' Standard Motor: Carbon Brush-Holder

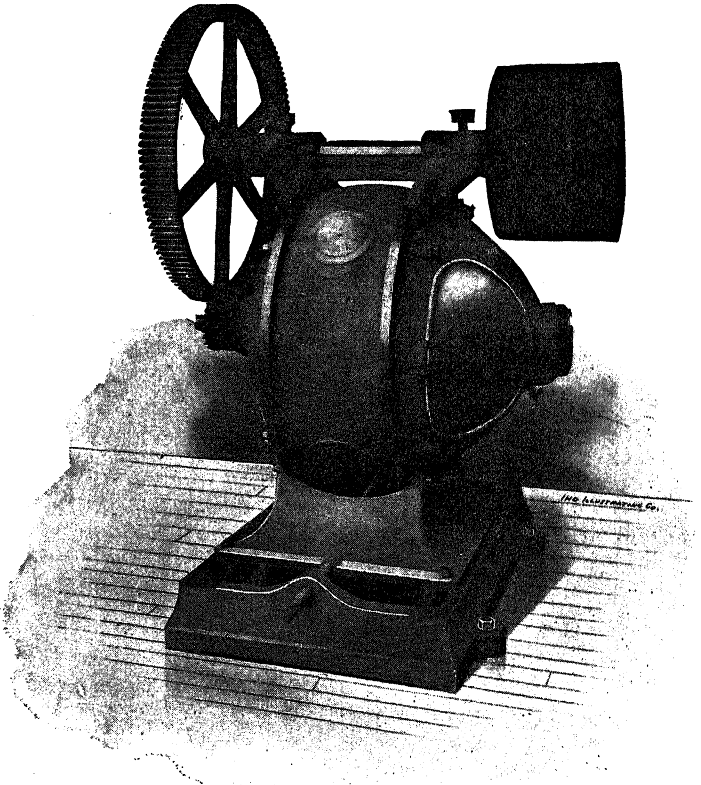


Fig. 203.—R. W. Blackwell & Co.'s Universal Motor—Back-Geared, fully enclosed

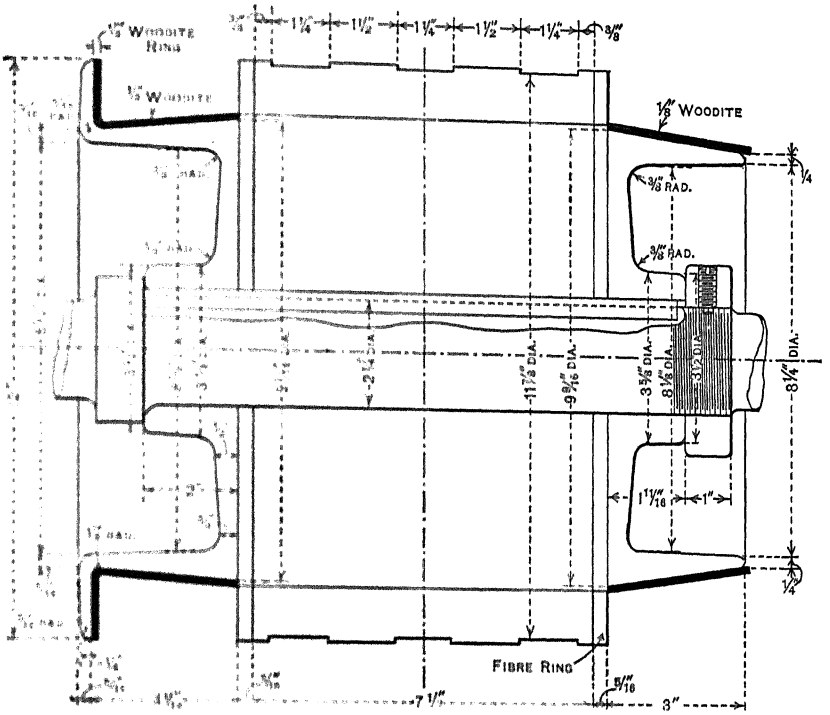


Fig. 204.—Longitudinal Section of Thames Ironworks' Motor Armature

As it has large magnetic inertia, it is adapted for giving a steady field, and is therefore specially useful for shunt-wound motors.

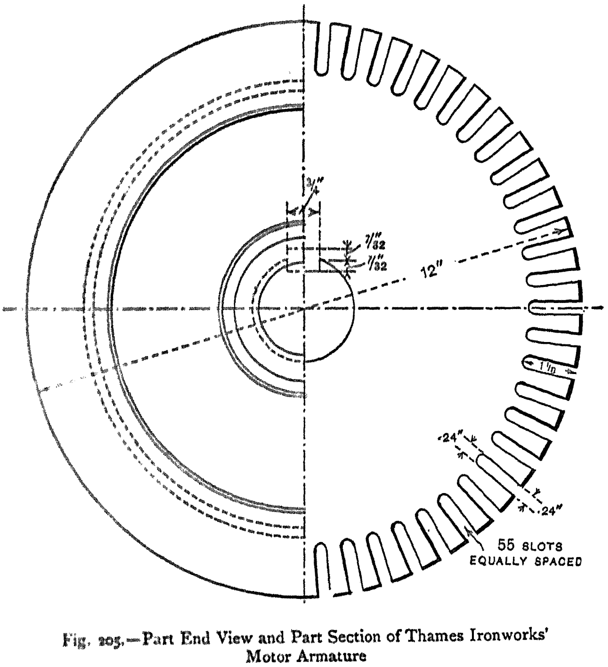


Fig. 205.—Part End View and Part Section of Thames Ironworks' Motor Armature

Figs. 204 and 205 show sections of the armature of a Thames Ironworks' motor with four poles. The cross-sections, radial depth of teeth, and other details are fairly illustrative of modern practice for machines of about 10 to 15 horse-power.

In practice, should it be necessary to conform to a given specification for speed, power, &c., in a number of motors, it is usual to compensate for slight differences in the permeability of magnet castings by varying the field turns or by slightly altering the air gap.

The armature core plates are, I believe, chiefly made from the best charcoal iron, although some are made from soft steel. My own experience favours iron, as I have had some unfortunate results with steel plates owing to the hysteresis loss being much above the average.

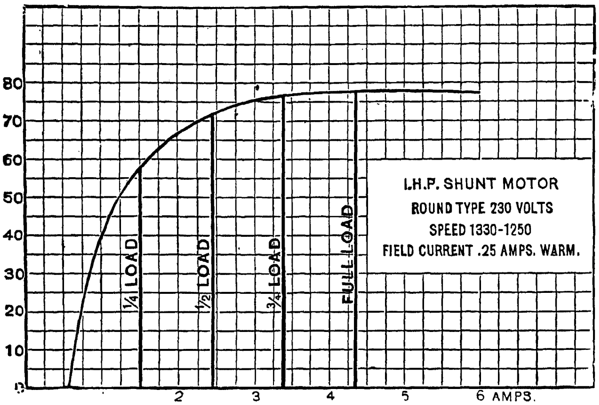


Fig. 206.—Efficiency Curve of Lundell 1-H.P. Motor

It is evident that the output for weight must be much less with the entirely enclosed motor, owing to the difficulty of dissipating the heat spent in the field and armature iron and windings. Hence this type is relatively costly, and is only used in special cases. The "protected ventilated" type has been found to be best suited for ordinary factory driving. The "enclosed" type is preferable for traction work, and generally for all outdoor applications.

In order to increase the cooling surface of enclosed motors some makers have devised radiating gills on the yokes or end pieces. They are troublesome to cast and difficult to keep clean, but they certainly tend to keep down the temperature.

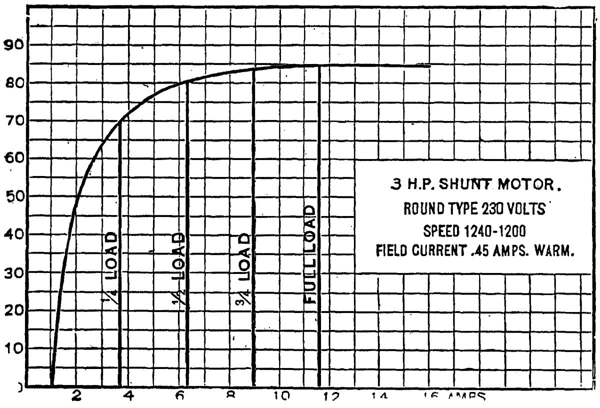


Fig. 207.—Efficiency Curve of Lundell 3-H.P. Motor

Efficiency of the Electric Motor.—The high degree of efficiency attained in the modern electric motor is best realized by continuous curves showing the efficiency from light to full and over load. Figs. 206, 207, and 208, taken from tests of Lundell motors, may be taken as typical of the best practice.

It is seen that even at ¼ load the efficiency is far higher than that attainable with a small gas- or steam-engine, and from ½ load to an overload of about 25 per cent the curves of motors of 3 horse-power and upwards are practically flat. As motors are usually worked within these limits it follows that for variable work the all-round efficiency is higher than that obtainable with any method of driving.

As already pointed out, the efficiency of the electric motor depends upon keeping the iron and copper losses and mechanical friction at a minimum, and the best results are attained when the energy wasted in heat is a minimum and the cooling effect is a maximum.

The copper losses are small when the number of amperes per square inch of cross-section is small. Practical considerations fix definite limits, which are from 2500 to 3000 amperes per square inch for armature and about 800 to 1000 for field-coils.

The iron losses depend on the weight and quality of the iron, the length of the magnetic paths, and the density of the magnetic flux. The former are defined by the geometrical lines of the motor, and the tendency of modern practice has been already indicated in the vogue for the multipolar variety with a round field.

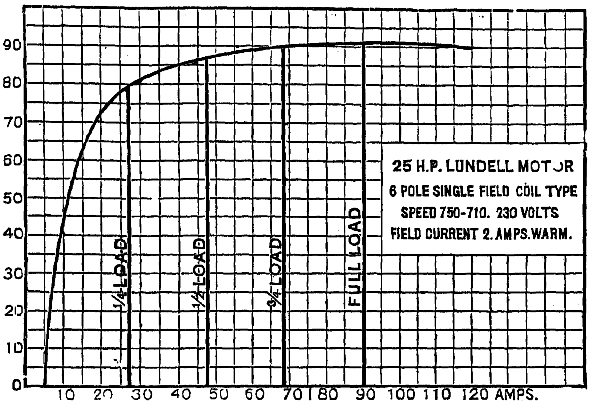


Fig. 208.—Efficiency Curve of Lundell 25-H.P. Motor

The magnetic flux is also fixed by practical considerations for armature cores at about 10,000 to 14,000 C.G.S. lines, for teeth at from 17,000 to 19,000 C.G.S. lines, measured at about one-third of the length from the root of the tooth, and in field-cores and yokes at about 12,000 to 15,000 C.G.S. lines. Special designs will, of course, depart from these figures.

The great importance of thorough ventilation is evident, as it lowers the resistance of the copper coils and increases the permeability of the iron by assisting to keep down the average temperature of the motor; and further, by doing this provides a margin for temporary overloads in excess of that permissible if the motor ran under the "rated" conditions at or near to the safe working temperature.

Ventilation is secured by air-ducts in the armature cores and field-poles, and sometimes by fans forming part of the armature construction.

Messrs Joseph Sankey & Sons' new iron alloy "Stalloy" is likely to effect important alterations in general designs, for the hysteresis and eddy-current losses are reduced to about one-half those in their alloy "Lohys". (See accompanying plate for comparative curves of watts lost per pound, for various values of B, and gauges of sheets.) With this material it will be possible to decrease the weight output and yet retain a high efficiency.

CHAPTER II

APPLICATIONS OF THE ELECTRIC MOTOR

It will be convenient to treat the applications of the electric motor under the following heads:—

(*a*) Domestic, small trades, pumps, fans, &c.
(*b*) Driving factories, &c.
(*c*) Driving lifts, cranes, &c.

(a) Domestic Uses, &c.

The motor has so far not found a large field in purely domestic work, although enthusiasts have drawn rosy pictures of the lightening of domestic drudgery by means of electric power, and political economists have sought for a cure for the real or fancied evils of centralization in the same source.

The fact is, the number of strictly domestic operations which can be economically carried out by "power" is limited; and further, small motors which are alone suitable for such purposes necessarily run at relatively high speeds. Single sewing-machines are conveniently and economically driven by friction-wheels from a $\frac{1}{16}$ horse-power motor.

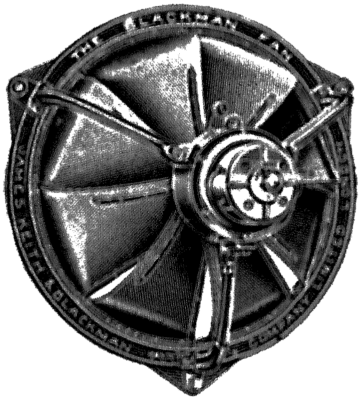


Fig. 209.—Blackman Protected Ventilated Motor Fan

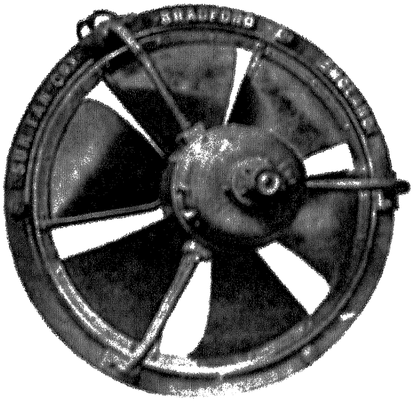


Fig. 210.—"Sun" Fan, 18 ins. diameter. Water-tight Motor, porthole type. Largely used for ship-work

Again, most of the supply companies serve consumers at 200 or more volts, and it is not possible to build small, efficient, and yet cheap motors for so high a pressure. Hence the capital outlay for motor plant for purely domestic purposes is often too large to permit of the economic driving of single sewing-machines, boot or knife cleaners, and the like kind of simple machines which are usually found in our homes.

A large field for small motors is found in fan driving. The recent census of "The Manufacture of Electrical Apparatus and Supplies" in the United States gives the aggregate of fan motors for the year 1900 at 12,766 horse-power, and this in spite of the small average size.

Among the best-known electric fans are the Blackman, Sun, and Sirocco. Illustrations of the two former are shown in figs. 209 and 210. They are typical of the best modern practice.

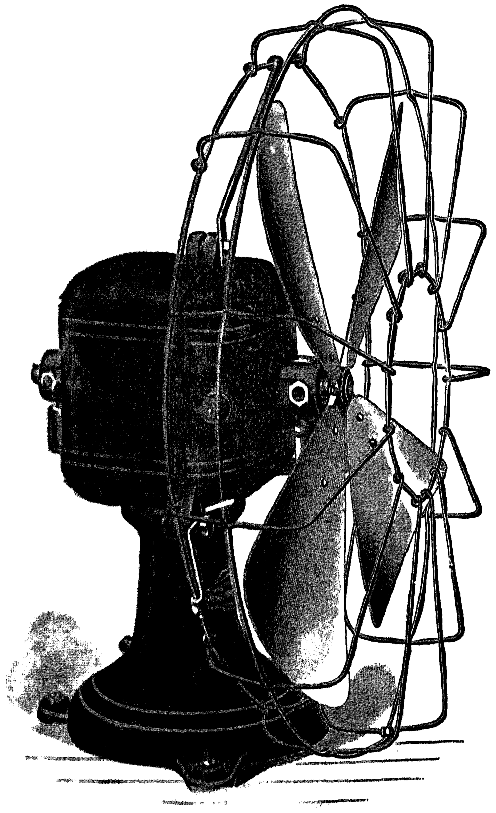


Fig. 211.—Desk Fan (General Electric Company)

Electric fan motors are almost without exception wound in series, as the load is constant for any speed and there is no risk of racing. The fan blades are built either to circulate a large volume of air against a small water gauge, or to drive or exhaust a smaller volume against a back pressure. Since the work in a fan, and therefore the speed and current, is affected by a wind acting with or against the fan, it is desirable to fit a suitable fuse or maximum current release to large fans. Otherwise there is much risk of burning the motor windings when a strong natural draught is opposed to the fan. This has happened occasionally with fans in exposed positions, or exhausting through long ducts without hoods at the outlets. In small fans gauze brushes are used in order to avoid the occasional irritating noise of carbons. Most types of fan motors are fitted with continuous lubricating devices, and the vertical shaft bearings are fitted with ball-bearing thrust-blocks.

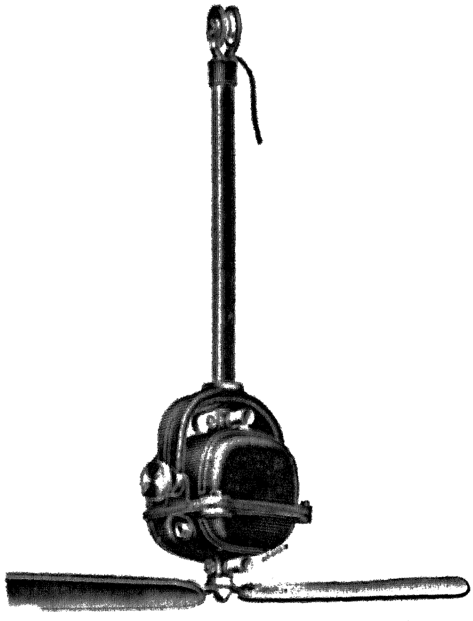


Fig. 212.—Ceiling Fan (General Electric Company)

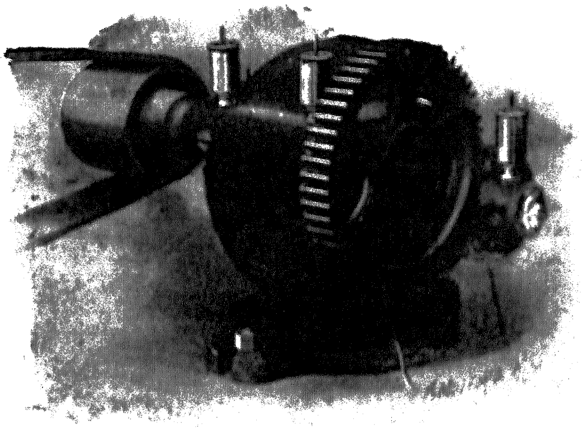


Fig. 213.—Eck "Hurricane" Protected Type Small Geared Motor, $\frac{1}{8}$ and $\frac{1}{4}$ H.P. for Direct Current

A reference to electric fans would be incomplete without mentioning the desk and ceiling fans (see figs. 211 and 212). Both of these types are essentially domestic, and in hot climates invaluable. The illustrations are from the lists of the General Electric Company, and are typical of the class.

One of the best lines of small motors, from $\frac{1}{12}$ to $\frac{1}{2}$ horse-power, is that of Messrs. Robert W. Blackwell & Co. The type is illustrated in fig. 213.

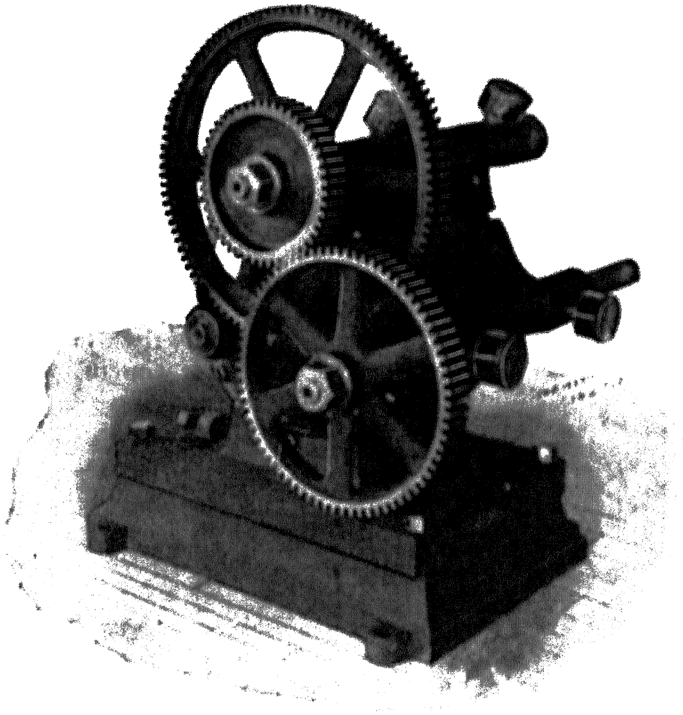


Fig. 214.—Blackwell "Jenny" Motor with Double Reduction Change Gear

It is protected and ventilated, and fitted with back gear. The speed of the countershaft can be varied from 600 to 400 revolutions per minute, according to the ratio of the gearing. The motors are wound up to 250 volts, and are supplied with regulating resistances controlling the speed within definite limits.

In addition to purely domestic applications small motors are most useful for dental machines and other special small machine-tools.

Fig. 214 shows a Blackwell double-reduction change-gear motor designed to reduce the speed of the pulley to about $\frac{1}{5}$ or $\frac{1}{6}$ that of the armature.

Fig. 215 shows a small motor with idle pulley attached to the frame. This is most useful for difficult drives, and will be appreciated by engineers and factory managers.

Figs. 216 and 217 show two applications of these small motors to the printing trade. These illustrations are self-explanatory to the expert printer, and are typical of numerous special machines in almost every trade. Indeed there seems to be no limit to the special applications of the electric motor.

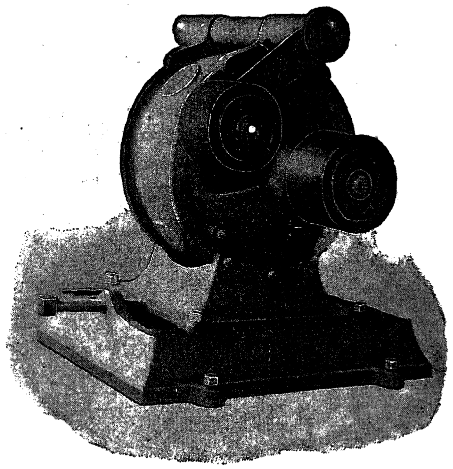


Fig. 215.—Blackwell "Jenney" Motor, complete, with Idler. Small size

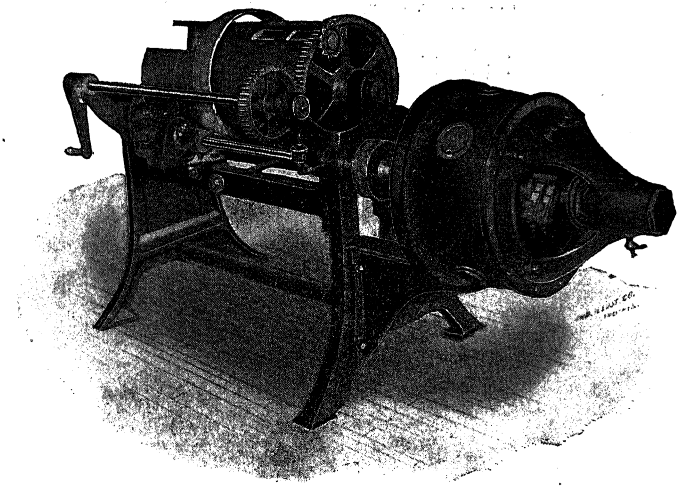


Fig. 216.—Blackwell "Jenney" Motor attached to a Hoe Tail Cutter, for the Printing Trade

The electric motor is largely used for pumping of all kinds, and this application is treated fully in the chapter on mining. Here it will be sufficient to illustrate two of the most common cases for electric drives. Fig. 218 shows a Bruce-Peebles motor coupled to twin centrifugal pumps

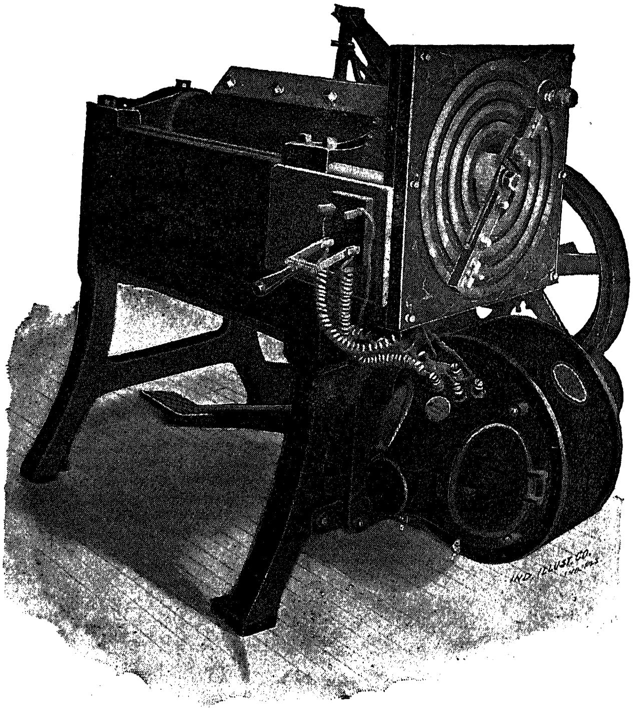


Fig. 217.—Blackwell "Jenney" Motor attached to Hoe Curve-Plate Shaving-Machine, for the Printing Trade

running in series, and fig. 219 one of the same firm's motors direct-geared to a set of three-throw vertical pumps. The excellent motor made by this firm has been mentioned already. Attention is directed now to the sound mechanical arrangement indicated in the two illustrations.

Steam-boiler feed-pumps are admittedly wasteful of steam, and many power-stations are now equipped with motor-driven pumps. The arrangement shown in fig. 220 is compact, efficient, and provides complete protection for the motor.

Fig. 221 shows a motor-driven punkah.

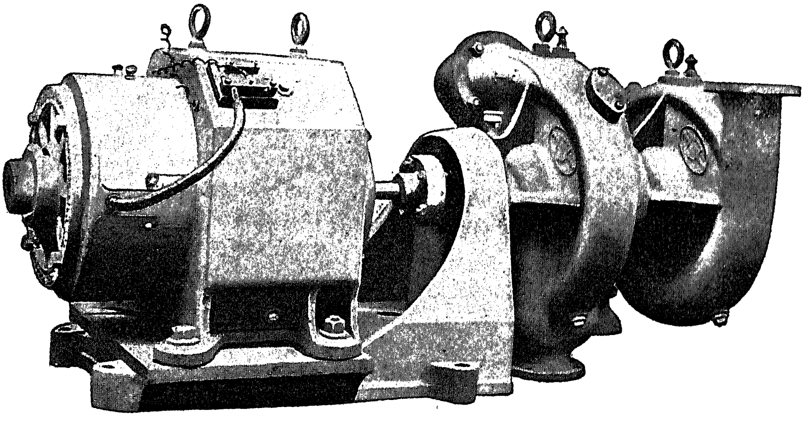


Fig. 218.—Bruce-Peebles Motor Direct-Coupled to Twin Centrifugal Pumps

The motor is to the right in the illustration, and drives through a highly efficient worm gearing a shaft arranged at a right angle with it.

On this shaft is mounted a magnetic clutch. The fixed part of the clutch, which in the illustration has a drum-like appearance, is keyed to the shaft, and the rope pulley which forms the armature of the clutch is free to rotate upon the shaft. The clutch magnet coil is connected in series with the motor through a two-way switch, which in one position cuts out the magnet coil and sets the rope pulley free. In the other position the full current passes through the magnet coil and holds the pulley to the clutch. The switch is moved automatically in time with the swing of the punkah by toothed-wheel gearing and a pinion fixed to the rope pulley as shown.

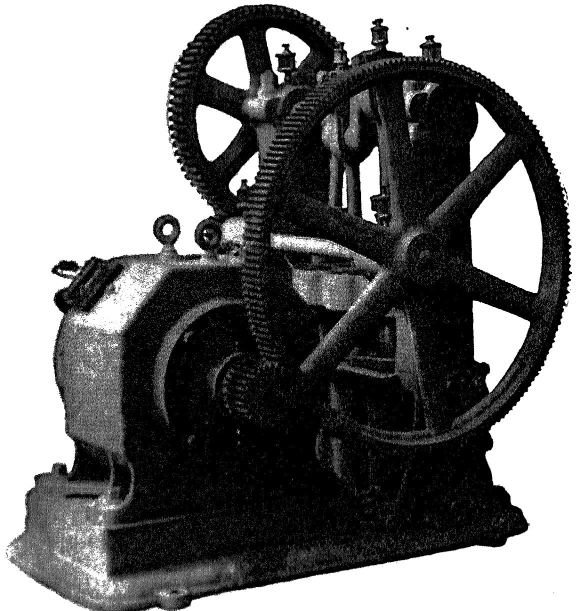


Fig. 219.—Bruce-Peebles Electric Pump with Double Gear

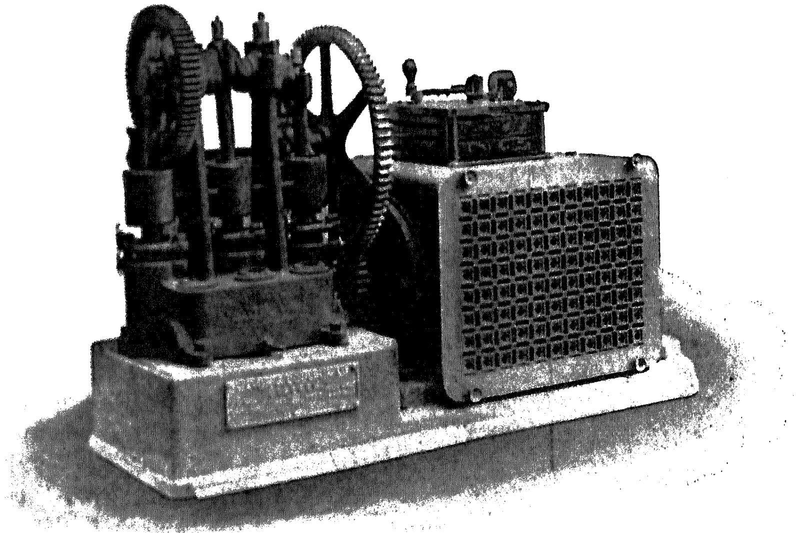


Fig. 220.—Mavor & Coulson Motor geared to Three-Throw Pumps

Two adjustable pins are fixed to the toothed wheel, which alternately catch the switch arm and throw it over. The length of rope and position of the pins are so adjusted that when the punkah is at the bottom of the swing the switch is in position to energize the electric magnet. The motor then winds up the rope until the punkah reaches the top of its swing, when the second pin throws over the switch arm, the clutch releases, and the weight of the punkah brings back the pulley and toothed wheel until the pin comes into contact with the switch arm. the magnet is again energized, and the cycle re-begins. The motor is thus kept constantly running in the same direction, and the reciprocating motion is produced automatically in step with the period of swing of the punkah. The worm shaft driven by the motor may be extended, and a row of punkahs, each controlled by a separate clutch on the shaft, may thus be driven by one motor.

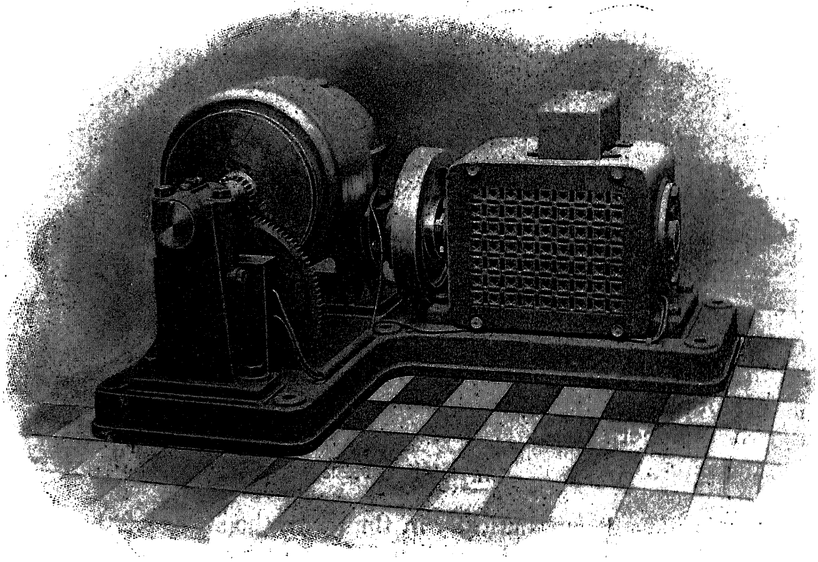


Fig. 221.—Mavor & Coulson Electric Motor-Driven Punkah

The motors are made in standard sizes—$\frac{1}{4}$ horse-power, $\frac{1}{2}$ horse-power, and 1 horse-power.

(b) Factory Driving

One of the best fields for electric driving is found in factories of all kinds where power is essential to efficiency and economy.

Indeed there are probably few, if any, operations performed by gas- or steam-engines which may not be accomplished at least with equal efficiency by the electric motor, and in the majority of cases better all-round results are obtainable.

The most apparent advantages of an electric drive for such purposes are—

(1) Good speed regulations for varying loads.
(2) High relative efficiency at all loads.

(3) Economy of space occupied by the motor.
(4) Economy in prime cost of power.
(5) Economy in transmission and in application.
(6) Absence of heavy foundations.
(7) Freedom from vibration, noise, or smells of any kind.
(8) Attendance reduced to a minimum.
(9) Skilled attention unnecessary, except for occasional surveys.
(10) Risk of breakdown comparatively slight.
(11) Wearing parts few in number and cheap to replace.
(12) Small capital outlay.
(13) Small consumption of energy, because the power is automatically adjusted to the load.

This list by no means exhausts the "points" of an electric drive; it merely indicates the chief features.

For factory purposes it does not often happen that motors of less than ½ horse-power are required, and more frequently machines of from 1 to 15 horse-power are found to be the most suitable. In laying out a new factory, or in altering an established one from gas or steam to electric driving, there are several broad issues to be considered and weighed very carefully before the scheme be decided on.

First, the operations to be performed should be considered. Whether they are mainly continuous or intermittent. Whether all the plant must be kept running, or whether part of it may be shut down for periods without seriously affecting the economy of the operations, &c.

For example, in a boot-factory the hand-work and machine operations are generally arranged so as to balance only when all the plant is in use. In an engineer's shop or a printing establishment, on the other hand, it is generally absolutely necessary to run separate machines or groups of similar machines independently. An efficient scheme will obviously include suitable arrangements to meet these several requirements.

It thus happens that in some instances one or two large motors may be conveniently arranged to drive a small factory through line shafting more economically than a number of smaller motors connected to separate drives. In other cases, such as printing establishments, it is usually found more economic, in spite of the increased capital outlay, to drive each large machine by a separate motor. The *Daily Express* newspaper, for example, has all the large presses separately driven, and the Linotypes are driven partly in groups and partly off separate motors. At the *Sussex Daily Paper* offices I found it more convenient to drive the Linotypes from a common shaft with two motors, one being a reserve, each motor having a fast and loose pulley.

In many instances, however, it appears to pay to drive Linotypes separately, and Messrs. R. W. Blackwell & Company have introduced from America a Jenney geared motor which is attached to the Linotype machines as shown in fig. 222, so as to form practically part of the machine. The power absorbed is only 100 watts, and the speed can be regulated by means of an auxiliary field winding between 63 and 70 revolutions per minute to suit the average operators. The motor actuates the machine through a clutch gear-wheel.

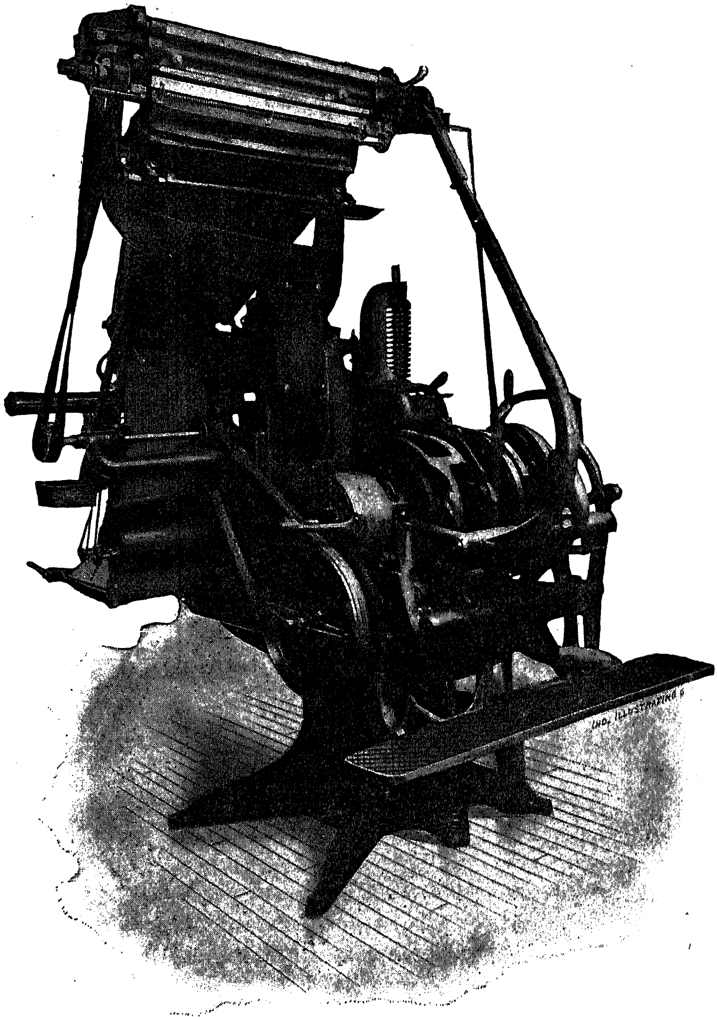


Fig. 222.—Complete Linotype Machine, with Blackwell "Jenney" Motor and Clutch Gear-Wheel fitted

Except in special cases it will be found that it does not pay to put in motors smaller than 5 brake horse-power or larger than 10 horse-power. Each motor should drive a length of shafting from which one or a number of similar machines may be run. Machines requiring more than 10 horse-power are generally driven more economically by separate motors, especially if they be intermittent in operation, as this reduces the power lost in shafting.

The power lost in main and line shafting and belting is usually much greater than the owners suppose. Careful tests have been made during recent years by competent observers in a number of industrial concerns with results that have caused much astonishment. The tests show a loss in shafting, bearings, and belting varying from 15 per cent to 80 per cent of the total indicated horse-power of the engines.

W. Arthur Ker stated, in a paper read before the Liverpool Engineering Society in 1900, that the constant loss in shafting in seventeen different workshops amounted to an average of 42 per cent of the full-load output.

Flather gives in *Dawson's Pocket-Book* a list of nineteen large workshops having losses of from 15 to 80 per cent, and an average of 38.6 per cent.

Fessenden found the mean of tests on 108 shops to average 69 per cent. Hawthorne in fifty-five tests found the average loss to be 26 per cent. Gibbs in a series of tests found 55 per cent of the power to be lost in friction.

Raworth, Selby-Bigge, Lacey, Clirehugh, and Sillars have also arrived at similar results, and I have also found the shafting losses in printing establishments, boot-factories, and other works to agree with the preceding.

Some recent figures are given in the following table, compiled by D. Selby-Bigge for a paper read before the Iron and Steel Institute at Düsseldorf, in the autumn of 1902:—

Nature of Work.	Total Horse-power.	Per cent to Drive Shafting.	State of Load.	Horse-power per 100 feet Shafting.	Horse-power per lb. Shafting.	Horse-power per lb. Bearing.	Horse-power per Belt.
Wire-drawing and polishing	400	39	½	14.0	0.58	0.37	1.76
Steel-stamping and polishing	74	77	⅓	9.8	0.35	0.84	2.4
Boiler and machine work ...	38	65	⅓	4.77	0.21	0.55	0.48
Heavy machine work ...	112	57	1	5.7	0.23	0.58	0.45
Light machine work ...	74	54	1	8.0	0.4	0.69	0.12
Manufacture of small tools	47	52	1	2.5	0.23	0.24	0.11
do. do.	190	57	1	4.36	0.43	0.39	0.21
Screw machines and screws	241	47	1	6.33	0.38	0.63	0.23

It must not be assumed that these high percentages necessarily imply uneconomic working; so much depends on the total power required and the nature of the operation. The electrical engineer, in investigating such problems, should carefully balance the cost of the existing system of driving against the interest, sinking fund, and maintenance of an electric drive. In some cases it will be found that the interest on capital outlay due to scrapping of old and purchase of new plant added to revenue charges will be so large as hardly to warrant a conversion, having regard to the interests of the shareholder, although there would be no question as to the proper course if starting *de novo*.

There are few published records of the variations in demand for power with a number of machine tools driven from a single-line shaft. Indeed a recording ammeter is necessary to show the variations accurately, and this instrument is not in common use. The diagrams in fig. 223, taken by Mr. W. A. Ker and published in the paper already referred to here, are therefore specially interesting. They were taken during the ordinary working of the shops of Messrs. C. & A. Musker (Limited), Tuebrook, and are indicative of everyday motor driving. It will be seen there is a considerable variation in power from minute to minute on each line shaft.

A point of much importance is the average number of machines likely to be running at once. Mr. W. A. Ker found, from a large number of observations of various line shafts, that two-thirds of the total number of machines was approximately the average number running at once. This agrees well with my own experience in various shops. I find the inequalities on the several lines of shafting tend to balance when there are a number of motors at work driving separate shafts, and given sufficient motors, the total average load does not vary much during normal working.

In a boot-factory converted by me from gas-engine to motor-driving, it was found that with two 15-horse-power motors, one 7-horse-power motor, two 5-horse-power motors, two 3-horse-power motors, and one very small motor, the main ammeter showed an average load of about 35 horse-power, with a maximum variation of less than 10 per cent and generally less than 5 per cent, although the load on all the motors varied from minute to minute in a much greater degree.

The net result with motor drives is that not only is the loss in heavy main shafting obviated, but, since a well-designed motor is capable of a 50-per-cent overload for short periods and over 100 per cent for very short periods, motors of smaller rated powers are permissible for the several line shafts than would be safe with steam- or gas-engines; and the aggregate load, and therefore the power of the engine and boilers, may be much less than with the old system.

These considerations help to explain how it is that much less boiler power is required to drive given works through dynamos and motors than with separate engines or engine driving through main and line shafting.

Two instances selected at random from Mr. Selby-Bigge's Düsseldorf paper will be instructive. At Messrs. Vickers Maxim's ship-building works at Barrow-in-Furness the average coal consumption per month during the winter of 1898, with steam-power, was 476 tons; during the corresponding period of 1899, with electric power, the monthly consumption was only 232 tons, being a saving of about one-half.

Again, Messrs. Richardsons, Westgarth, & Company state that the output of their tools is now, with electric power, 30 per cent in excess of what it was with steam. With the old shafts and belts the workmen were limited to, say, two or three speeds as a rule, whereas with an electric drive their choice of speed was much greater. This is a point that is not generally appreciated even by engineers.

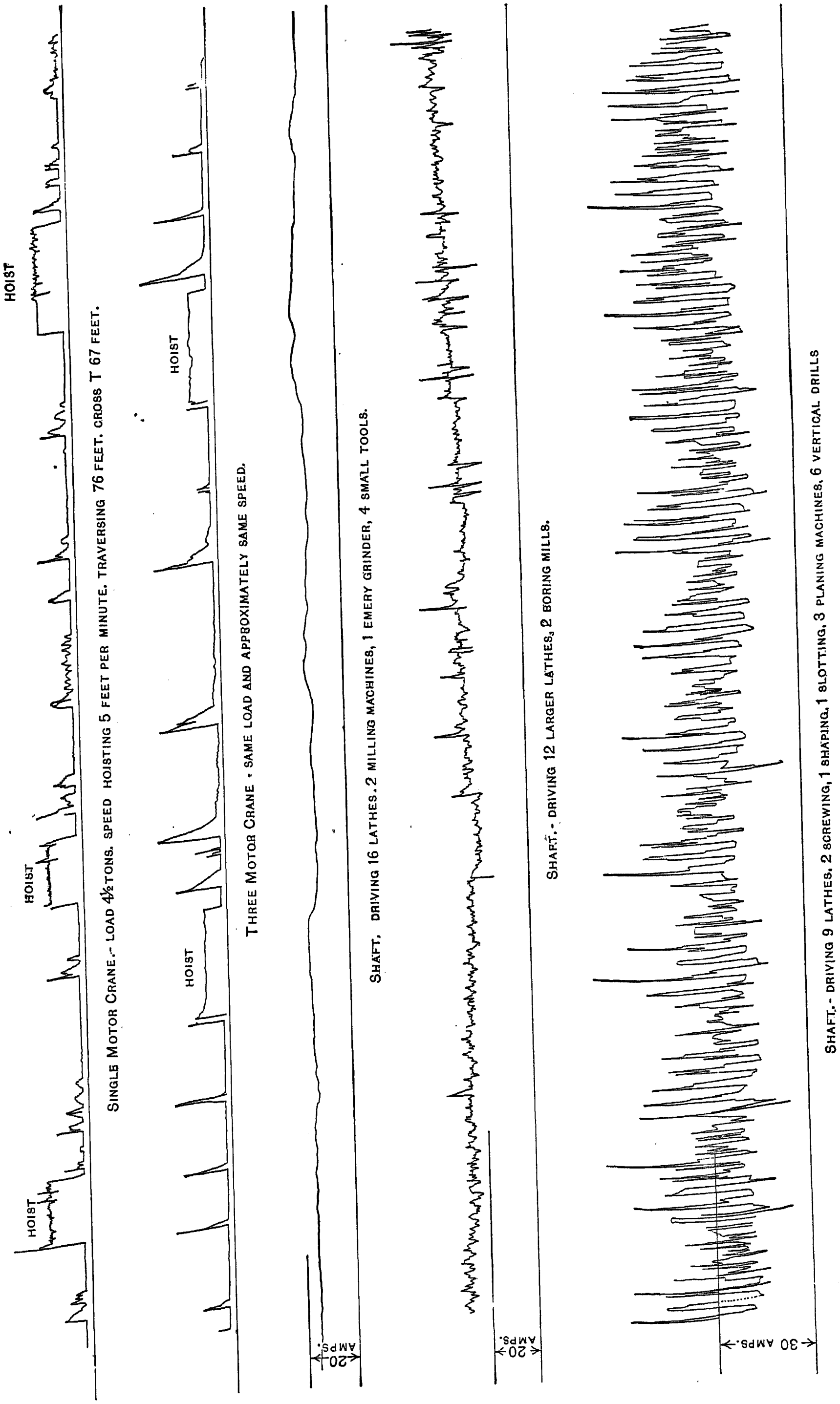


Fig. 223.—Power Diagrams of Motor Drives, taken by Mr. W. A. Ker

(c) Electric Lifts, Cranes, &c.

The electric motor is used largely to drive all kinds of lifting and hoisting machinery, and is found to be in many respects more convenient than rope or shaft driving, and even steam or hydraulic power, and also is generally cheaper.

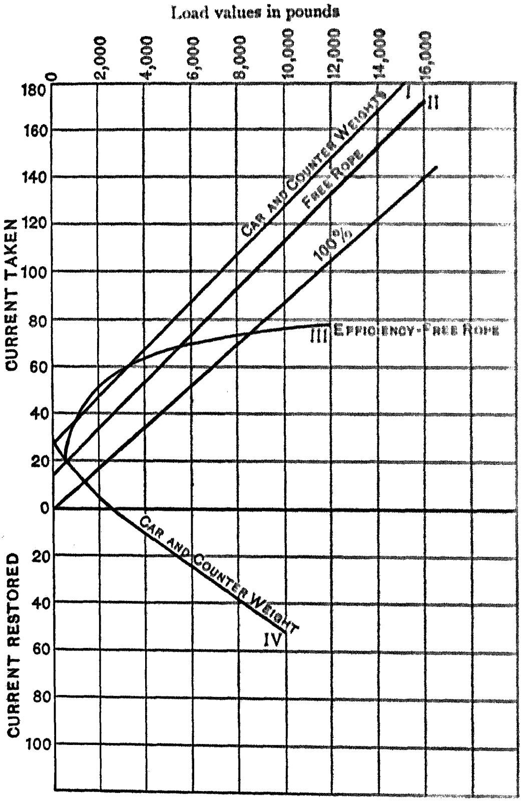


Fig. 224.—Curves of Central London Railway Lift

The ease with which electric power can be economically transmitted, in spite of distance or position, obviously renders it specially applicable to a number of purposes for which other forms of energy are unsuitable, or impossible on the score of either cost of installation or running charges.

This is abundantly proved by the vogue for electric-hoisting apparatus in hotels, warehouses, factories, railway-stations, goods yards, &c.; while in battle-ships, cruisers, and merchant-ships auxiliary steam, hydraulic, and pneumatic engines are being gradually replaced by electric motors.

For the great majority of hoisting apparatus the motor is entirely iron-clad, and the field-magnets are compound wound, with the series turns assisting the shunt winding. This gives a high initial torque at starting, with small expenditure of energy. The series turns are frequently short-circuited when the required speed is attained, and the motor then running with shunt excitation cannot race. This arrangement conforms to the mechanical requirements of most hoisting problems, and usually gives a higher efficiency than is possible with any other. Some modern lifts, however, are actuated by shunt motors; but it is difficult to see any advantage in the arrangement except simplification of the controller details. The economy of an electric lift is seen from the following:—

COMPARATIVE COST OF WORKING HYDRAULIC AND ELECTRIC LIFTS.

Type of Lift.	Load in cwts.	Source of Power.	Travel in feet.	Cost of average round trip up and down.	Number of trips per penny.	Remarks.
Electric	7	Manchester Corporation	50	0.159*d.*	6.3	Observed. Conditions unfavourable. Current at 2*d.*
Hydraulic suspended high-pressure	7	Do.	50	0.29	3.45	Calculated from published scale.
Hydraulic suspended low-pressure	7	Do.	50	0.445	2.2	Calculated at 6*d.* per 1000 gallons. Pressure, 50 lbs.
Electric	9	Private supply	50	0.066	15.0	Observed. Conditions ordinary. Current at 2*d.*
Hydraulic suspended high-pressure	9	London Hydraulic Power Company	50	0.277	3.6	Calculated from published scale.
Electric	9	Glasgow Corporation	50	0.061	16.4	Observed. Current at 2*d.*
Hydraulic suspended high-pressure	9	Do.	50	0.212	4.7	Calculated from published scale.
Do.	12	London Hydraulic Power Company	50	0.35	2.86	Observed.
Do	9	Do.	50	0.287	3.5	Observed
Do.	12	Do.	50	0.42	2.4	Observed.

Table from a paper read before the Iron and Steel Institute by D. Selby-Bigge at Düsseldorf in the autumn of 1902.

The averages of the tabulated data are:—
Electric lifts = 0.095*d.* per round trip and 12.6 round trips per penny.
Hydraulic lifts = 0.326*d.* „ „ 3.24 „ „

It is thus seen that the electric lifts average rather less than one-third the hydraulic lifts as regards cost of power.

TESTS OF AN EASTON & COMPANY'S ELECTRIC GOODS LIFT.

By ALBION T. SNELL. 1902.

Load.	Direction.	Time in seconds.	Amperes.	Watts.	B.O.T. units per trip.
Light	up	50	too small to read		
„	down	„	8.5	3.740	.052
5 cwts.	up	„	1.0	440	.006
„	down	„	4.0	1.760	.024
10 cwts.	up	„	4.0	1.760	.024
„	down	„	2.0	880	.012
15 cwts.	up	„	8.5	3.740	.052
„	down	„	too small to read		

In the preceding tests the average speed, including starting and stopping, was 66 feet per minute, the maximum being about 80 feet per minute. The average energy per round trip (up or down) did not exceed 0.06 of a Board of Trade unit, which at 2*d.* per unit is equal to 0.12*d.*

Since the average journey is probably only half the full distance, the cost per average journey (up or down) is 0.03*d.*, say 33 average journeys per 1*d.*

Mr. G. Rosenbusch, A.M.I.C.E., of Messrs. Waygood & Otis (Limited), has been good enough to give me the benefit of his and of his firm's large experience in connection with electric-lift work, and I am thus enabled to give the following description of the electric-lift motor with some most instructive blocks:—

The considerations governing the motor are, that it must start quietly under heavy load, run at nearly constant speed under varying conditions of loading, and have a large overload factor. The motor speed is governed by the method adopted for transmitting the rotatory motion of the armature to the vertical movement of the cage. The reducing mechanism for operating the slower-speed apparatus must also be efficient and noiseless. Of the several mechanical combinations available, worm gearing has been chosen in most cases as the most suitable on account of noiseless running, high-ratio speed reduction, compactness, and sound mechanical construction. It is efficient when properly made, and numbers of recent tests show that worm gearing is now made of over 80 per cent efficiency, and in some cases of approximately 90 per cent. The motor is coupled in line with the worm shaft, and actuates a winding drum or friction sheave on the worm-wheel shaft.

It is desirable that the net load to be moved, neglecting friction, should be as nearly as possible always zero, because then the work done is a minimum. In other words, the live load, together with the necessary dead load of cage and accessories, should be counterbalanced. This means that the drum in turning operates two separate masses in opposite directions—namely, the cage with its live load, and a counterbalance equal in weight to the cage plus the average load carried, so that the cage is overbalanced. In lifts of large capacity this overbalance is of considerable magnitude, sometimes reaching from 5000 to 6000 lbs. The maximum net load is thus reduced below the live load, and a smaller capacity motor can be used, ensuring economy in prime cost and reducing the friction losses, which are constant.

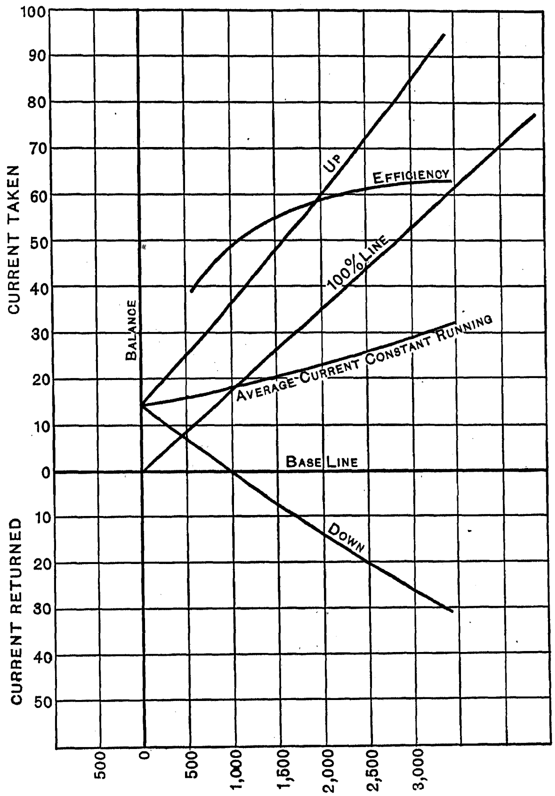


Fig. 225.—Curves of Normal Average Size Electric Lift

This method of loading, together with the resultant possible conditions of equilibrium, determine the kind of motor to be used. For the net load may assist the motor when an empty cage ascends or a heavy load descends; or it may be against the motor when a heavy load ascends or an empty cage descends; or it may, in various intermediate states, approximate to equilibrium. It follows, then, that current may be taken of varying amounts from zero to maximum, or current may even be generated, irrespective of the direction of motion of the cage. Fig. 224 is a curve of a test of a lift on the Central London Railway, and fig. 225 that of an average-sized lift. They show the large variation of power under normal working conditions to which lifts are subjected. These two sets of curves are worthy of special study, since they show the order of the amount of current which can be generated by modern lifts under conditions of normal running. Both figures have current values for the ordinates and load values in lbs. for the abscissæ.

In fig. 225, where the cage and live load balance the counterweight, a current of 15 amperes is required to overcome friction. As the live load is increased, the current varies positively or negatively as the cage is ascending or descending. With an out-of-balance load of 1000 lbs. coming down the current is zero. Similarly, 2000 lbs. in the same direction *generates* 15 amperes. And an ascending out-of-balance load of 1000 lbs. absorbs 35 amperes, and 2000 lbs. about 60 amperes. The preceding quantities are roughly estimated from the scale of the curves.

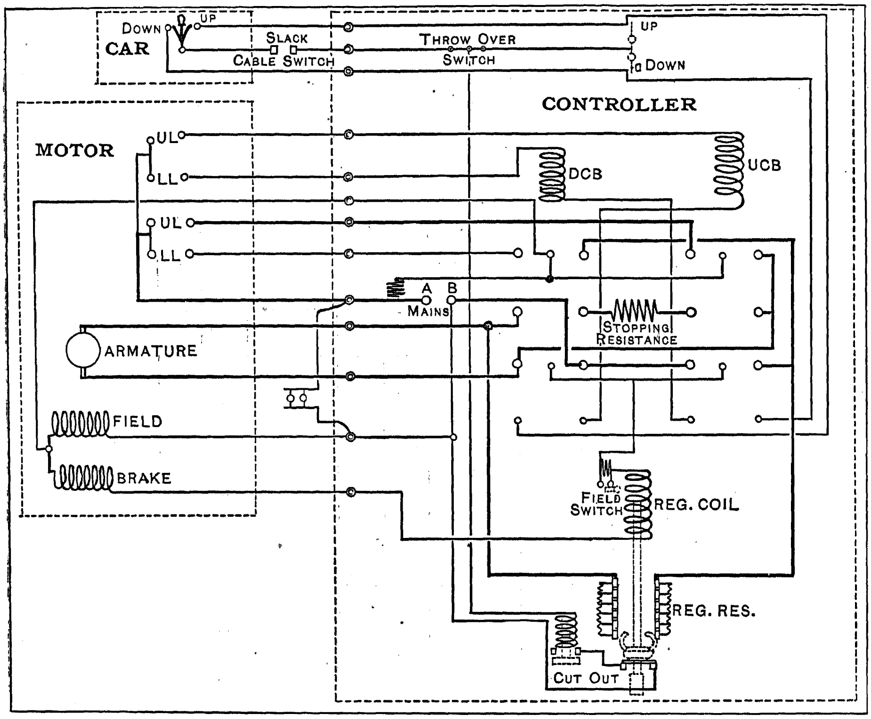


Fig. 226 —Diagram of Connections of Waygood & Otis Electric Lift with Rosenbusch Controller

The "up" and "down" values of current are in these cases not quite proportional to the relative load, owing probably to the different efficiency of the motor under the two conditions of a receiving and generating machine, and also to the friction load being probably different under the two opposite conditions. The economic importance of generating current, however, is sufficiently evident, and is doubtlessly appreciated in many cases.

Although, when the energy is taken from a supply company or a municipal authority, it is possible there may be difficulties raised on the part of the suppliers, since the current generated would tend to reverse the meter, and thus indirectly to register a credit to the consumer—an obvious source of trouble.

Clearly a series-wound motor is not suitable, as the lift would be constantly running away like a tram-car on a 100-per-cent grade. Recourse is, therefore, had to compound or shunt motors. Compound motors with differential gearing have the disadvantage of giving varying speeds, depending upon the effect the loading has on the series excitation, the extreme case being a very heavy descending load, which produces a demagnetizing effect due to the current generated. On the other hand, the series windings, if cumulative, increase the torque when starting, thus increasing the rate of acceleration and yet restricting the starting current. The series coils are generally cut out when the lift is under way; but with heavy descending loads, and the rheostat partly in circuit so that the motor is not running at the full-line voltage, there is a tendency to an unequal rate of travel of the cage.

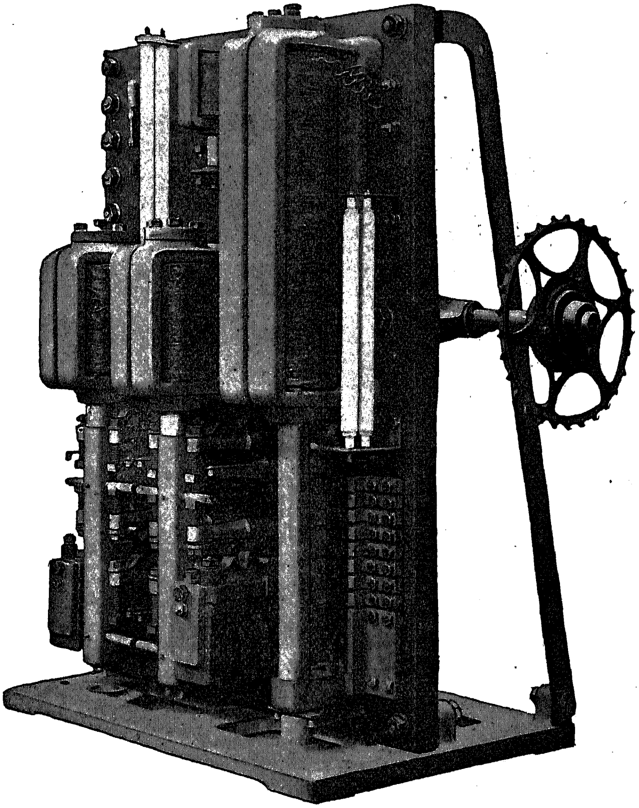


Fig. 227.—Elevation of Rosenbusch Controller for Electric Lifts

Series coils, however, necessitate two windings, with consequently less insulation space, and require an increase in the number of contacts on the controller, the latter being necessary to cut out the coil when full speed is attained.

A simple shunt motor with a strong field, so that the excitation may be varied within wide limits, either automatically in the case of small lifts or by a switch in larger ones, is therefore sometimes preferable.

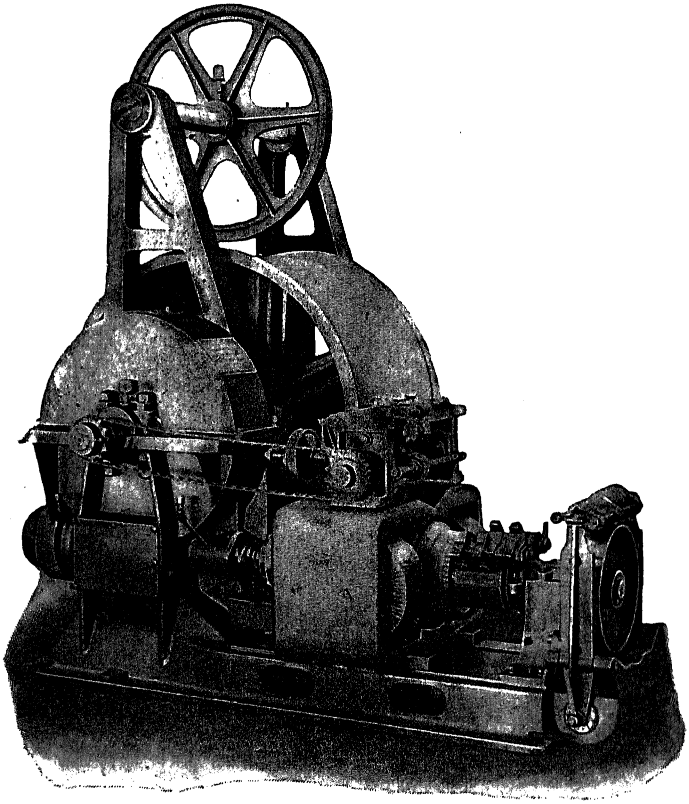
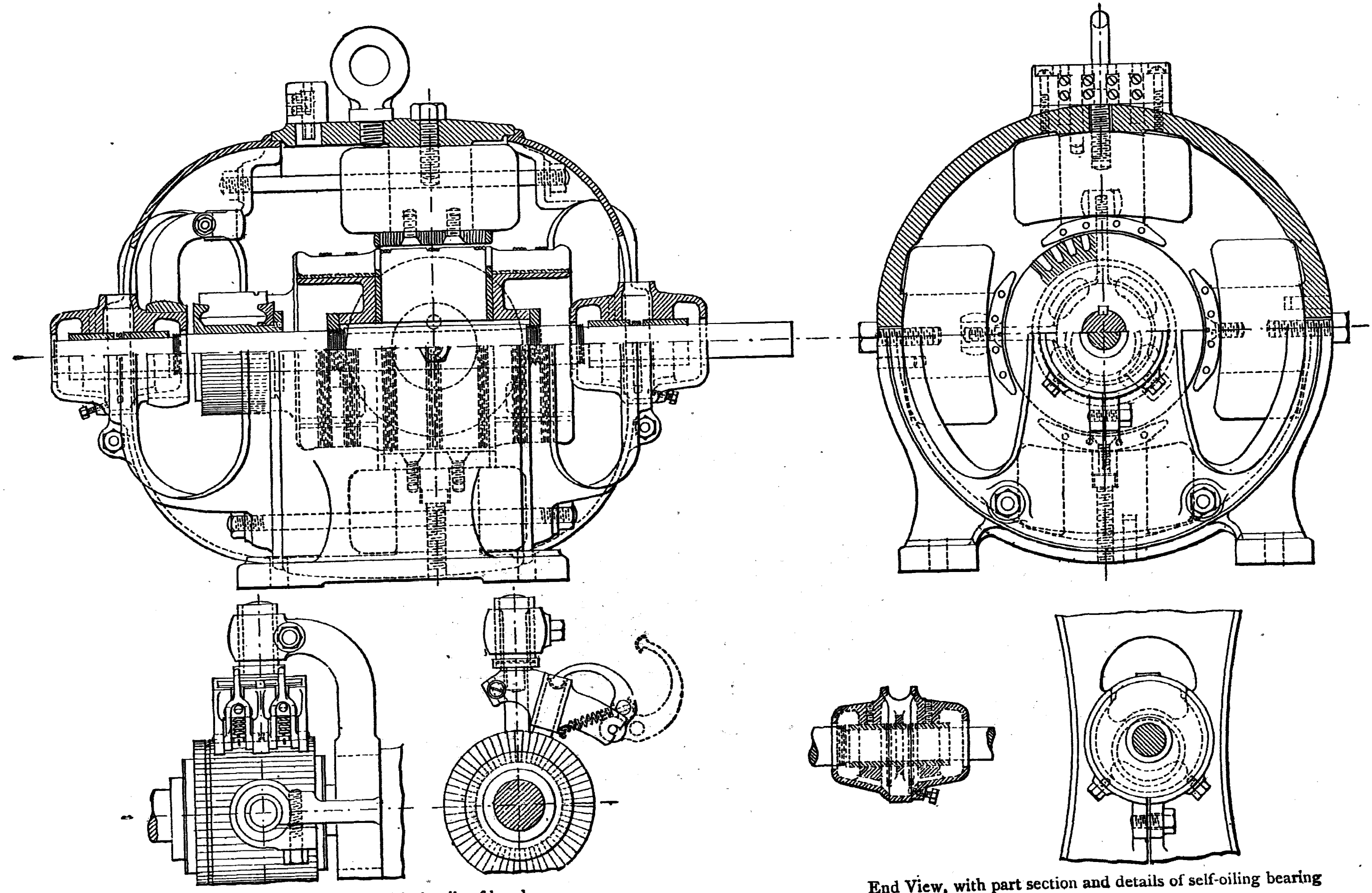


Fig. 228.—An American Type of Electric Lift Equipment with Unenclosed Motor

This gives a strong field, large torque, and rapid acceleration at starting, with high efficiency when under way.

Also a shunt motor designed with a relatively low armature resistance will give almost constant speeds for both "up" and "down", under most conditions of loading, in that the C R drop of the armature is small, and therefore the number of volts lost over the armature resistance is a small percentage to be subtracted from, or added to, the line voltage, the motor running as a motor or a dynamo respectively.

A motor combining the above features, with sundry improvements introduced by Mr. Rosenbusch in connection with the brakes, produce probably the best practice of the present day for lift work. The

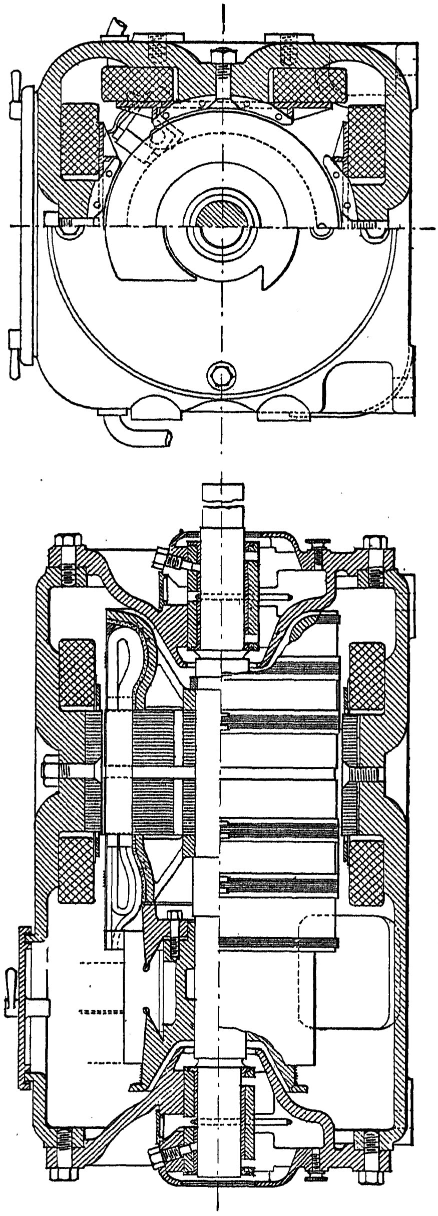


Longitudinal Section, with details of brush-gear

End View, with part section and details of self-oiling bearing

Fig. 229.—Waygood & Otis Motor for Small Slow-Speed Lifts

diagram in fig. 226, together with the illustration of the controller in fig. 227, show the main features.



End View of Section

Longitudinal Section

Fig. 230.—Waygood & Otis Motor for High-Speed Lifts, with Electric Braking

Generally, a motor designed to meet the above requirements is of the four-pole type with laminated pole tips, having a high degree of magnetic saturation so as to keep down magnetic inertia and to allow the field to build up rapidly. It is provided with high-grade carbon brushes in radial holders, a large commutator, and simple windings. Mechanically, it must have the same factors of safety as are provided for traction designs. Its commercial rating is generally stated at a fixed horse-power, but practically it is designed on the lines of a traction-motor or for an assumed load factor or duty-curve rating. It is capable of large momentary sparkless overloads, which are not determined by its heating rating, but are limited simply by the sparking limit, the strong field giving a preponderance of field ampere turns over the armature turns, with resultant large torque and sparklessness when running slowly.

An open motor is most frequently used for lift work in America (see fig. 228, which is an illustration of an electric lift gear built in New York), because open motors are cheaper, more efficient, and more get-at-able than the enclosed type. Further, it is easily coupled to a worm shaft, and a bearing can be saved.

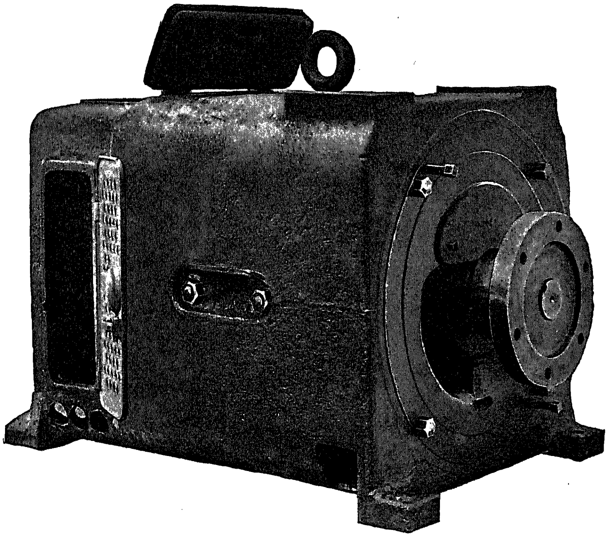


Fig. 231.—Central London Railway Electric Lift Motor, showing Inspection Covers and Coupling

The motors used are generally of the type described by the illustrations in figs. 229 and 230. The former shows the construction of one of the motors designed by Messrs. Waygood & Otis, Limited, for general use with small lifts of slow speeds, the motors running at from 800 to 950 revolutions per minute. The latter shows one of the same firm's designs for lifts of higher speeds, where electric braking is desired. In this type the speed can be varied from 450 to 750 revolutions per minute by field excitation alone.

Figs. 231 and 232 are illustrations of the motors used on the Central London Railway lifts, and figs. 233 and 234 are test curves illustrating their working.

Special interest is attached to these electric lifts on account of their capacity and the continuous work to which they are subjected for hours every night and morning. The figures showing the motors assembled and apart are sufficient evidence of plain solid construction, with an entire absence of useless finish to the external parts. The test curves indicate a well-balanced design with small copper losses in both armature windings and shunt field coils (there are no series windings); while the losses due to air and mechanical friction, to foucault currents and to hysteresis, are approximately equal to the total copper loss when the motor is carrying full load. This is frequently found to be the case when the various elements of the design have the best proportions. It is interesting to note that the friction and iron losses have a maximum value at about one-quarter load, and steadily decrease as the load is increased; whereas the copper losses, of course, increase uniformly with the load.

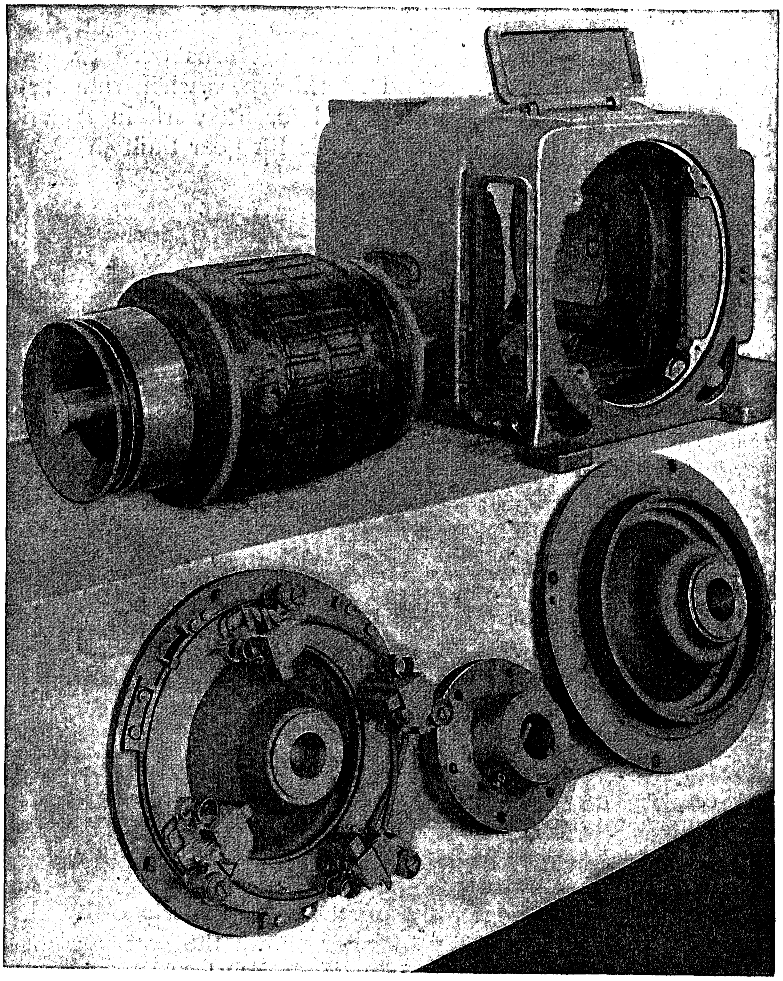


Fig. 232.—Central London Railway Electric Lift Motor taken Apart

The efficiency curve indicates an excellent all-round performance,

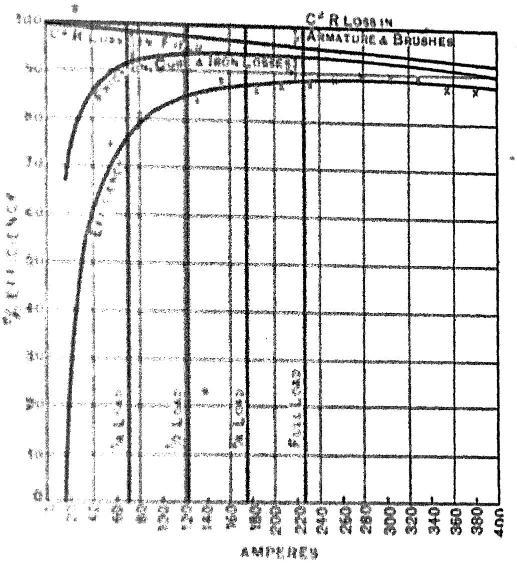


Fig. 233.—Efficiency Curve of Motor illustrated in Figs. 231 and 232. The losses are given by the ordinates between the efficiency curves and the two top curves

varying from 75 per cent efficiency at one-quarter load to 88 per cent at full load, after which the curve is practically flat up to an overload of 100 per cent.

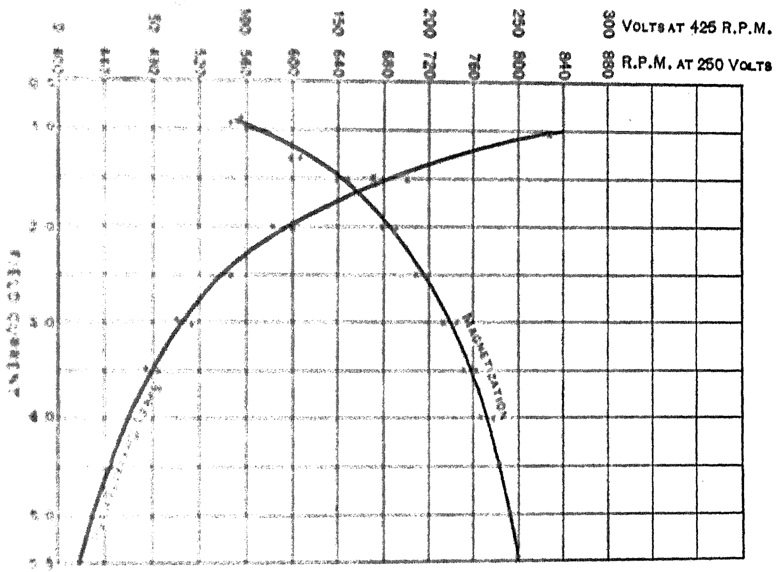


Fig. 234.—Speed Characteristic and Magnetization Curves of Motors illustrated in Figs. 231 and 232

A higher maximum efficiency could no doubt have been obtained at the expense of safety by decreasing the air-gap; but as the curve would have been less flat on the top, the average efficiency within the limits of daily work would certainly not have been greater.

Fig. 234 shows the variation of the magnetization for different values of the shunt current. An increase of from 1 to 2 amperes causes a rise of about 75 per cent in the magnetization, while a further increase to 5 amperes merely raises the curve 30 per cent. The normal working excitation appears to be well over the knee of the curve. These motors seem to be designed for a full load requiring about 225 amperes at 250 volts, or, say, 56 kilowatts.

The regulation of these lifts is effected by a pilot switch in the cage governing a pilot motor, which actuates the large controller, effecting the changes on the large motor connections.

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